Vorticity and its novel effects in heavy-ion collisions

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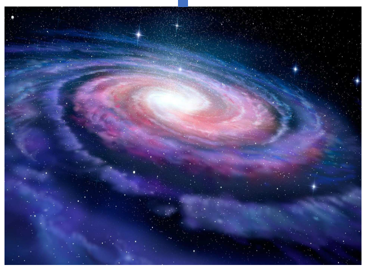
Outline

- Fluid vorticity in heavy-ion collisions
- Spin polarization and subatomic spintronics
- Chiral vortical effect and rotating matter
- Summary

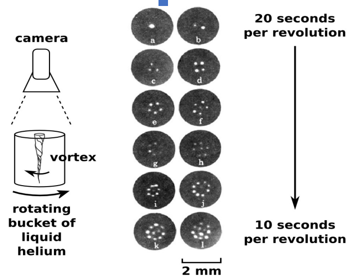
Fluid vorticity

- Vortices: common phenomena in fluids across a very broad hierarchy of scales

$10^{21} m$ | $10^{2} - 10^{-2} m$ | $10^{-5} - 10^{-8} m$ | $10^{-15} m$

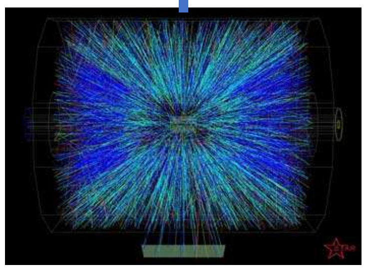


Rotating galaxies



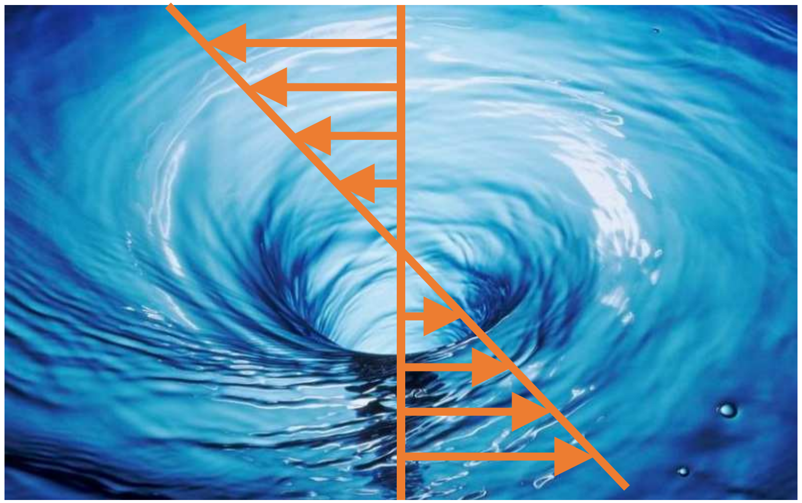
Superfluid helium

Tornados, ocean vortices, …



Quark gluon matter

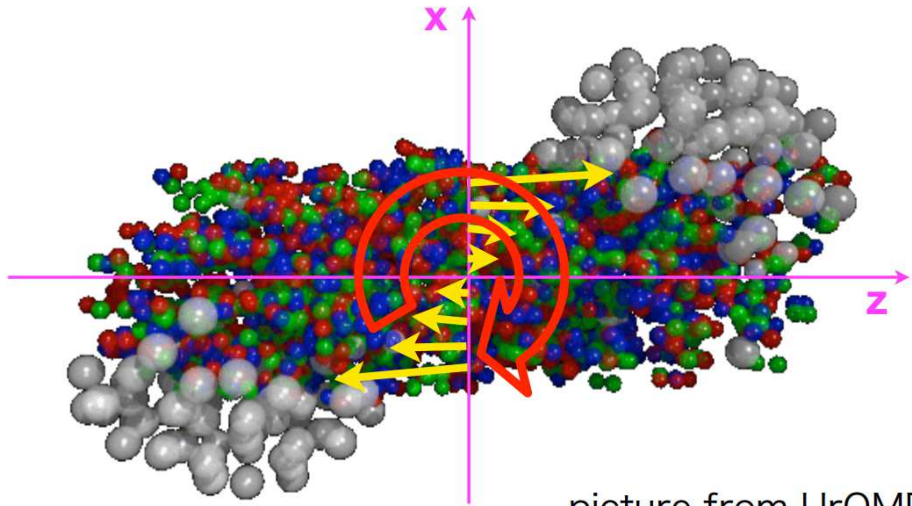
Fluid vorticity



$$\boldsymbol{\omega} = \boldsymbol{\nabla} \times \boldsymbol{v}$$

Local angular velocity

Vorticity in HIC



picture from UrQMD

Global angular momentum

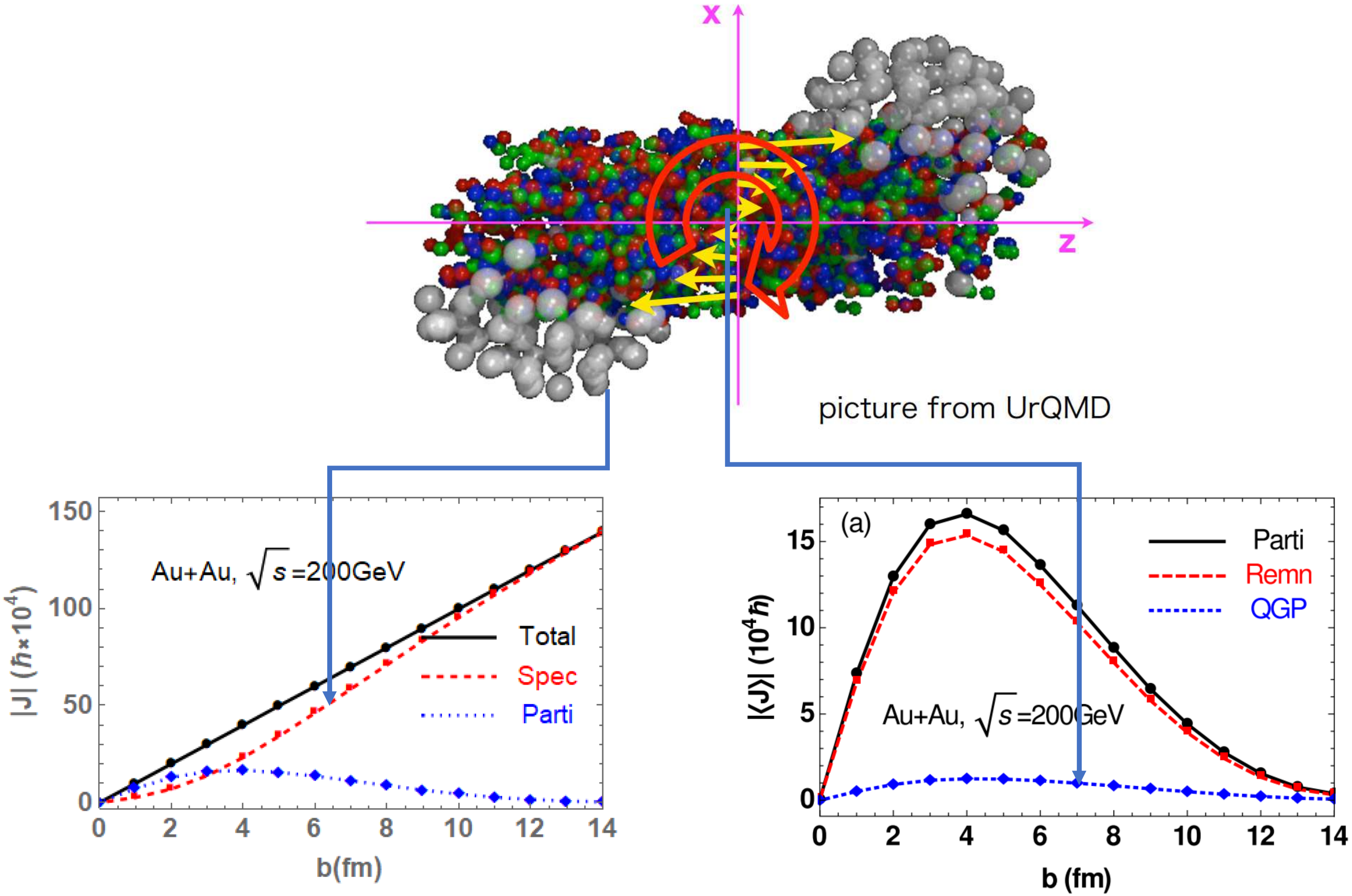
$$J_0 \sim \frac{Ab\sqrt{s}}{2} \sim 10^6 \hbar$$

