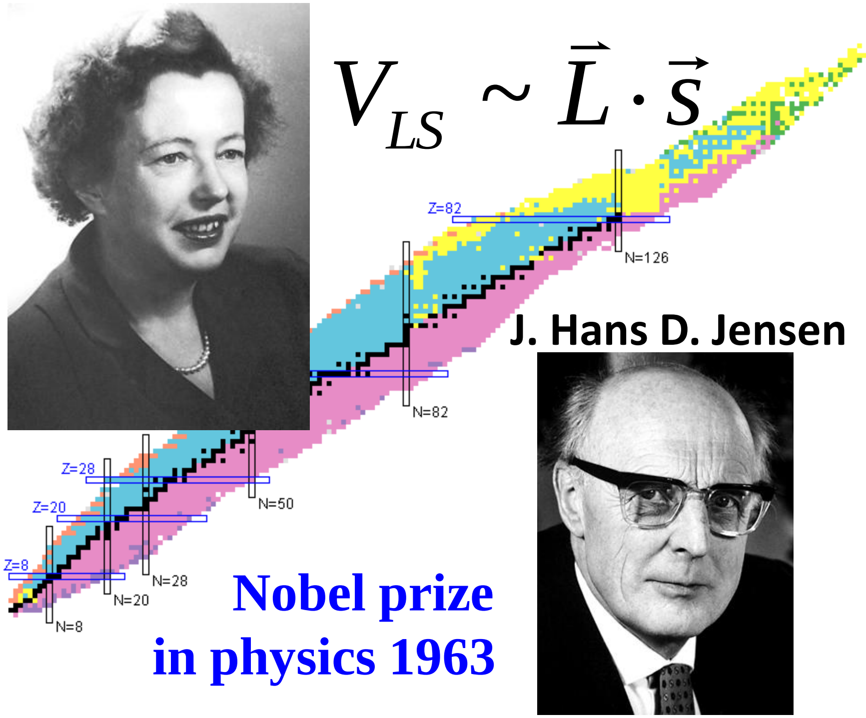
Spin dynamics in heavy-ion collisions

Jun Xu (徐骏)
Shanghai Institute of Applied Physics,
Chinese Academy of Sciences

Symposium on Intermediate-energy heavy-ion collisions (iHIC2018)
Tsinghua University, Beijing, Apr. 8th – 10th , 2018

Spin-orbit coupling and spin dynamics

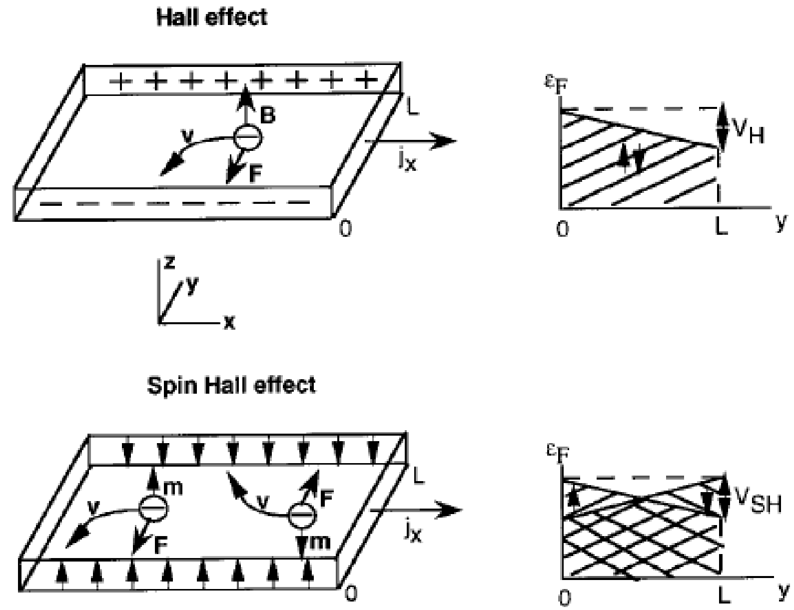
Maria Goeppert Mayer



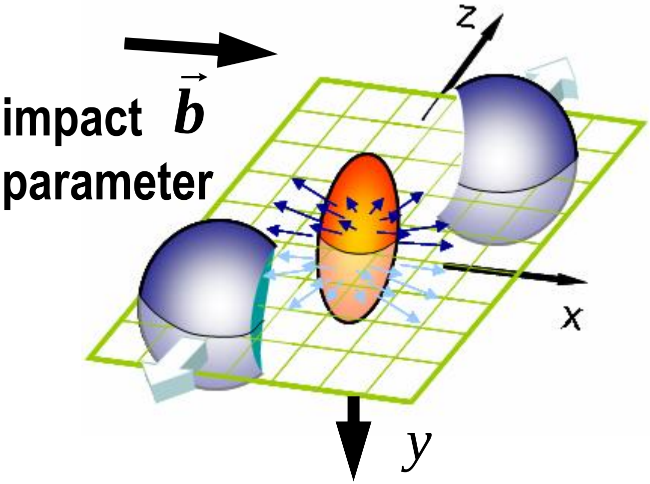
$$V_{LS} \sim \vec{L} \cdot \vec{s}$$

J. Hans D. Jensen

Nobel prize
in physics 1963



Spin hall effect
in heavy-ion collisions



1） orbit angular momentum or vorticity $\vec{L} \cdot \vec{s}$

2） magnetic field $\vec{\mu} \cdot \vec{B}$

Perpendicular to the reaction plane

Different types of spin-orbit couplings

$$H^{SO} = A(\vec{p})\sigma_x - B(\vec{p})\sigma_y + C(\vec{p})\sigma_z = \vec{b}\cdot\vec{\sigma}$$

2D system	A(p)	B(p)	C(p)
Rashba	$\beta_R p_y$	$\beta_R p_x$	
Dresselhaus [001]	$\beta_D p_x$	$\beta_D p_y$	
Dresselhaus [110]	βp_x	$-\beta p_x$	
Rashba - Dresselhaus	$\beta_R p_y - \beta_D p_x$	$\beta_R p_x - \beta_D p_y$	
Cubic Rashba (hole)	$i\frac{\beta_R}{2}(p_-^3 - p_+^3)$	$\frac{\beta_R}{2}(p_-^3 + p_+^3)$	
Cubic Dresselhaus	$\beta_D p_x p_y^2$	$\beta_D p_y p_x^2$	
Wurtzite type	$(\alpha + \beta p^2)p_y$	$(\alpha + \beta p^2)p_x$	
Single-layer graphene	$v p_x$	$-v p_y$	
Bilayer graphene	$\frac{p_-^2 + p_+^2}{4m_e}$	$\frac{p_-^2 - p_+^2}{4m_e i}$	
3D system	A(p)	B(p)	C(p)
Bulk Dresselhaus	$p_x(p_y^2 - p_z^2)$	$p_y(p_x^2 - p_z^2)$	$p_z(p_x^2 - p_y^2)$
Cooper pairs	Δ	0	$\frac{p^2}{2m} - \epsilon_F$
Extrinsic			
$\beta = \frac{i}{\hbar}\lambda^2 V(p)$	$q_y p_z - q_z p_y$	$q_z p_x - q_x p_z$	$q_x p_y - q_y p_x$
Neutrons in nuclei			
$\beta = i W_0(n_n + \frac{n_p}{2})$	$q_z p_y - q_y p_z$	$q_x p_z - q_z p_x$	$q_y p_x - q_x p_y$

K. Morawetz, Phys. Rev. B, 2015 $\vec{\sigma}\cdot(\nabla\rho\times\vec{p})$

For spin ½ particles

Single-particle energy:

$$\hat{\varepsilon}(\vec{r},\vec{p}) = \varepsilon(\vec{r},\vec{p})\hat{I} + \vec{h}(\vec{r},\vec{p})\cdot\vec{\sigma}$$

Spin-independent   Spin-dependent

Distribution function:

$$\hat{f}(\vec{r},\vec{p}) = f_0(\vec{r},\vec{p})\hat{I} + \vec{g}(\vec{r},\vec{p})\cdot\vec{\sigma}$$

Spin-averaged f:

$$f(\vec{r}\vec{p},t,0) = f_{1,1}(\vec{r}\vec{p},t) + f_{-1,-1}(\vec{r}\vec{p},t)$$

Spin-polarized f:

$$\tau(\vec{r}\vec{p},t,z) = f_{1,1}(\vec{r}\vec{p},t) - f_{-1,-1}(\vec{r}\vec{p},t)$$

Spin polarization in x axis:

$$\tau(\vec{r}\vec{p},t,x) = f_{-1,1}(\vec{r}\vec{p},t) + f_{1,-1}(\vec{r}\vec{p},t)$$

Spin polarization in y axis:

$$\tau(\vec{r}\vec{p},t,y) = -i[f_{-1,1}(\vec{r}\vec{p},t) - f_{1,-1}(\vec{r}\vec{p},t)]$$

Single-particle Hamiltonian from Skyrme interaction

Skyrme spin-orbit interaction:

$$V_{so} = iW_0(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2)\vec{k}'$$

Single-particle Hamiltonian:

$$h_q^{so}(\vec{r}, \vec{p}) = h_1 + h_4 + (\vec{h}_2 + \vec{h}_3) \cdot \vec{\sigma}.$$

Y.M. Engel et al., NPA (1975)

$$h_1 = -\frac{W_0}{2}\nabla_{\vec{r}} \cdot [\vec{J}(\vec{r}) + \vec{J}_q(\vec{r})]$$

Spin-orbit density

time-even

$$\vec{J}(\vec{r}) = \int d^3p \frac{\vec{p}}{\hbar} \times \vec{\tau}(\vec{r}, \vec{p}).$$

$$\vec{h}_2 = -\frac{W_0}{2}\nabla_{\vec{r}} \times [\vec{j}(\vec{r}) + \vec{j}_q(\vec{r})]$$

current density

time-odd

$$\vec{j}(\vec{r}) = \int d^3p \frac{\vec{p}}{\hbar} f(\vec{r}, \vec{p}),$$

$$\vec{h}_3 = \frac{W_0}{2}\nabla_{\vec{r}}[\rho(\vec{r}) + \rho_q(\vec{r})] \times \vec{p}$$

number density

time-even

$$\rho(\vec{r}) = \int d^3p f(\vec{r}, \vec{p}),$$

$$h_4 = -\frac{W_0}{2}\nabla_{\vec{r}} \times [\vec{s}(\vec{r}) + \vec{s}_q(\vec{r})] \cdot \vec{p}$$

Spin density

time-odd

$$\vec{s}(\vec{r}) = \int d^3p \vec{\tau}(\vec{r}, \vec{p}),$$

$f(\vec{r}, \vec{p})$ **and** $\tau(\vec{r}, \vec{p})$ **are calculated from the test-particle method.**

From spin-dependent TDHF to spin-dependent Boltzmann equation

TDHF => Liouville equation

$$i\hbar\langle\mathbf{r}|\dot{\hat\rho}|\mathbf{r}''\rangle^{\uparrow\uparrow} = \int d^3r' \left(\langle\mathbf{r}|\hat h|\mathbf{r}'\rangle^{\uparrow\uparrow}\langle\mathbf{r}'|\hat\rho|\mathbf{r}''\rangle^{\uparrow\uparrow} - \langle\mathbf{r}|\hat\rho|\mathbf{r}'\rangle^{\uparrow\uparrow}\langle\mathbf{r}'|\hat h|\mathbf{r}''\rangle^{\uparrow\uparrow} + \langle\mathbf{r}|\hat h|\mathbf{r}'\rangle^{\uparrow\downarrow}\langle\mathbf{r}'|\hat\rho|\mathbf{r}''\rangle^{\downarrow\uparrow} - \langle\mathbf{r}|\hat\rho|\mathbf{r}'\rangle^{\uparrow\downarrow}\langle\mathbf{r}'|\hat h|\mathbf{r}''\rangle^{\downarrow\uparrow}\right)$$

$$i\hbar\langle\mathbf{r}|\dot{\hat\rho}|\mathbf{r}''\rangle^{\uparrow\downarrow} = \int d^3r' \left(\langle\mathbf{r}|\hat h|\mathbf{r}'\rangle^{\uparrow\uparrow}\langle\mathbf{r}'|\hat\rho|\mathbf{r}''\rangle^{\uparrow\downarrow} - \langle\mathbf{r}|\hat\rho|\mathbf{r}'\rangle^{\uparrow\uparrow}\langle\mathbf{r}'|\hat h|\mathbf{r}''\rangle^{\uparrow\downarrow} + \langle\mathbf{r}|\hat h|\mathbf{r}'\rangle^{\uparrow\downarrow}\langle\mathbf{r}'|\hat\rho|\mathbf{r}''\rangle^{\downarrow\downarrow} - \langle\mathbf{r}|\hat\rho|\mathbf{r}'\rangle^{\uparrow\downarrow}\langle\mathbf{r}'|\hat h|\mathbf{r}''\rangle^{\downarrow\downarrow}\right)$$

$$i\hbar\langle\mathbf{r}|\dot{\hat\rho}|\mathbf{r}''\rangle^{\downarrow\uparrow} = \int d^3r' \left(\langle\mathbf{r}|\hat h|\mathbf{r}'\rangle^{\downarrow\uparrow}\langle\mathbf{r}'|\hat\rho|\mathbf{r}''\rangle^{\uparrow\uparrow} - \langle\mathbf{r}|\hat\rho|\mathbf{r}'\rangle^{\downarrow\uparrow}\langle\mathbf{r}'|\hat h|\mathbf{r}''\rangle^{\uparrow\uparrow} + \langle\mathbf{r}|\hat h|\mathbf{r}'\rangle^{\downarrow\downarrow}\langle\mathbf{r}'|\hat\rho|\mathbf{r}''\rangle^{\downarrow\uparrow} - \langle\mathbf{r}|\hat\rho|\mathbf{r}'\rangle^{\downarrow\downarrow}\langle\mathbf{r}'|\hat h|\mathbf{r}''\rangle^{\downarrow\uparrow}\right)$$

$$i\hbar\langle\mathbf{r}|\dot{\hat\rho}|\mathbf{r}''\rangle^{\downarrow\downarrow} = \int d^3r' \left(\langle\mathbf{r}|\hat h|\mathbf{r}'\rangle^{\downarrow\uparrow}\langle\mathbf{r}'|\hat\rho|\mathbf{r}''\rangle^{\uparrow\downarrow} - \langle\mathbf{r}|\hat\rho|\mathbf{r}'\rangle^{\downarrow\uparrow}\langle\mathbf{r}'|\hat h|\mathbf{r}''\rangle^{\uparrow\downarrow} + \langle\mathbf{r}|\hat h|\mathbf{r}'\rangle^{\downarrow\downarrow}\langle\mathbf{r}'|\hat\rho|\mathbf{r}''\rangle^{\downarrow\downarrow} - \langle\mathbf{r}|\hat\rho|\mathbf{r}'\rangle^{\downarrow\downarrow}\langle\mathbf{r}'|\hat h|\mathbf{r}''\rangle^{\downarrow\downarrow}\right)$$

Wigner transformation on both sides

$$i\hbar\dot f^{\uparrow\uparrow} = i\hbar\{h^{\uparrow\uparrow}, f^{\uparrow\uparrow}\} + h^{\uparrow\downarrow}f^{\downarrow\uparrow} - f^{\uparrow\downarrow}h^{\downarrow\uparrow} + \frac{i\hbar}{2}\{h^{\uparrow\downarrow}, f^{\downarrow\uparrow}\} - \frac{i\hbar}{2}\{f^{\uparrow\downarrow}, h^{\downarrow\uparrow}\} - \frac{\hbar^2}{8}\{\{h^{\uparrow\downarrow}, f^{\downarrow\uparrow}\}\} + \frac{\hbar^2}{8}\{\{f^{\uparrow\downarrow}, h^{\downarrow\uparrow}\}\} + \cdots$$

$$i\hbar\dot f^{\uparrow\downarrow} = f^{\uparrow\downarrow}(h^{\uparrow\uparrow} - h^{\downarrow\downarrow}) + \frac{i\hbar}{2}\{(h^{\uparrow\uparrow} + h^{\downarrow\downarrow}), f^{\uparrow\downarrow}\} - \frac{\hbar^2}{8}\{\{(h^{\uparrow\uparrow} - h^{\downarrow\downarrow}), f^{\uparrow\downarrow}\}\} - h^{\uparrow\downarrow}(f^{\uparrow\uparrow} - f^{\downarrow\downarrow})$$
$$+ \frac{i\hbar}{2}\{h^{\uparrow\downarrow}, (f^{\uparrow\uparrow} + f^{\downarrow\downarrow})\} + \frac{\hbar^2}{8}\{\{h^{\uparrow\downarrow}, (f^{\uparrow\uparrow} - f^{\downarrow\downarrow})\}\} + \cdots,$$

$\uparrow\leftrightarrow\downarrow$ **for other two equations**

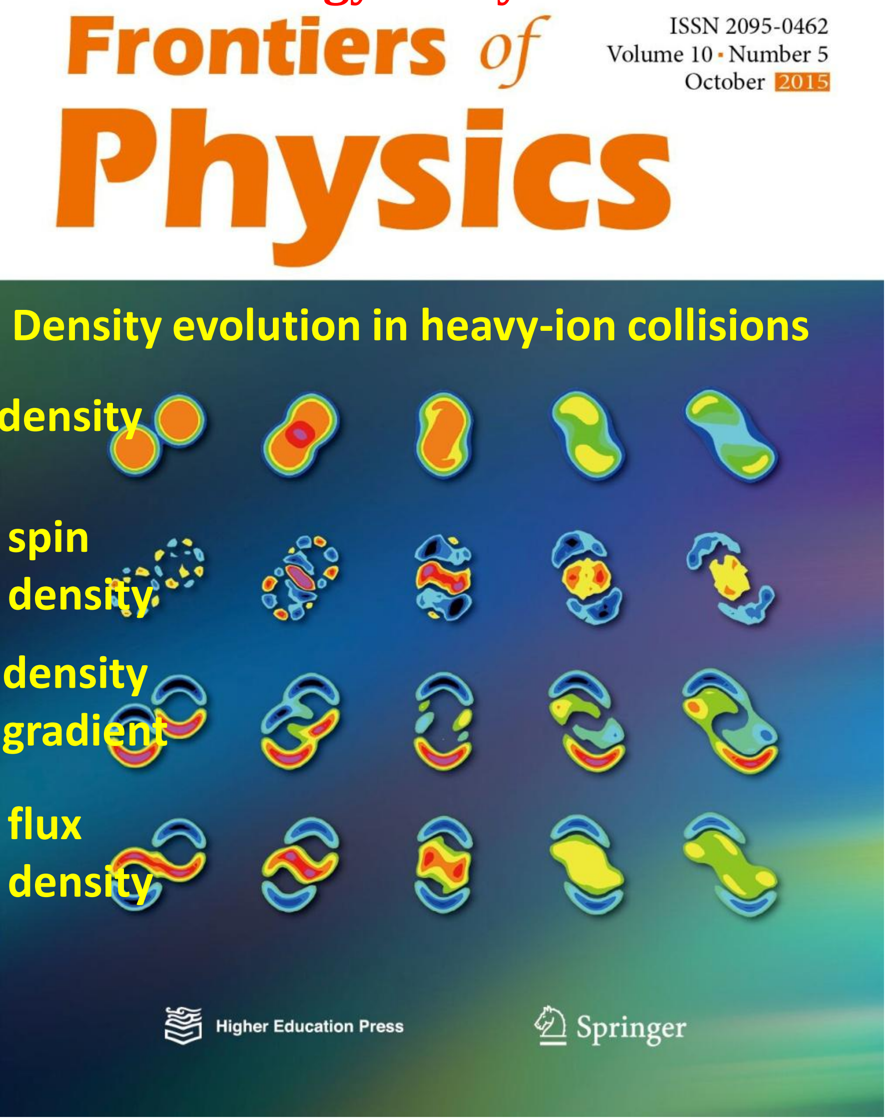
E.B. Balbutsev, I.V. Molodtsova, and P. Schuck, Nucl. Phys. A (2011); Phys. Rev. C (2013)

cut higher-order terms → spin-dependent Boltzmann equation

$$\frac{\partial \hat f}{\partial t} + \frac{i}{\hbar}\left[\hat\varepsilon, \hat f\right] + \frac{1}{2}\left(\frac{\partial\hat\varepsilon}{\partial\vec p}\cdot\frac{\partial\hat f}{\partial\vec r} + \frac{\partial\hat f}{\partial\vec r}\cdot\frac{\partial\hat\varepsilon}{\partial\vec p}\right) - \frac{1}{2}\left(\frac{\partial\hat\varepsilon}{\partial\vec r}\cdot\frac{\partial\hat f}{\partial\vec p} + \frac{\partial\hat f}{\partial\vec p}\cdot\frac{\partial\hat\varepsilon}{\partial\vec r}\right) = I_c$$

H. Smith and H.H. Jensen, transport phenomena (Oxford University press, Oxford, 1989)

Spin dynamics in intermediate- and low-energy heavy-ion collisions



invited review, selected as cover story

JX, B.A. Li, W.Q. Shen, and Y. Xia, Front. Phys. (2015)

SIBUU12

Boltzmann-Uehling-Uhlenbeck equation

$$\frac{\partial f}{\partial t} + v\cdot\nabla_r f - \nabla U\cdot\nabla_p f = -\int \frac{d^3p_2\, d^3p_1'\, d^3p_2'}{(2\pi)^9}\sigma v_{12}[f f_2(1-f_{1'})(1-f_{2'}) - f_{1'}f_{2'}(1-f)(1-f_2)](2\pi)^3\delta^3(p+p_2-p_1'-p_2')$$

test-particle method

C.Y. Wong, PRC (1982)

equations of motion

$$\frac{d\vec r}{dt} = \frac{\vec p}{m} \qquad \frac{d\vec p}{dt} = -\nabla U$$

Spin-dependent Boltzmann-Uehling-Uhlenbeck eq.

2×2 **matrix eq.**

$$\frac{\partial \hat f}{\partial t} + \frac{i}{\hbar}\left[\hat\varepsilon,\hat f\right] + \frac12\left(\frac{\partial\hat\varepsilon}{\partial\vec p}\cdot\frac{\partial\hat f}{\partial\vec r}+\frac{\partial\hat f}{\partial\vec r}\cdot\frac{\partial\hat\varepsilon}{\partial\vec p}\right) - \frac12\left(\frac{\partial\hat\varepsilon}{\partial\vec r}\cdot\frac{\partial\hat f}{\partial\vec p}+\frac{\partial\hat f}{\partial\vec p}\cdot\frac{\partial\hat\varepsilon}{\partial\vec r}\right) = I_c$$

test-particle method

Y. Xia, JX, B.A. Li, and W.Q. Shen, Phys. Lett. B (2016)

spin-dependent equations of motion

$$\frac{d\vec r}{dt} = \frac{\vec p}{m} + \nabla_p(\varepsilon + \vec h\cdot\vec n) \qquad \frac{d\vec p}{dt} = -\nabla(\varepsilon + \vec h\cdot\vec n)$$