(RHIC Au+Au 200 GeV, b=10 fm)

Local vorticity

$$\omega \sim ?$$

Vorticity in HIC



picture from UrQMD

Deng-XGH, 2016

Velocity field in parton model

- To calculate the vorticity, we need to know the velocity

Definition of velocity field

$$v_1^a(x) = \frac{1}{\sum_i \Phi(x,x_i)} \sum_i \frac{p_i^a}{p_i^0} \Phi(x,x_i) = \frac{J^a}{J^0}$$ **~ Particle flow velocity**

$$v_2^a(x) = \frac{\sum_i p_i^a \Phi(x,x_i)}{\sum_i [p_i^0 + (p_i^a)^2/p_i^0]\Phi(x,x_i)} = \frac{T^{0a}}{T^{00}+T^{aa}}$$ **~ Energy flow velocity**

Smearing function Phi

$$\Phi_G(x,x_i) = \frac{K}{\tau_0\sqrt{2\pi\sigma_\eta^2 2\pi\sigma_r^2}} \exp\left[-\frac{(x-x_i)^2+(y-y_i)^2}{2\sigma_r^2} - \frac{(\eta-\eta_i)^2}{2\sigma_\eta^2}\right]$$

Parameters are so chosen that with hydro, it is consistent with data (Pang-Wang-Wang 2012)

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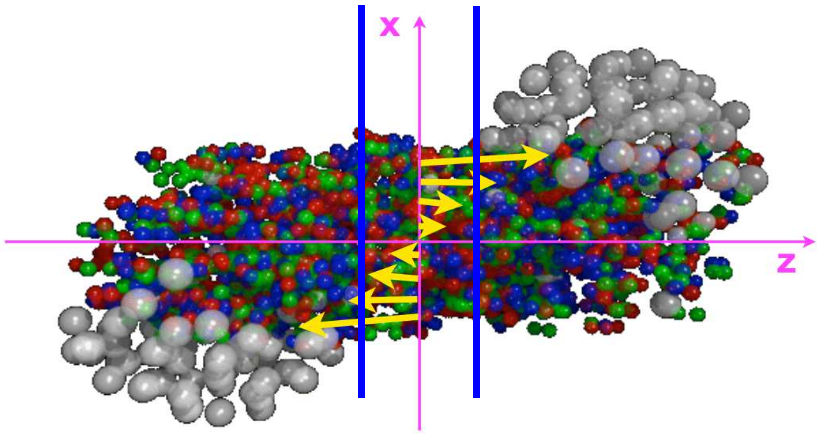
Definition of vorticity field (for each definition of v)