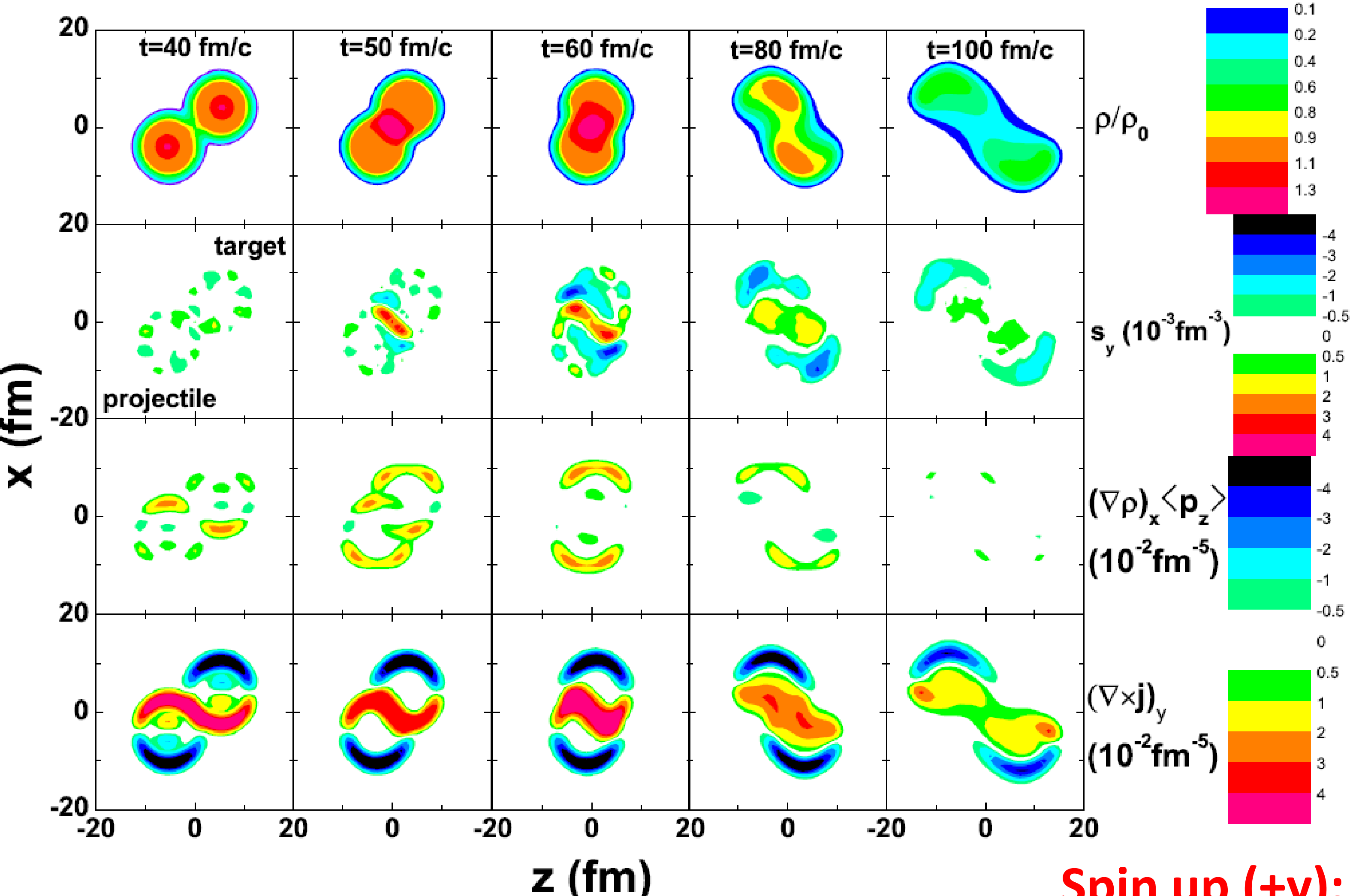
$$\frac{d\vec n}{dt} = 2\vec h\times\vec n$$

$\vec n \sim \vec g$ or $\vec\tau$

spin expectation direction

Local spin polarization

Au+Au@100MeV/A b = 8 fm W_0 = 150 $MeVfm^5$



Time-even:

$$\vec{\sigma} \,//\, \vec{p}\times\nabla\rho$$

Time-odd:

$$\vec{\sigma} \,//\, \nabla\times\vec{j}$$

$$(\nabla\rho)_x\langle p_z\rangle \wedge (\nabla\times j)_y$$

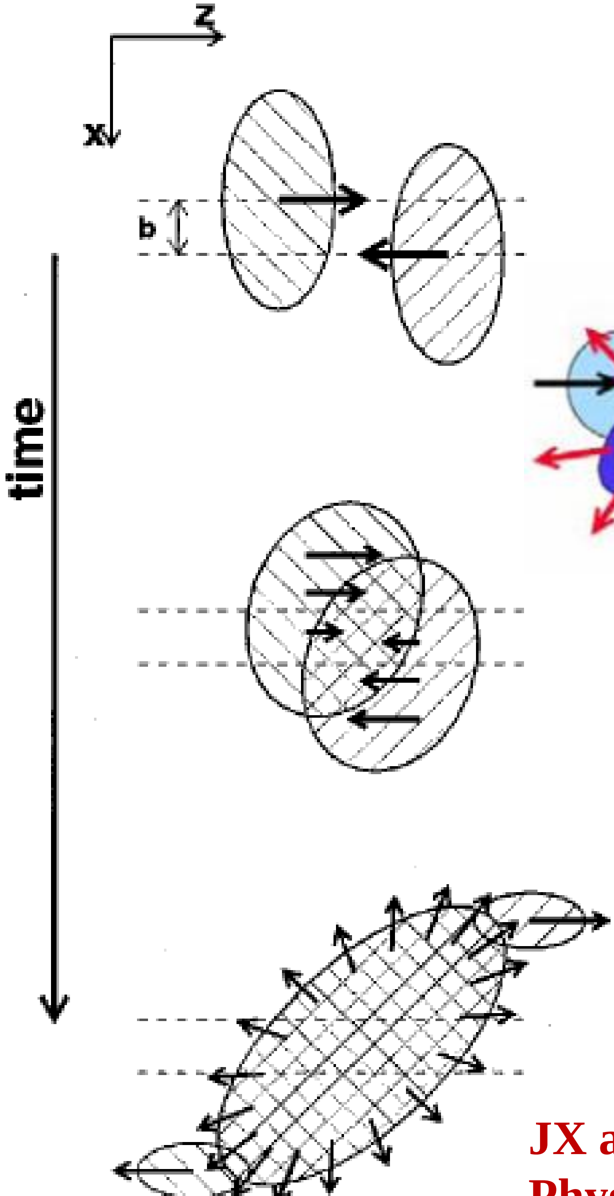
Spin up (+y): attractive

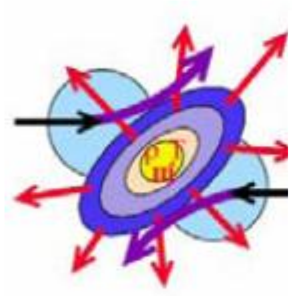
Spin down (-y): repulsive

Time-odd terms overwhelm time-even terms

Transverse flow $\langle p_x \rangle \sim y$

sensitive to nuclear interaction





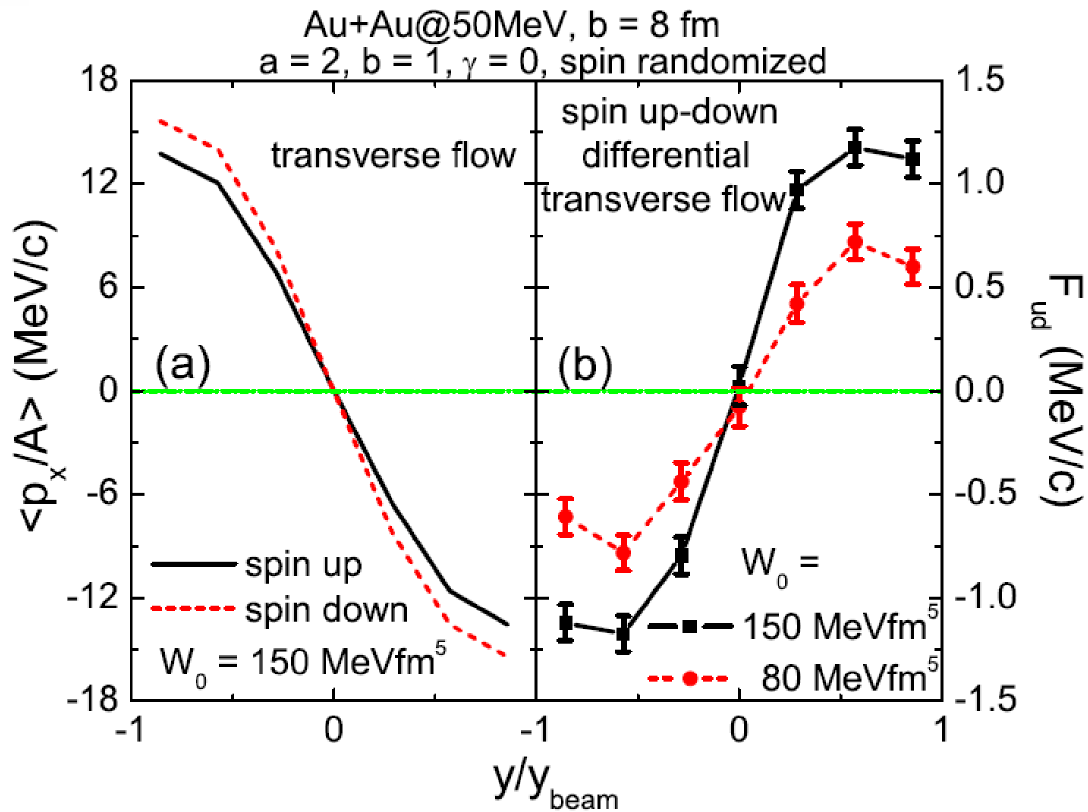
Spin up-down differential transverse flow

$$U = U_0 + \sigma U_{spin}$$

$$\sigma = 1(\uparrow) or -1(\downarrow)$$

$$F_{ud}(y) = \frac{1}{N(y)} \sum_{i=1}^{N(y)} \sigma_i (p_x)_i$$

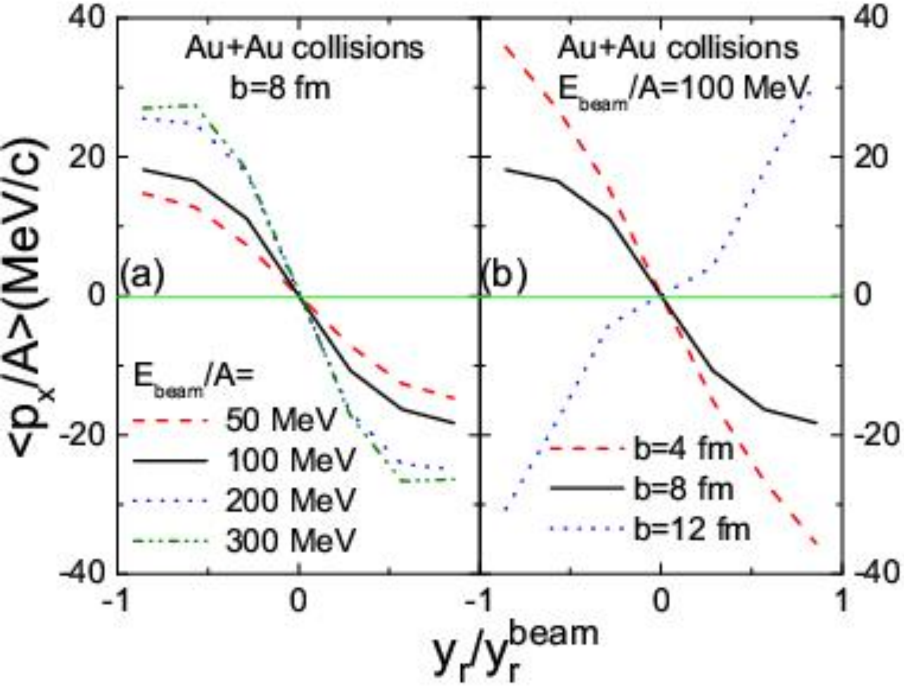
reflects different transverse flows of spin-up and spin-down nucleons

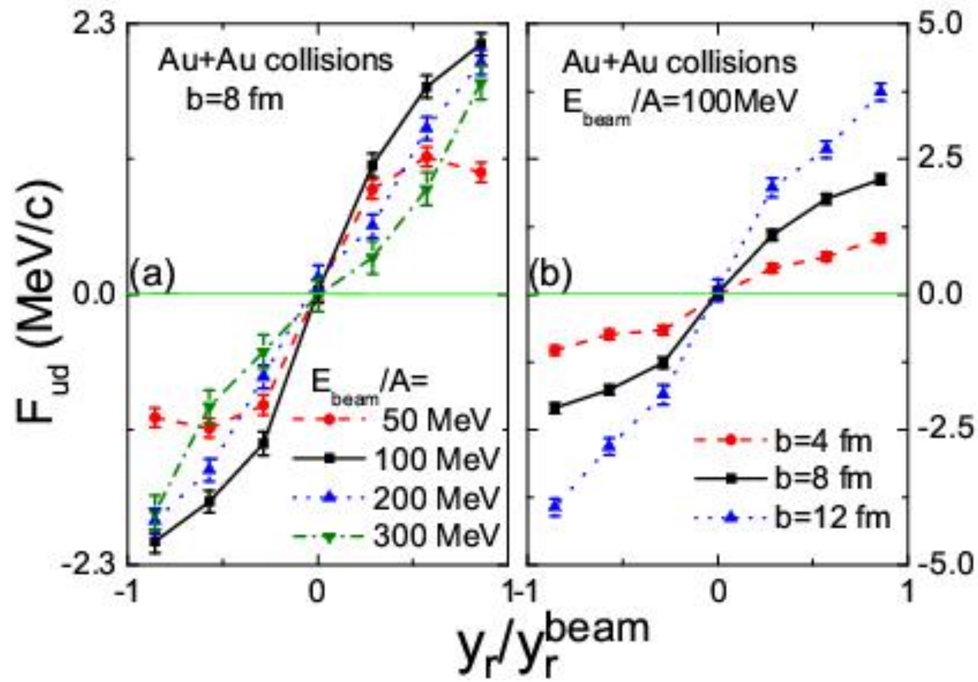


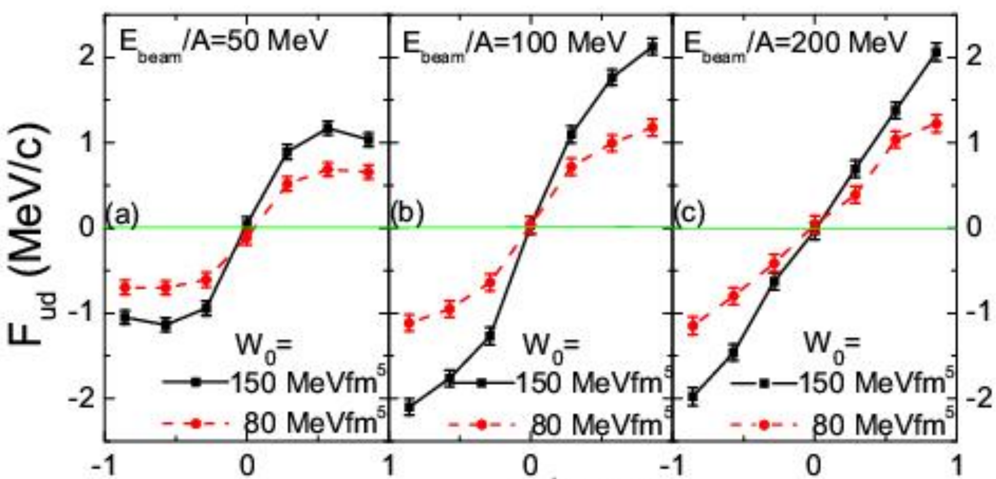
JX and B.A. Li
Phys. Lett. B (2013)

F_{ud} is sensitive to W_0, the strength of the spin-orbit interaction.

Energy and impact parameter dependence







The transverse flow

repulsive NN scatterings

attractive mean-field potential

Spin up-down differential transverse flow

density gradient (surface)

angular momentum (current)

violent NN scatterings

Y. Xia, JX, B.A. Li, and W.Q. Shen, Phys. Rev. C (2014)

System size dependence

Directed flow:

$$v_1 = \langle \cos(\phi) \rangle = \left\langle \frac{p_x}{p_T} \right\rangle$$

heavier system

higher density, pressure

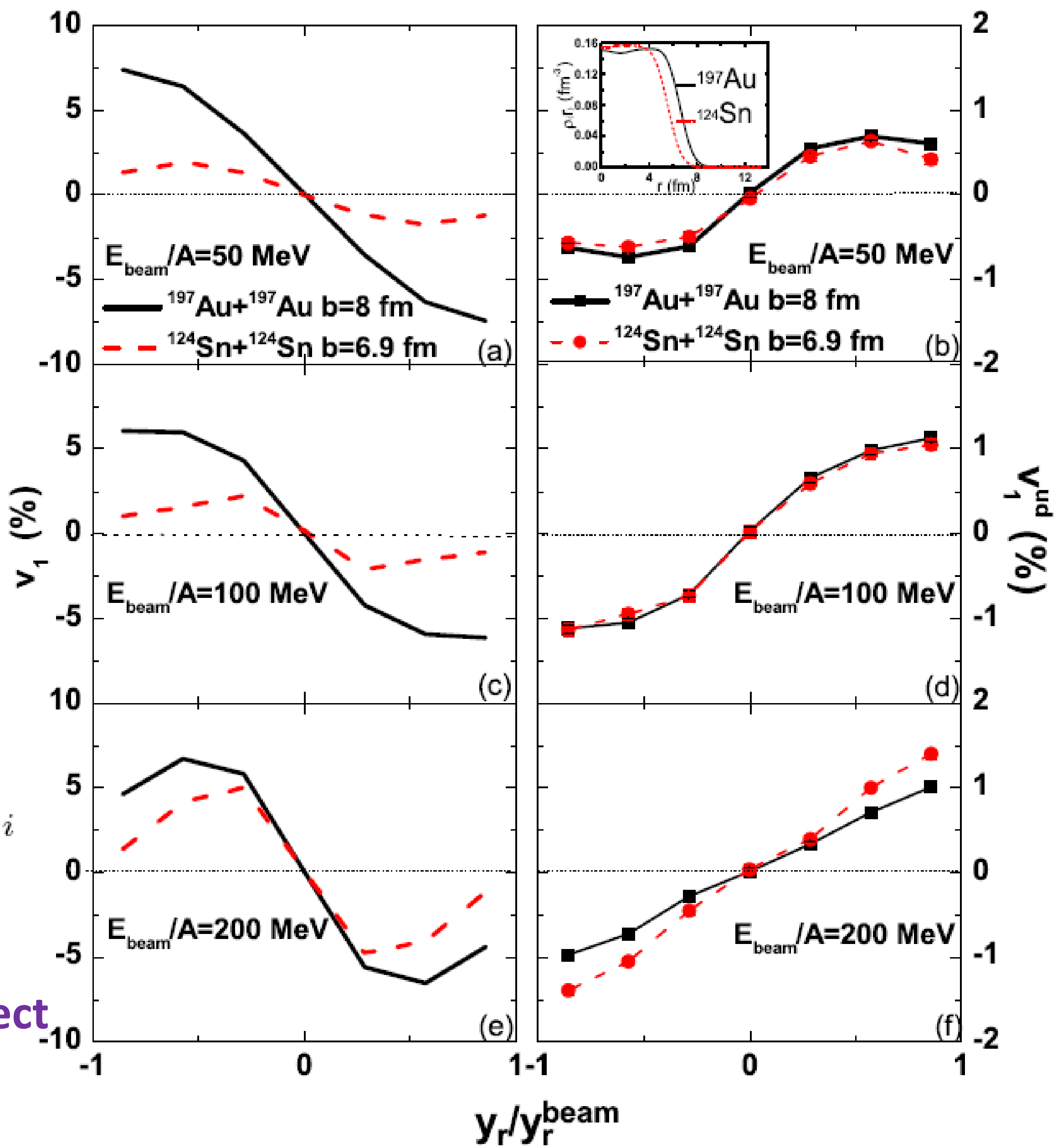
Larger v_1

Spin up-down differential directed flow:

$$v_1^{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left(\frac{p_x}{p_T}\right)_i$$

Surface effect

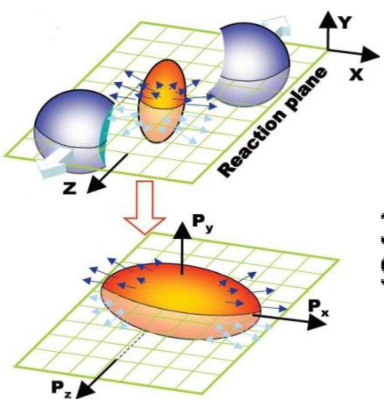
NN scatterings wash out spin effect



Effects of spin-orbit interaction on v_2

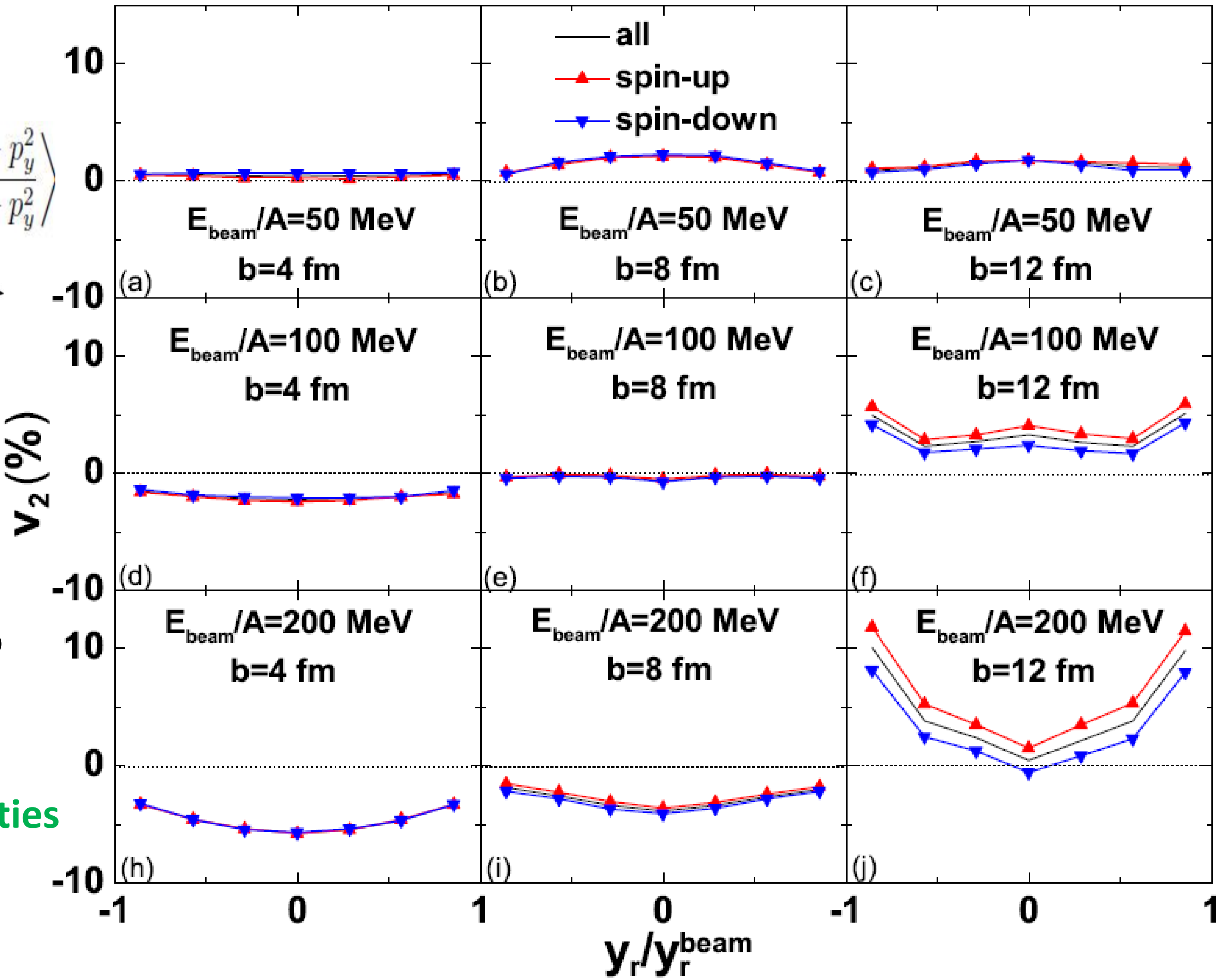
Elliptic flow:

$$v_2 = \langle \cos(2\phi) \rangle = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