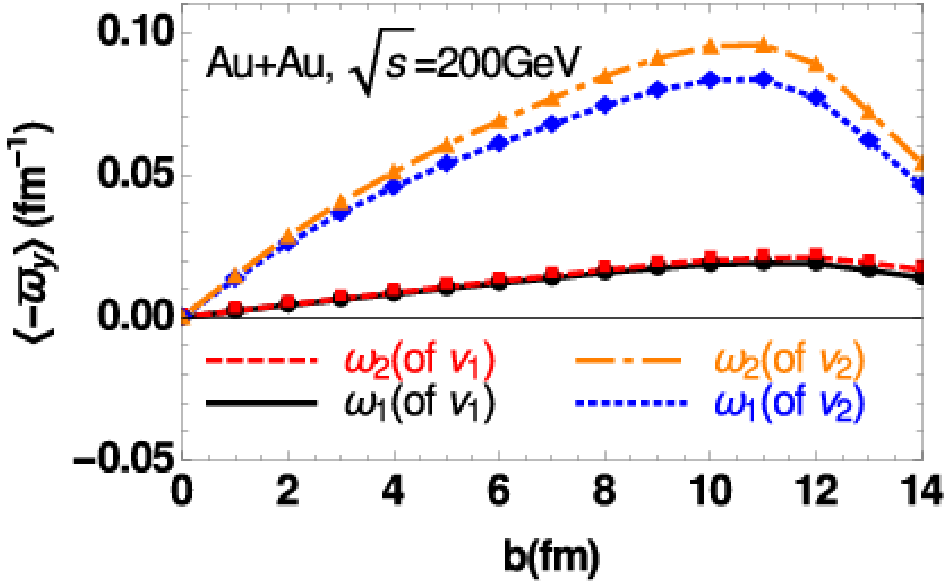
$$\omega_1 = \nabla \times v,$$ **~ nonrelativistic definition**

$$\omega_2^\mu = \epsilon^{\mu\nu\rho\sigma} u_\nu \partial_\rho u_\sigma$$ **~ relativistic definition**

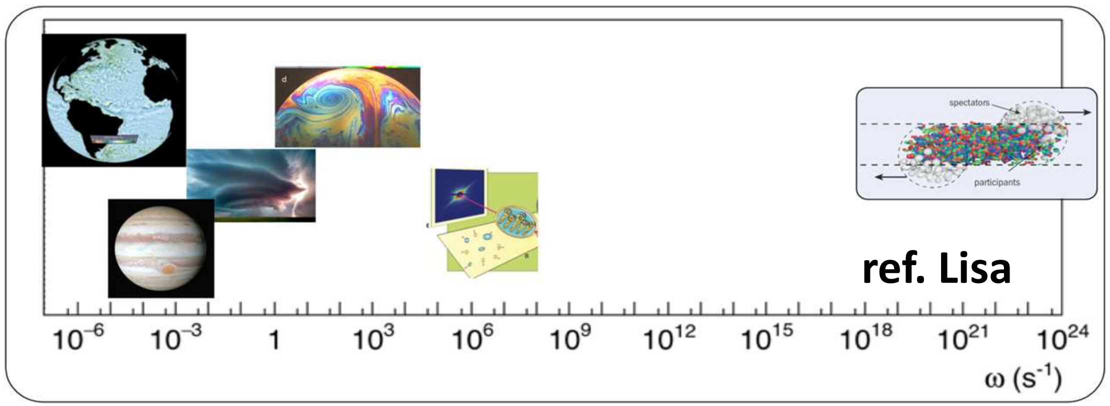
Vorticity in HIC



Mid rapidity



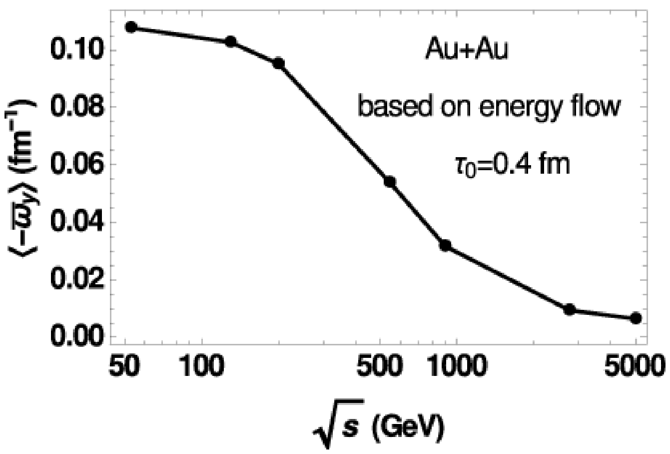
Vorticity in Au+Au@RHIC at $b = 10$ fm is $10^{20} - 10^{21} s^{-1}$

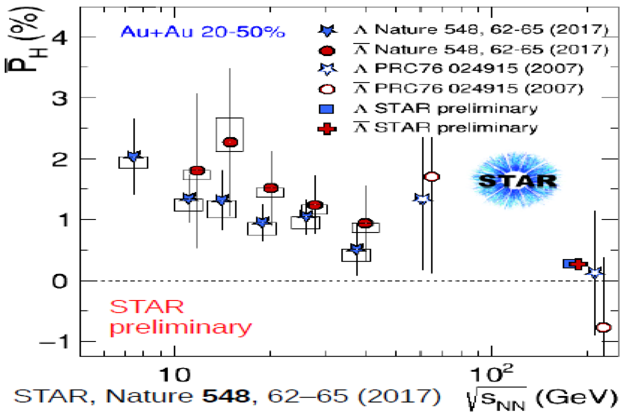


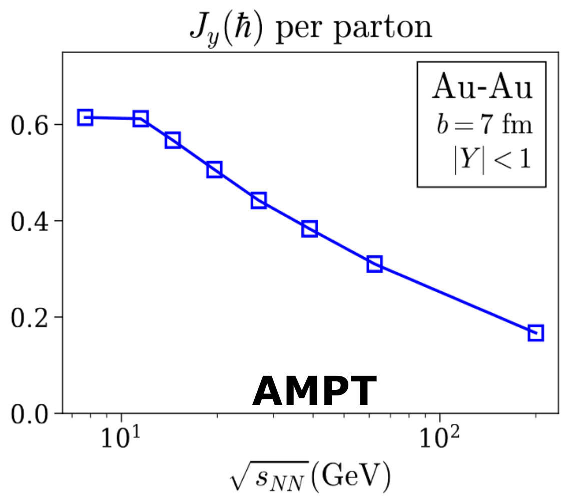
See also: Becattini etal 2015,2016; Jiang-Lin-Liao 2016;Pang-Petersen-Wang-Wang 2016; Xia-Li-Wang 2017,2018; Sun-Ko 2017; … …