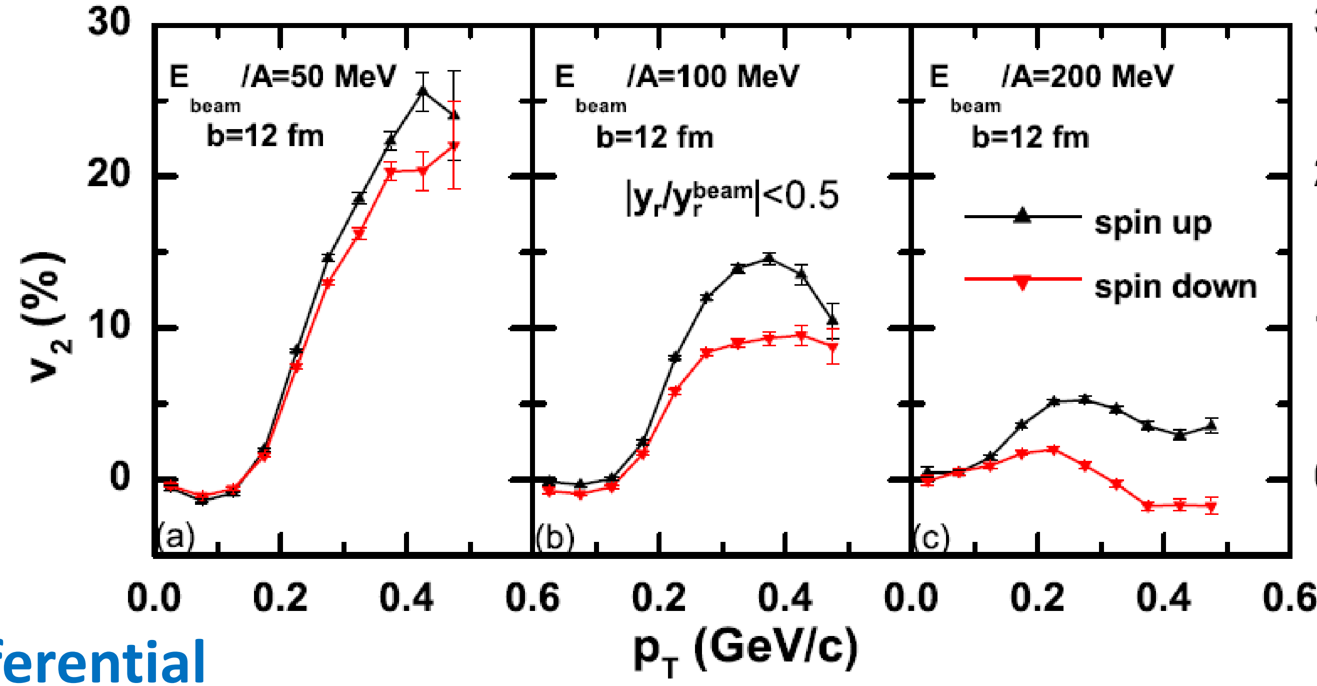


+: in-plane hydro
-: squeeze out

Spin splitting
at large centralities



Spin splitting at higher p_T

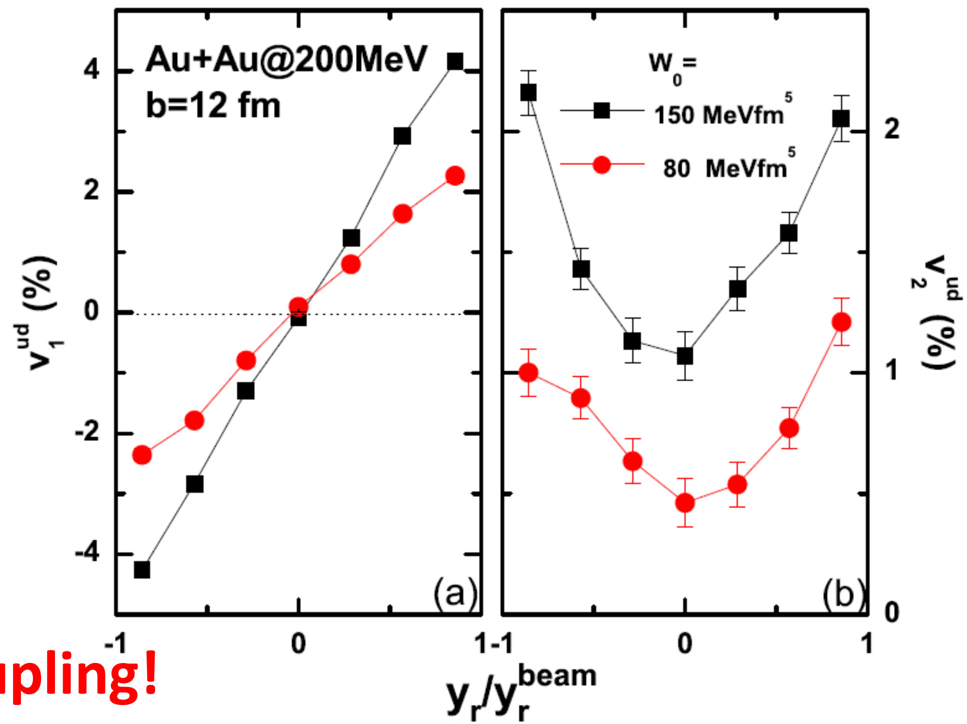


Spin up-down differential directed flow:

$$v_1^{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left(\frac{p_x}{p_T}\right)_i$$

Spin up-down differential elliptic flow:

$$v_2^{ud}(y_r) = \frac{1}{N(y_r)} \sum_{i=1}^{N(y_r)} \sigma_i \left(\frac{p_x^2 - p_y^2}{p_T^2}\right)_i$$



Both are sensitive probes of SO coupling!

- Skyrme-Hartree-Fock model

$$V_{so} = iW_0(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2)\vec{k}'$$

Hartree-Fock method

$$\vec{W}_q = \frac{W_0}{2}\left(\nabla\rho + \nabla\rho_q\right)$$

q=n,p

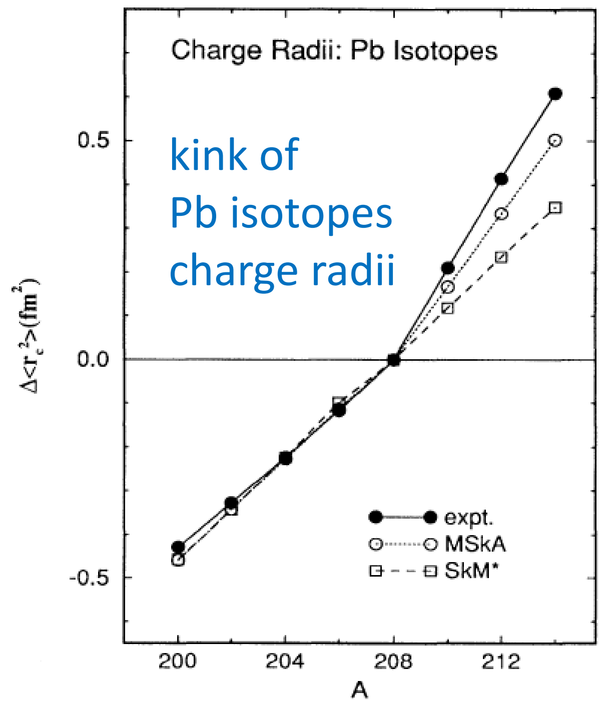
- Relativistic mean field model

Dirac equation

Non-relativistic expansion

$$\vec{W}_q = \frac{C}{(2m - C\rho)^2}\nabla\rho, C = \frac{g_\sigma^2}{m_\sigma^2} + \frac{g_\omega^2}{m_\omega^2}$$

P. G. Reinhard and H. Flocard, NPA, 1995



the isospin dependence of the SO potential

$$\vec{W}_q = \frac{W_0}{2}(1 + \chi_w)\nabla\rho_q + \frac{W_0}{2}\nabla\rho_{q'}.(q \neq q')$$

M. M. Sharma *et al.*, Phys. Rev. Lett., 1995

the density dependence of the SO potential

$$v_{ij} = v_{ij}^0 + (i/\hbar^2)W_1(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)\cdot \mathbf{p}_{ij} \times (\rho_{q_i} + \rho_{q_j})^\gamma \delta(\mathbf{r}_{ij})\mathbf{p}_{ij}.$$

$$\vec{W}_q = \frac{W_0}{2}\nabla(\rho+\rho_q) + \frac{W_1}{2}[(\rho)^\gamma \nabla(\rho-\rho_q) + (2+\gamma)(2\rho_q)^\gamma \nabla\rho_q] + \frac{W_1}{4}\gamma\rho^{\gamma-1}(\rho-\rho_q)\nabla\rho$$

W_1 and γ fitted to reproduce the density dependence of the SO potential from the RMF model

Similar spin-orbit field in semi-infinite nuclear matter

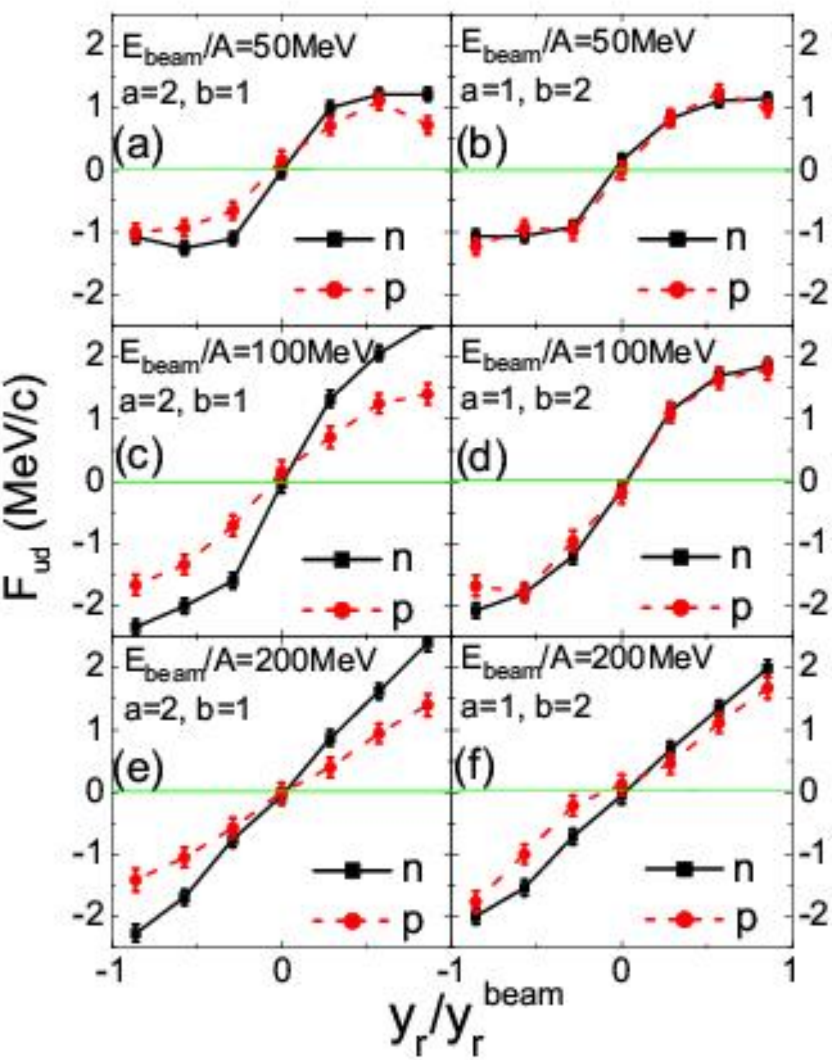
J. M. Pearson and M. Farine, Phys. Rev. C 50, 185 (1994).

Generally $$\vec{W}_q = W_0 \left(\frac{\rho}{\rho_0}\right)^\gamma \left(a\nabla\rho_q + b\nabla\rho_{q'}\right) \qquad (q \neq q')$$

density dependence — isospin dependence

W_0= 80 ~ 150 MeVfm^5, γ, a, and b still under debate

T. Lesinski *et al.*, Phys. Rev. C 76, 014312 (2007).
M. Zalewski *et al.*, Phys. Rev. C 77, 024316 (2008).
M. Bender *et al.*, Phys. Rev. C 80, 064302 (2009).



isospin dependence of SO coupling

$$\frac{W_0}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma}\left(a\nabla\rho_q + b\nabla\rho_{q'}\right)+\ldots$$

Globally a neutron-rich system

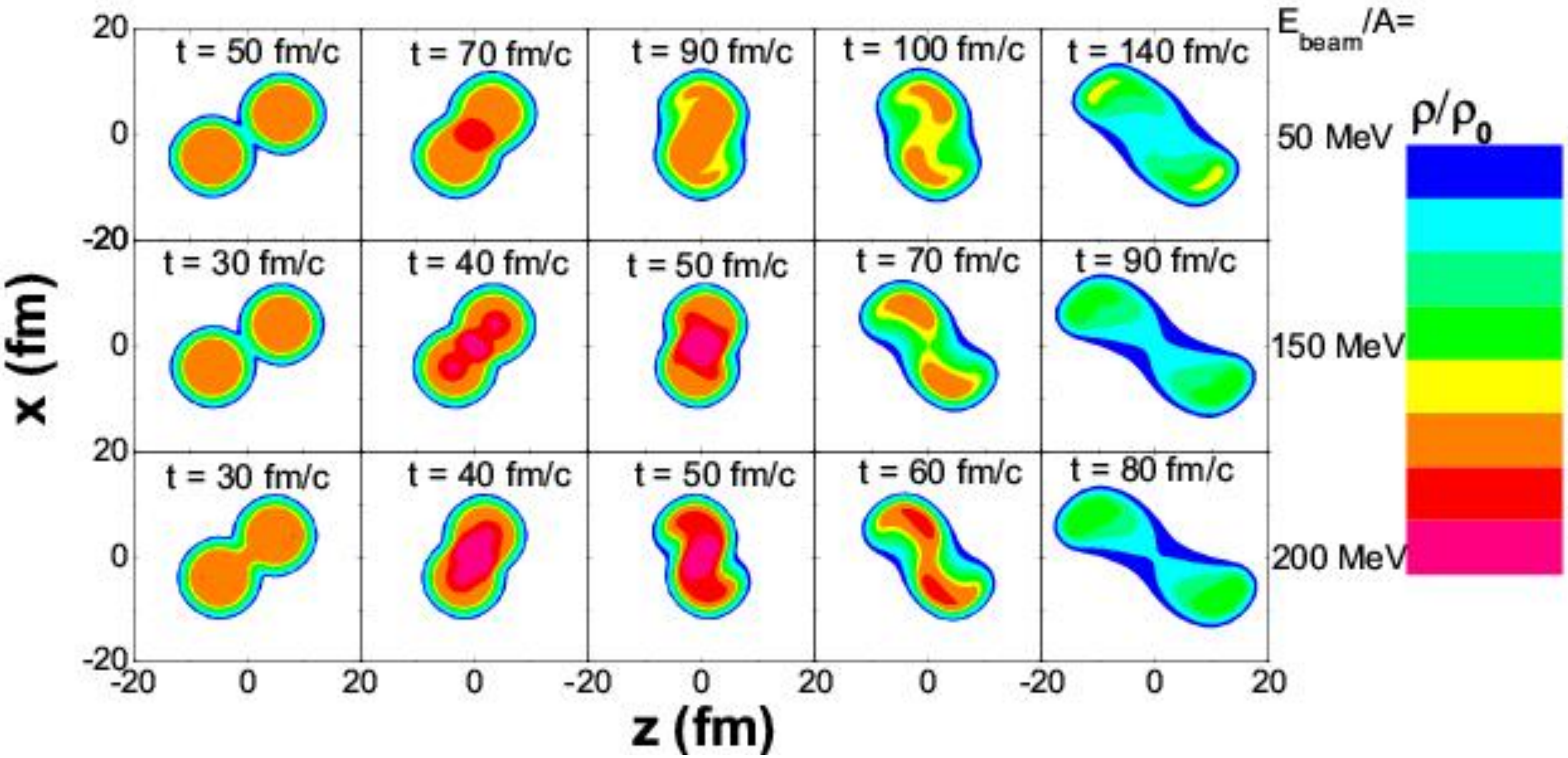
$$\left|\nabla\rho_n\right| > \left|\nabla\rho_p\right|$$

$$\left|\nabla\times\vec{j}_n\right| > \left|\nabla\times\vec{j}_p\right|$$

By comparing the spin up-down differential transverse flow for neutrons and protons using different isospin dependence of SO coupling.

$$F' = \left[\frac{dF_{ud}}{d(y_r/y_r^{beam})}\right]_{y_r=0} \qquad \delta' = \frac{F_n' - F_p'}{F_n' + F_p'}$$

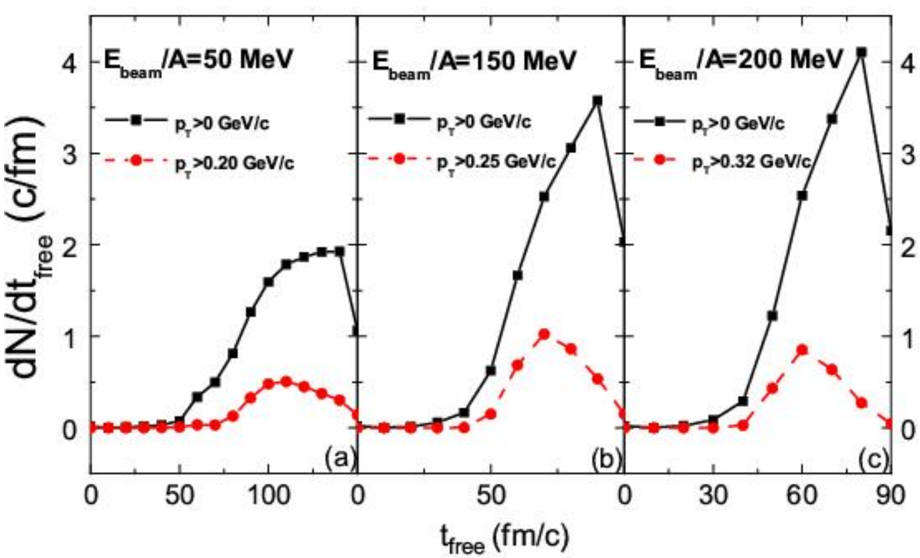
	$E_{beam}=50$ (AMeV)		$E_{beam}=100$ (AMeV)		$E_{beam}=200$ (AMeV)	
	$a/b=2$	$a/b=1/2$	$a/b=2$	$a/b=1/2$	$a/b=2$	$a/b=1/2$
F_n'	4.17 ± 0.09	3.41 ± 0.53	5.62 ± 0.35	4.43 ± 0.24	2.60 ± 0.50	2.37 ± 0.28
F_p'	2.59 ± 0.36	3.58 ± 0.34	2.55 ± 0.33	3.74 ± 0.75	1.68 ± 0.23	1.10 ± 0.39
δ'	0.23 ± 0.06	-0.02 ± 0.09	0.38 ± 0.06	0.08 ± 0.10	0.21 ± 0.08	0.36 ± 0.09



$\langle \rho_h \rangle < \rho_0$

$\langle \rho_h \rangle \approx \rho_0$

$\langle \rho_h \rangle > \rho_0$



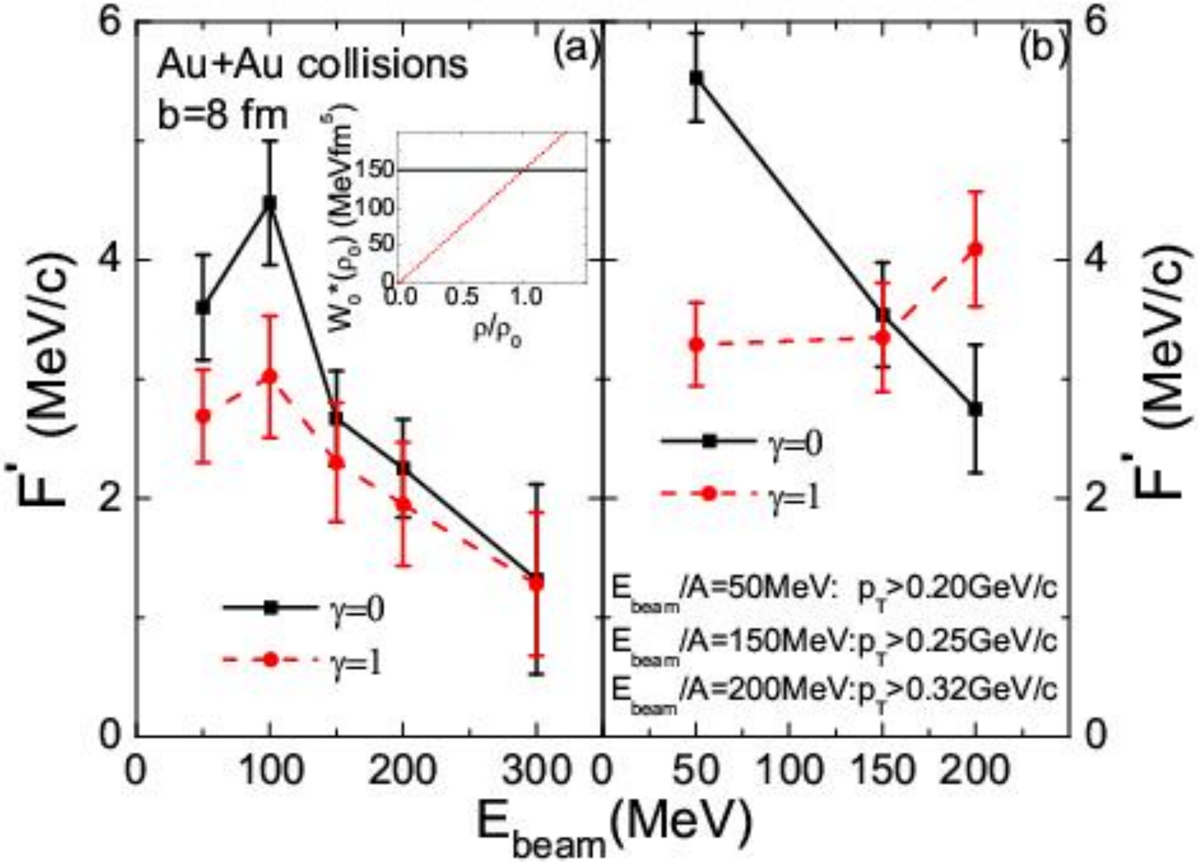
Free nucleons: $\rho < \rho_0 / 8$

Different densities are reached at different beam energies

Nucleons of high transverse momentum (p_T) are emitted at early stages.

density dependence of SO coulping

$$\frac{W_0}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma}\left(a\nabla\rho_q + b\nabla\rho_{q'}\right)+\ldots$$



$$F' = \left[\frac{dF_{ud}}{d(y_r/y_r^{beam})}\right]_{y_r=0}$$

Low-p_T nucleons: emitted at later stages carry information of **lower densities**