Vorticity in HIC

- Collision energy dependence





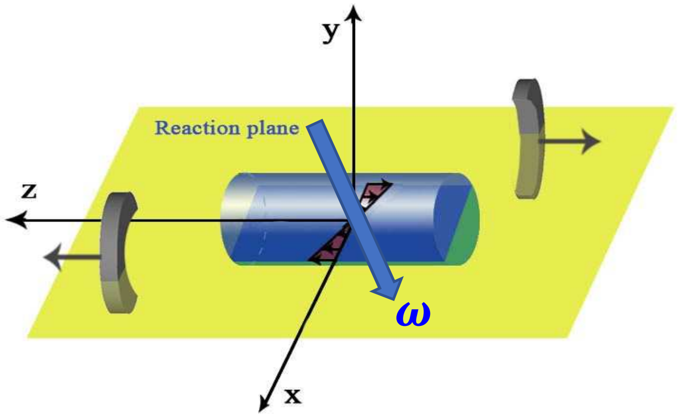


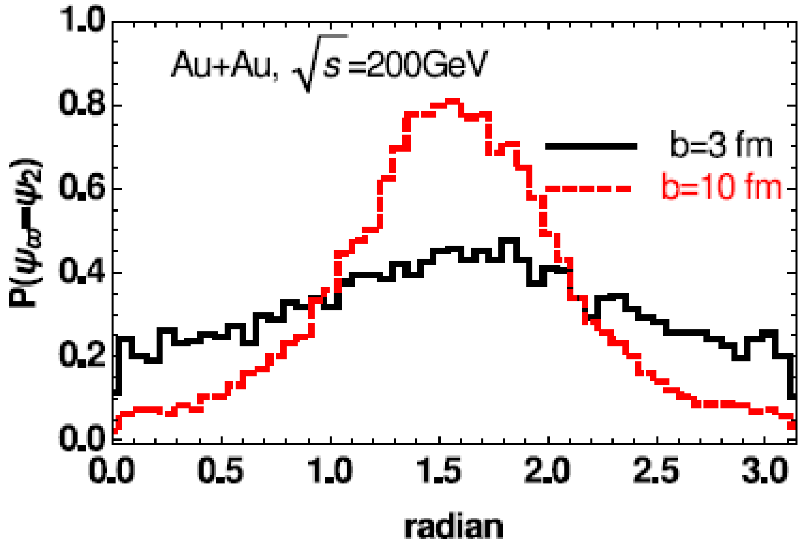
- Consistent with the Lambda polarization result of STAR (see below)
- With increasing energy, more AM carried by high-rapidity particles, midrapidity closer to Bjorken expansion
- Indicates stronger vortical effect at lower energy (beam energy scan, NICA, HIAF)