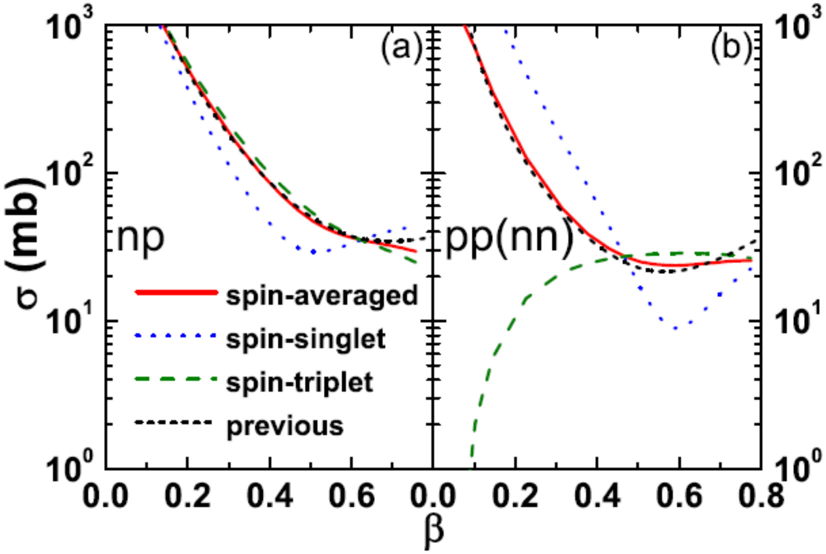
high-p_T nucleons: emitted at early stages carry information of **higher densities**

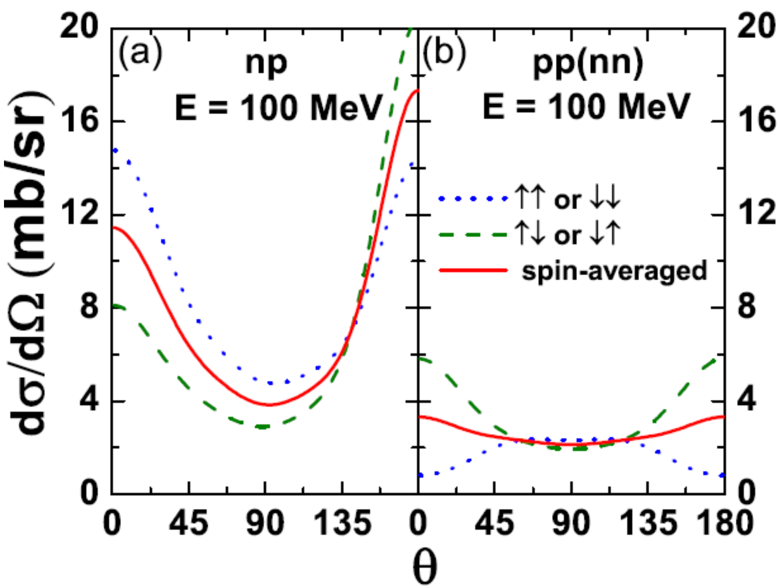
The strength of the SO coupling at a certain density can be extracted from HIC at the corresponding collision energy.

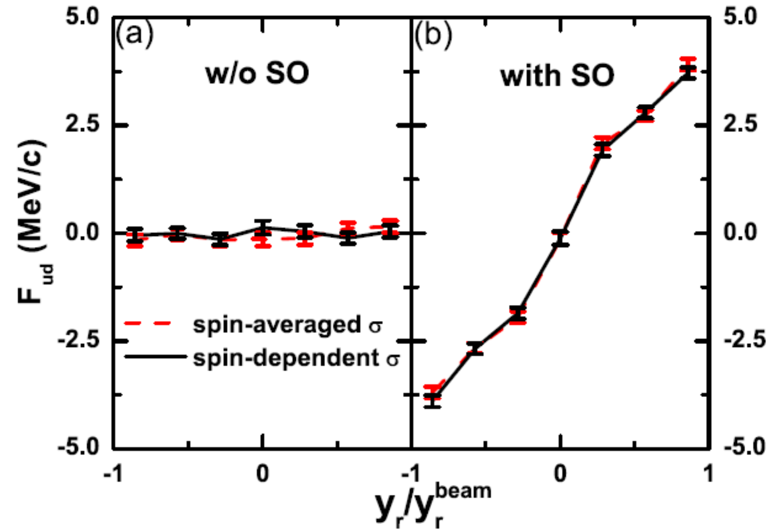
In this way the strength and density dependence of SO coupling can be **disentangled**.

Effects of spin-dependent cross section

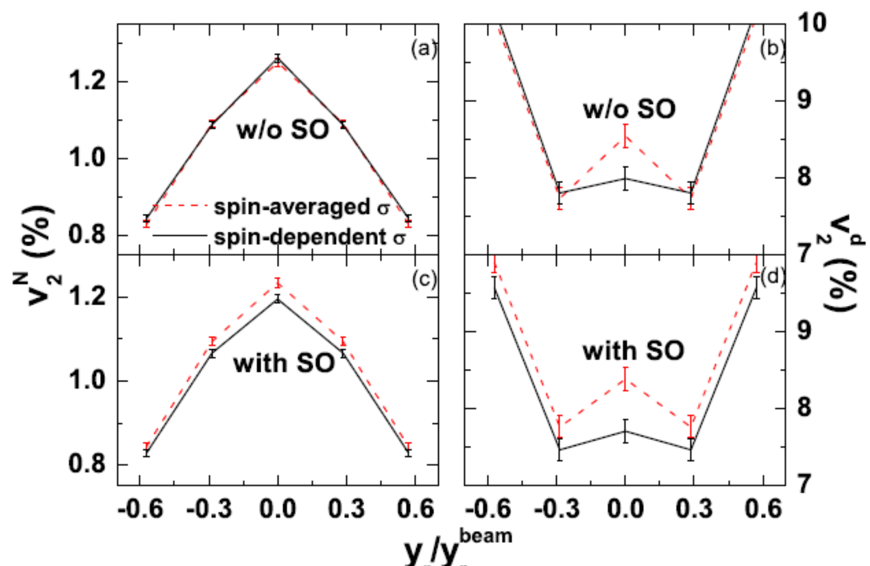
Spin-dependent cross section
from phase-shift analysis
of NN scatterings in free space







Spin up-down differential flow not affected



Slightly affect the overall v_2, especially for clusters

Y. Xia, JX, B.A. Li, and W.Q. Shen, PRC (2017)

Spin effects on low-energy nuclear reactions

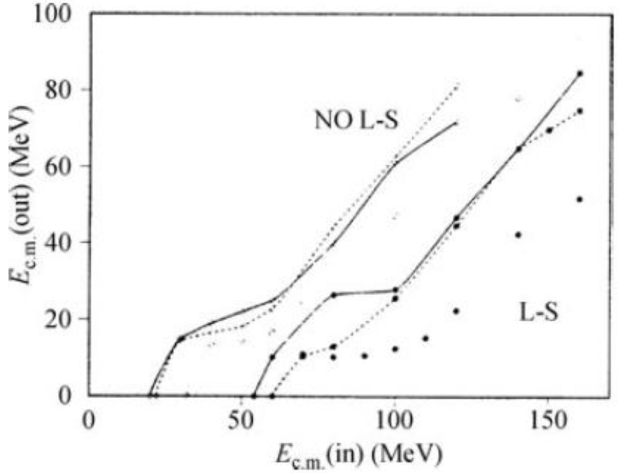
Effects of spin-orbit coupling illustrated by $O^{16}+O^{16}$ from TDHF:

Fusion threshold

TABLE I. Thresholds for the inelastic scattering of $^{16}O + ^{16}O$ system.

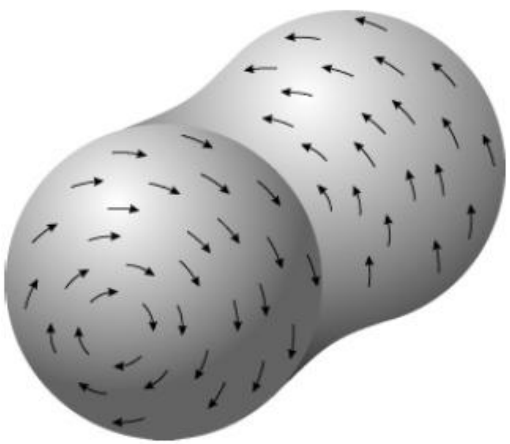
Force	Skyrme II (MeV)	Skyrme M* (MeV)
Spin orbit	68	70
No spin orbit	31	27

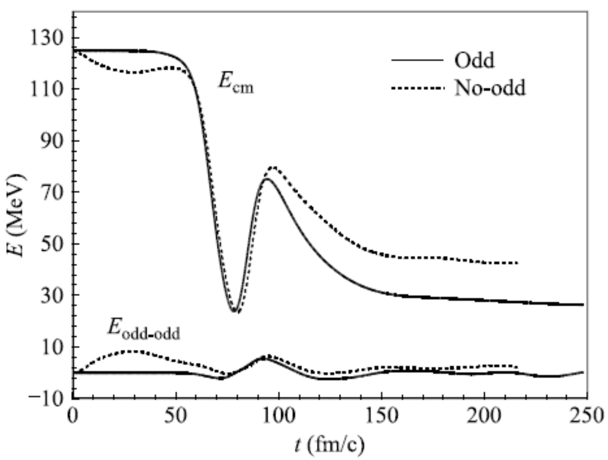
A.S. Umar *et al.*, Phys. Rev. Lett., 1986



P.G. Reinhard *et al.*, Phys. Rev. C, 1988

Spin twist & time-odd terms





J. A. Maruhn *et al.*, Phys. Rev. C, 2006

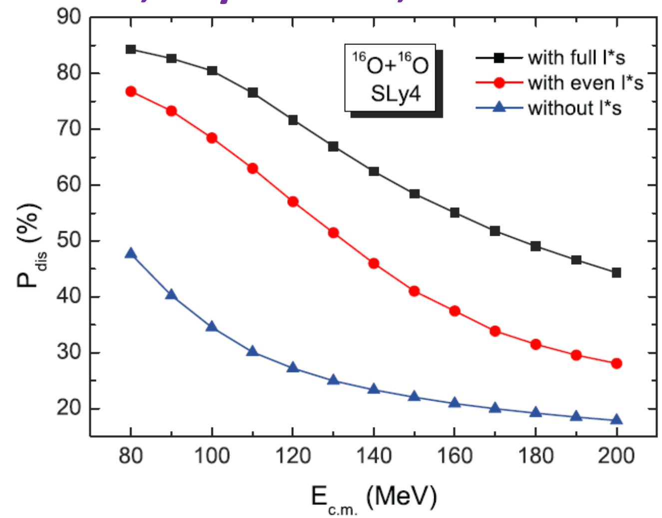
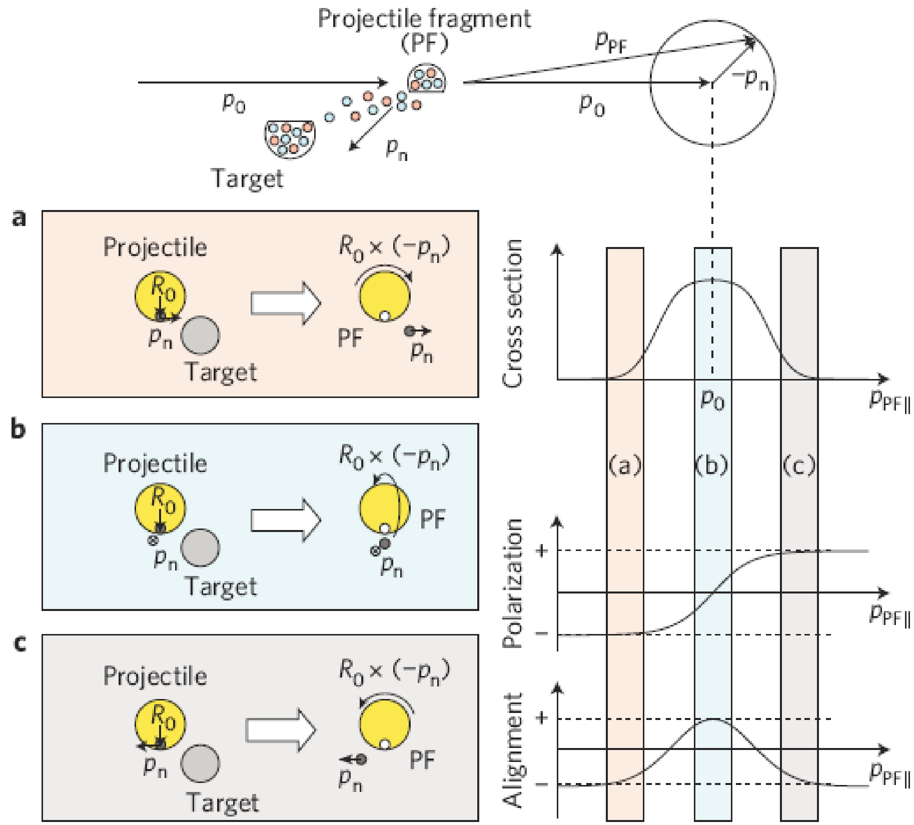


FIG. 2. (Color online) Percentage of energy dissipation as a function of center-of-mass energy for head-on collisions of $^{16}O + ^{16}O$ with parametrization SLy4. The black, red, and blue lines represent the TDHF calculations involving full l*s, time-even l*s, and no l*s force.