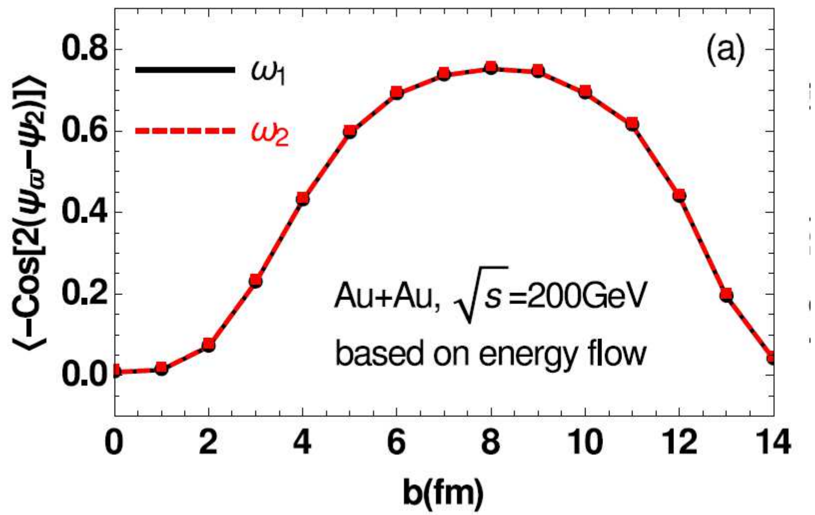
Xia-Li-Wang 2017

Vorticity in HIC

- Event-by-event azimuthal fluctuation



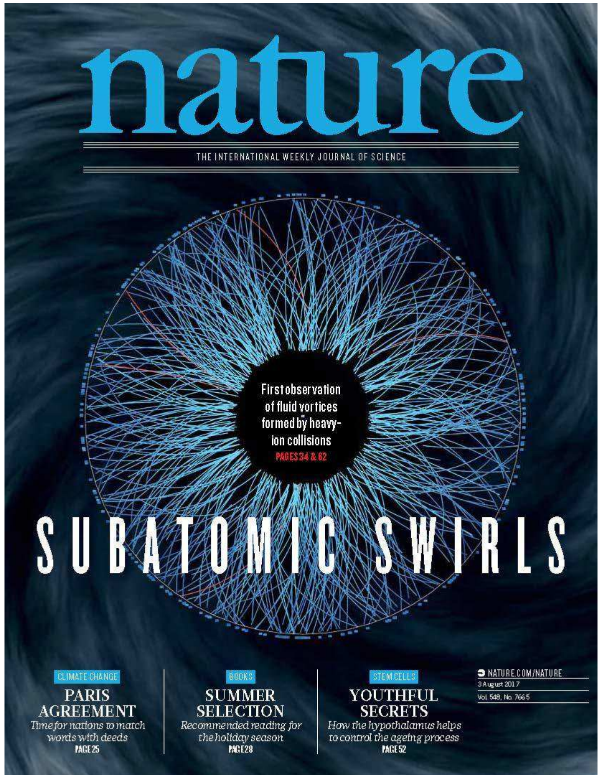


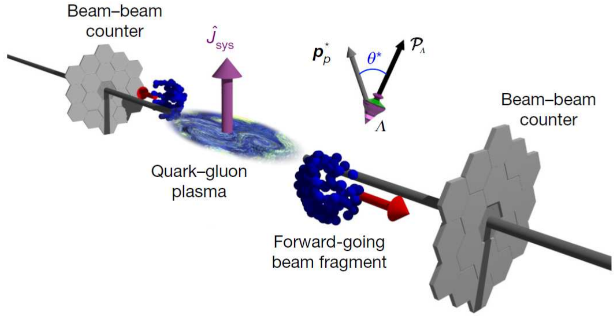


- For small and very large b, fluctuation so strong that correlation with PP is lost
- Moderate b, Gaussian around pi/2
- Suppress the correlation with the matter geometric plane

How to measure the vorticity?

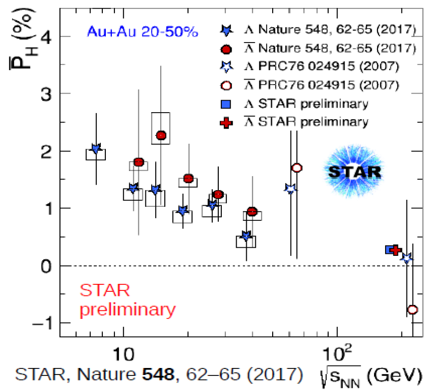
The most vortical fluid!





Liang-Wang 2005

Averaged vorticity from 7.7 GeV-200 GeV: $\omega \approx (9 \pm 1) \times 10^{21} s^{-1}$ "Most vortical fluid!"



LETTER

doi:10.1038/nature23004

Global Λ hyperon polarization in nuclear collisions

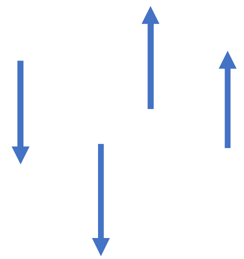
The STAR Collaboration*

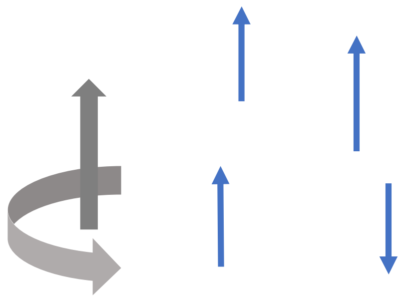
Spin-vorticity coupling

Liang-Wang 2005

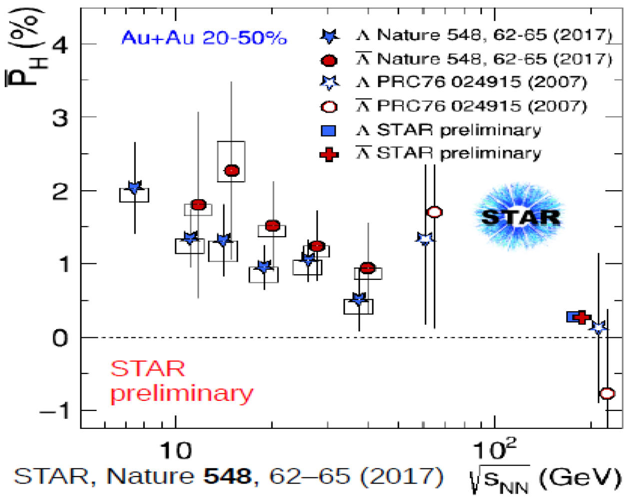
$$H = H_0 - \boldsymbol{\omega} \cdot \boldsymbol{J}$$

$$\frac{dN}{d\boldsymbol{p}} \sim e^{-(H_0 - \boldsymbol{\omega}\cdot\boldsymbol{J})/T}$$





$$\mathrm{P} = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} \sim \frac{\langle\omega\rangle}{T}$$

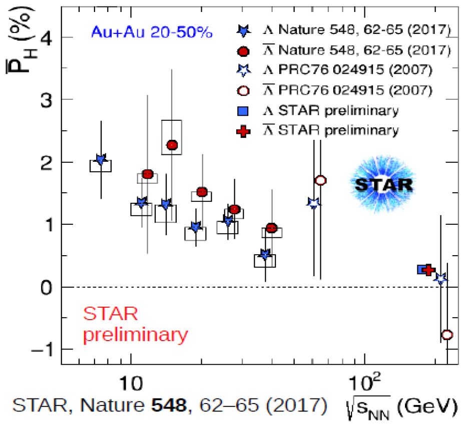


Possible magnetic-field contribution. A way to measure B?

$$H = H_0 - \boldsymbol{\omega} \cdot \boldsymbol{J} - \boldsymbol{m} \cdot \boldsymbol{B}$$

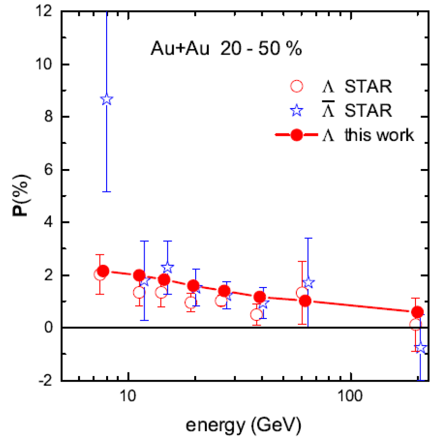
Vortical structure in QGP

- Measured is the space-averaged vorticity near mid rapidity.



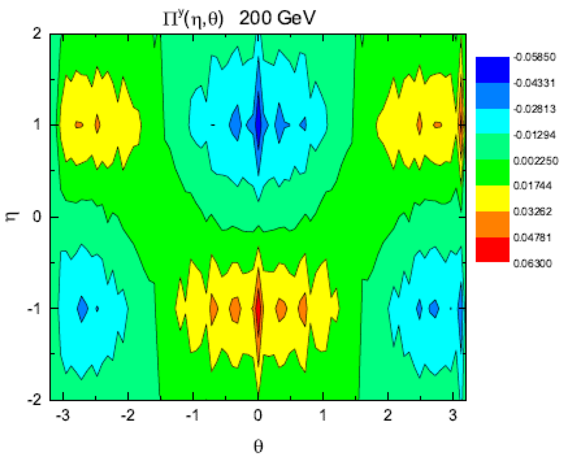
AMPT

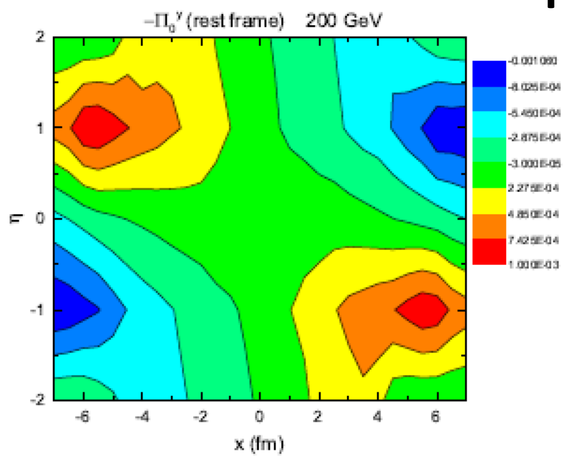
Due to global AM

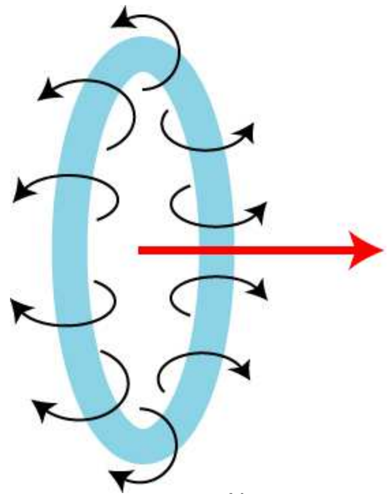


- How is the vortical structure?

Another source for vorticity: collective expansion





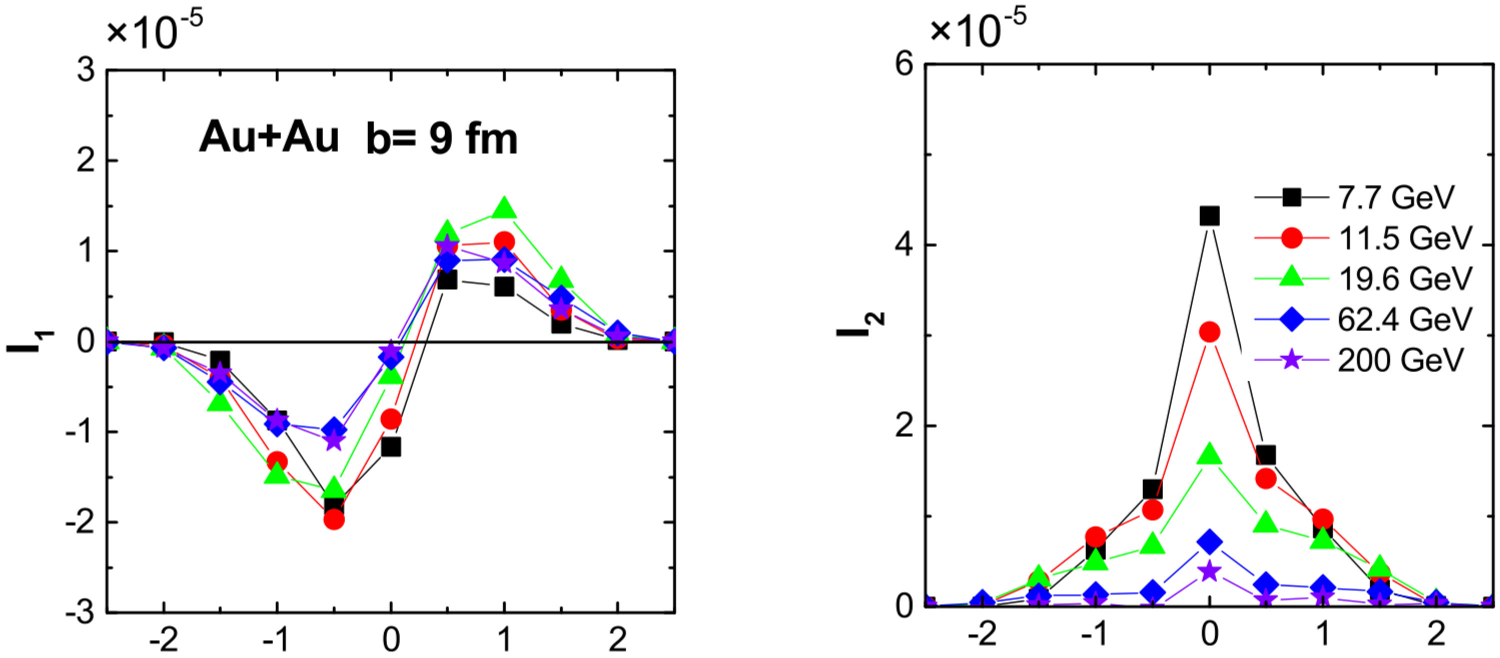


Wei-Deng-XGH 2018

Spin harmonic flow

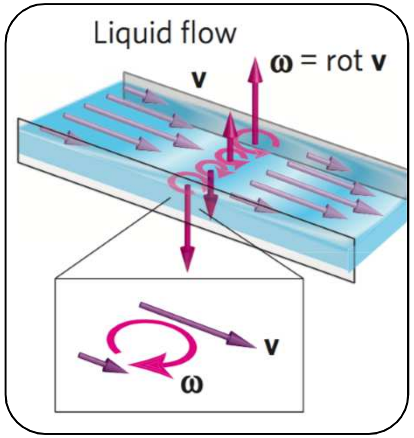
$$I_1(\eta) = \int \frac{d\theta}{2\pi} \langle cos[(\theta - \Phi_1)] P_y(\eta, \theta) \rangle$$

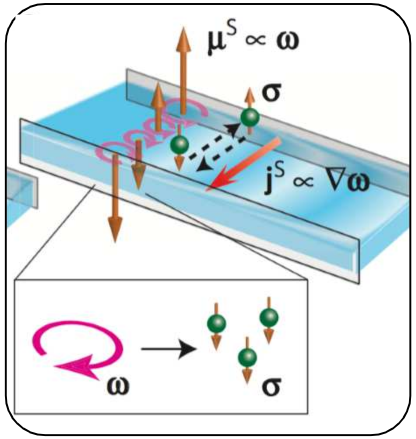
$$I_2(\eta) = \int \frac{d\theta}{2\pi} \langle cos[2(\theta - \Phi_2)] P_y(\eta, \theta) \rangle$$

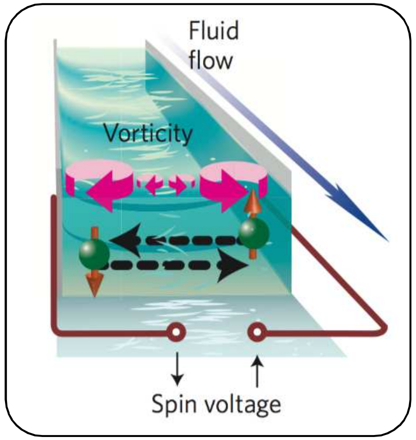


Subatomic spintronics

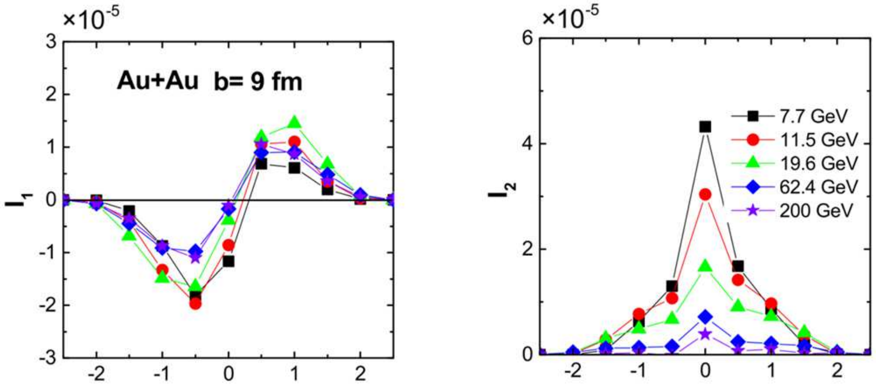
- **Spin hydrodynamic generation in Hg** (Takahashi, et al. Nat. Phys. (2016))



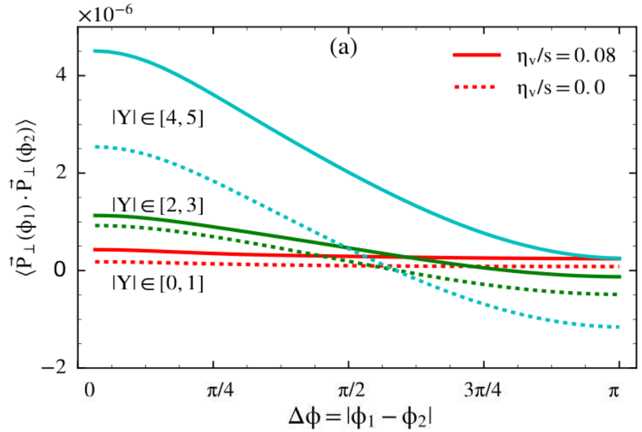




- **Subatomic spintronics in HIC: a new probe for QGP**



Harmonic flow of spin, to apear



Spin-spin correlation, Pang et al 2016

Chiral vortical effect and ground state of rotating dense matter

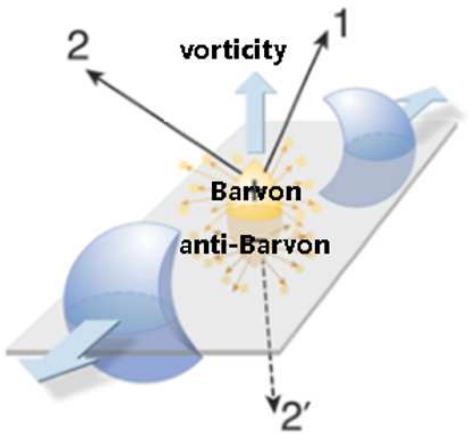
Chiral vortical effect

- Chiral fermions + fluid vorticity ⇒ chiral vortical effect (CVE)

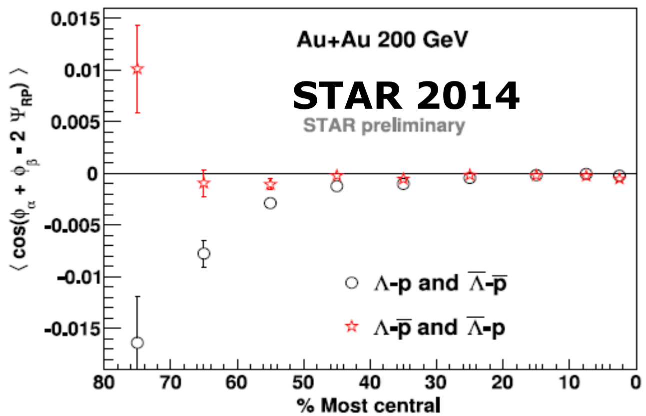
(Erdmenger etal 2008; Barnerjee etal 2008, Son, Surowka 2009; Landsteiner etal 2011):

$$\vec{J}_R = \frac{1}{4\pi^2}\mu_R^2\vec{\omega} + \frac{T^2}{12}\vec{\omega}, \qquad \vec{J}_L = -\frac{1}{4\pi^2}\mu_L^2\vec{\omega} - \frac{T^2}{12}\vec{\omega}$$

Event-by-event baryon separation w.r.t. reaction plane



The vortical gamma correlator



However, there are background effects.

Axial chiral vortical effect

- **Axial current induced by vorticity:**

$$\boldsymbol{j}_a^5 = N_c \left(d_{abc} \frac{\mu_b \mu_c}{2\pi^2} + b_a \frac{T^2}{6} \right) \boldsymbol{\Omega} \,,$$

$$d_{abc} = \frac{1}{2} \mathrm{Tr} \left[\tau_a \{ \tau_b, \tau_c \} \right] , \qquad b_a = \mathrm{Tr}(\tau_a)$$

- **Low-energy effective lagrangian for aCVE (anomaly matching):**

$$\mathcal{L}_{\mathrm{EFT}} = \frac{N_c}{2 f_\pi} \left(\frac{d_{abc}}{2\pi^2} \mu_b \mu_c + \frac{b_a}{6} T^2 \right) \boldsymbol{\nabla} \pi_a \cdot \boldsymbol{\Omega}$$

Chiral anomaly

Chiral-grav. mixed anomaly?

- **Look for consequences of a CVE for low T dense quark matter under rotation**

A chiral soliton lattice (I)

- The Hamiltonian for the neutral pion ($\phi \equiv \pi^0/f_\pi$)

$$\mathcal{H} = \frac{f_\pi^2}{2}\left[(\partial_r\phi)^2 + \frac{1-(\Omega r)^2}{r^2}(\partial_\theta\phi)^2 + (\partial_z\phi)^2\right] + m_\pi^2 f_\pi^2(1-\cos\phi) - \frac{\mu_{\rm B}\mu_{\rm I}}{2\pi^2}\Omega\partial_z\phi$$

- The ground state is given by

$$\langle\partial_r\pi_0\rangle = \langle\partial_\theta\pi_0\rangle = 0 \qquad \partial_z^2\phi = m_\pi^2\sin\phi$$

- Its solution is given by zero or the Jacobi elliptic function

$$\cos\frac{\phi(\bar{z})}{2} = {\rm sn}(\bar{z},k) \quad \text{with} \quad \bar{z} \equiv z m_\pi/k$$

with period

$$\ell = \frac{2kK(k)}{m_\pi}$$

with $K(k)$ the 1st complete elliptic integral

A chiral soliton lattice (II)

- This is a one dimensional chiral soliton lattice. It is the ground state when

$$|\Omega| \geq \Omega_{\mathrm{CSL}} \equiv \frac{8\pi m_\pi f_\pi^2}{\mu_{\mathrm{B}}|\mu_{\mathrm{I}}|}$$

- Each lattice cell carries topological charges

$$\frac{J_z}{A} = \frac{\mu_{\mathrm{B}}\mu_{\mathrm{I}}}{\pi}, \quad \frac{N_{\mathrm{B}}}{A} = \frac{\mu_{\mathrm{I}}\Omega}{\pi}, \quad \frac{N_{\mathrm{I}}}{A} = \frac{\mu_{\mathrm{B}}\Omega}{\pi}$$

  with crossed correlation between baryon and isospin

- The energy density

$$\frac{\mathcal{E}_{\mathrm{tot}}}{V} = 2m_\pi^2 f_\pi^2 \left(1 - \frac{1}{k^2}\right) < 0$$

Topological Barnett-Einstein-de Haas effect

- Introducing also the magnetic field (chiral limit)

$$\mathcal{H} = \frac{1}{2}(\boldsymbol{\nabla}\pi_0)^2 - \frac{\mu_{\rm B}}{4\pi^2 f_\pi}\boldsymbol{\nabla}\pi_0 \cdot (2\mu_{\rm I}\boldsymbol{\Omega} + \boldsymbol{B})$$

- Integrating out pion

$$\mathcal{H}_{\rm mix} = -\frac{\mu_{\rm B}^2 \mu_{\rm I}}{8\pi^4 f_\pi^2}\boldsymbol{\Omega}\cdot\boldsymbol{B}$$

- This induces cross-correlated response between rotation and magnetic field

$\boldsymbol{j} = \chi_{jB}\boldsymbol{B}$ ~ Einstein-de Haas effect

$\boldsymbol{m} = \chi_{m\Omega}\boldsymbol{\Omega}$ ~ Barnett effect

with $\chi_{jB} = \chi_{m\Omega} = \dfrac{\mu_{\rm B}^2 \mu_{\rm I}}{8\pi^4 f_\pi^2}$

Summary

- **Heavy-ion collisions generate subatomic swirls which show the fastest vortices**
- **High statistics spin polarization measurement provide a way to detect the vortical structure in QGP**
- **Chiral vorticial effect leads to modification to QCD ground state at high isospin and baryon density**

Thank you!

Outlook

- **Magnetic field: intensive study since 2008, boosted by the proposal and experimental search of chiral magnetic effect**
- **Vorticity and rotation: intensive study since 2014-2016, boosted by the experimental study of the CVE and especially the spin polarization of Lambda**
- **A new axis representing the QCD phase diagram, a lot of new problems should be attacked.**

Thank you!