G.F. Dai, L. Guo, E.G. Zhao, and S.G. Zhou, Phys. Rev. C, 2014

Spin related experiments

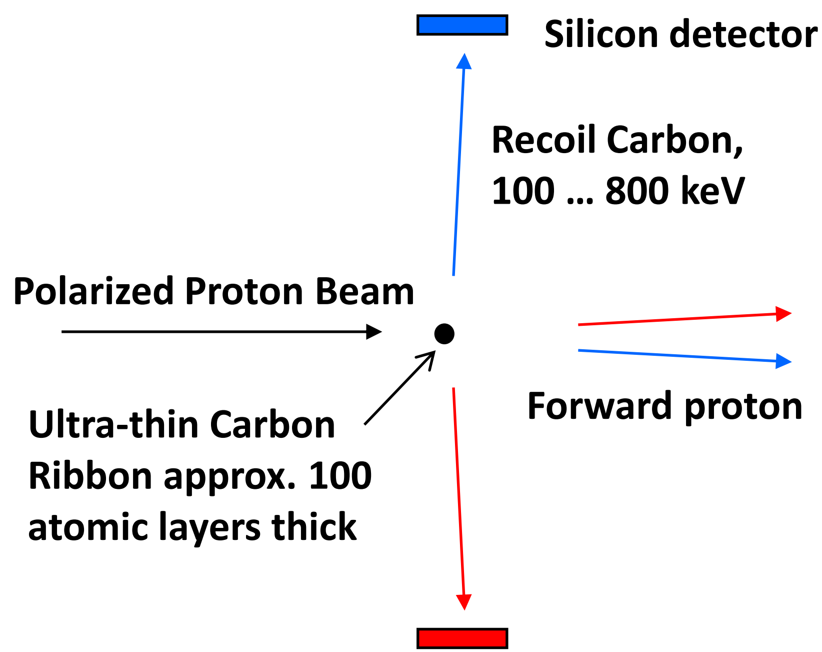
Spin-polarized beam from
pick-up or removal reactions



The spin alignment of projectile fragment can be measured through the angular distribution of its γ or β decay.

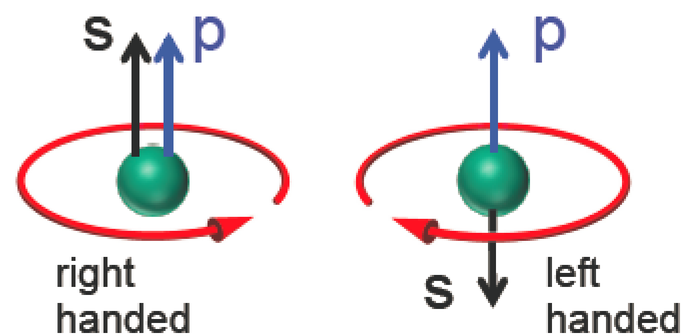
Analyzing power measurement

$$A = \frac{\sqrt{LU}\sqrt{RD} - \sqrt{LD}\sqrt{RU}}{\sqrt{LU}\sqrt{RD} + \sqrt{LD}\sqrt{RU}}$$



Spin effects on relativistic heavy-ion collisions - chiral dynamics

$$\hat{H} = \gamma^0\gamma^k(-p_k + A_k) + \gamma^0 M + A_0$$



for massless particles (M=0)

$$h = \pm\vec{\sigma}\cdot(\vec{p} - \vec{A}) + A_0 = c\vec{\sigma}\cdot\vec{k} + A_0$$

Weyl SOC

Spin dynamics

$$\vec{B} = \nabla\times\vec{A}$$

$$\vec{E} = -\nabla A_0$$

$$\frac{d\vec{r}}{dt} = c\vec{\sigma}$$

$$\frac{d\vec{k}}{dt} = c\vec{\sigma}\times\vec{B} + \vec{E}$$

$$\frac{d\vec{\sigma}}{dt} = 2c\vec{k}\times\vec{\sigma}$$

chiral dynamics

using $\vec{\sigma} = c\hat{k} - \frac{\hbar}{2k^2}\hat{k}\times\frac{d\hat{k}}{dt}$

X.G. Huang, Scientific Report (2016)

$$\sqrt{G}\frac{d\vec{r}}{dt} = \hat{k} + c\frac{\hbar}{2k^2}\vec{B} + c\frac{\hbar}{2k^3}\vec{E}\times\vec{k}$$

$$\sqrt{G}\frac{d\vec{k}}{dt} = \vec{k}\times\vec{B} + c\frac{\hbar\vec{k}}{2k^3}(\vec{E}\cdot\vec{B}) + \vec{E}$$

$$\sqrt{G} = 1 + c\frac{\vec{B}\cdot\vec{k}}{2k^3}$$

M.A. Stephenov and Y. Yin, PRL (2012)

J.W. Chen, S. Pu, Q. Wang, and X.N. Wang, PRL (2013)

D.T. Son and N. Tamamoto, PRD (2013)

Box system with periodic bound condition

$$V = 10^3\ fm^3$$

$$\rho = 10\ fm^{-3}$$

$$eB_y = 5m_\pi^2$$

$$T = 300 MeV$$

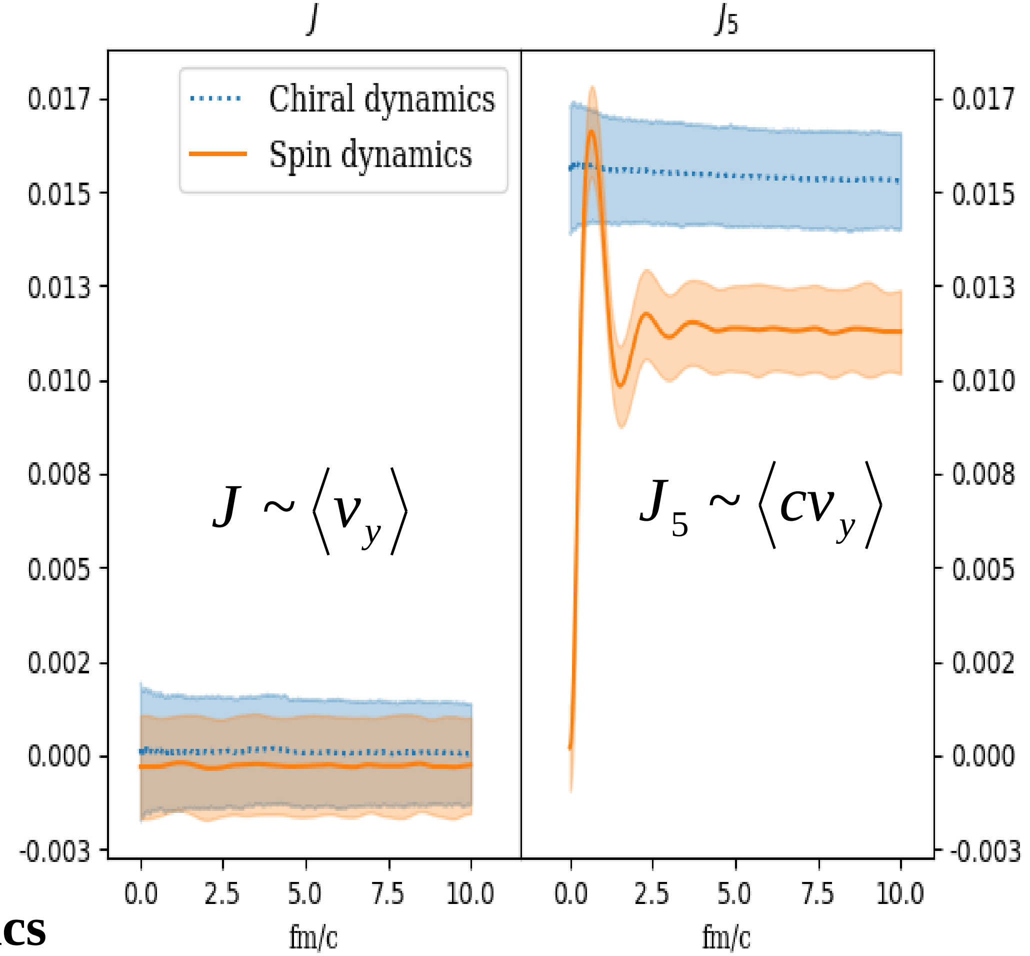
$$\vec{E} = 0$$

$Q(+)55\%$

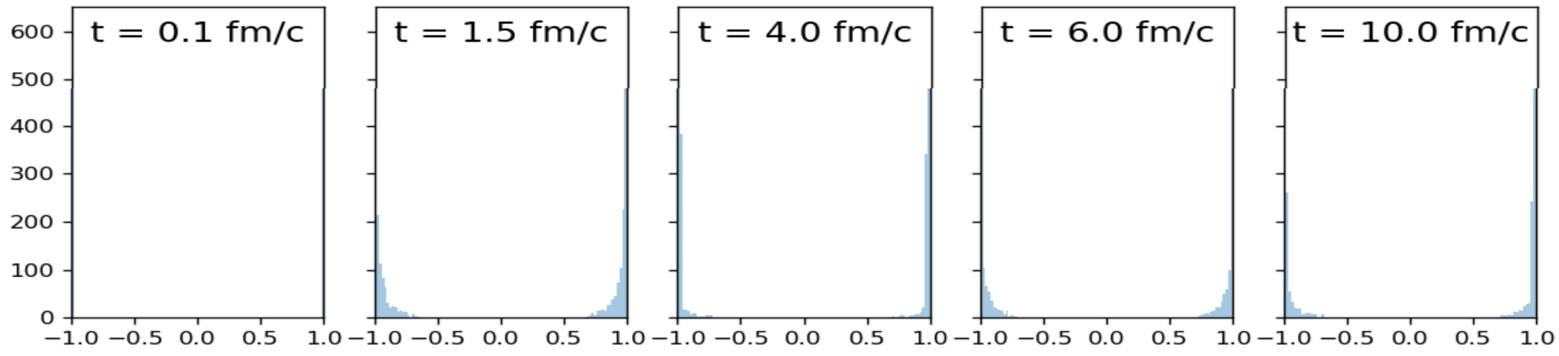
$Q(-)45\%$

$\mu_5 = 0$

preliminary



$\arccos(\hat{k} \cdot \vec{\sigma})$ **in spin dynamics**



$$f \Rightarrow \sqrt{G}\, f$$

Final momentum sampled according to $\sqrt{G(k_3)}\sqrt{G(k_4)}$

Y.F. Sun and C.M. Ko, PRC (2017)

Spin polarization in Boltzmann limit:

$$\langle \vec{\sigma} \rangle = \left\langle c \frac{d\vec{r}}{dt} \right\rangle = \frac{\hbar \vec{B}}{4T^2}$$

Consistent with that from the quantum kinetic approach

R.H. Fang, L.G. Pang, Q. Wang, and X.N. Wang, PRC (2016)

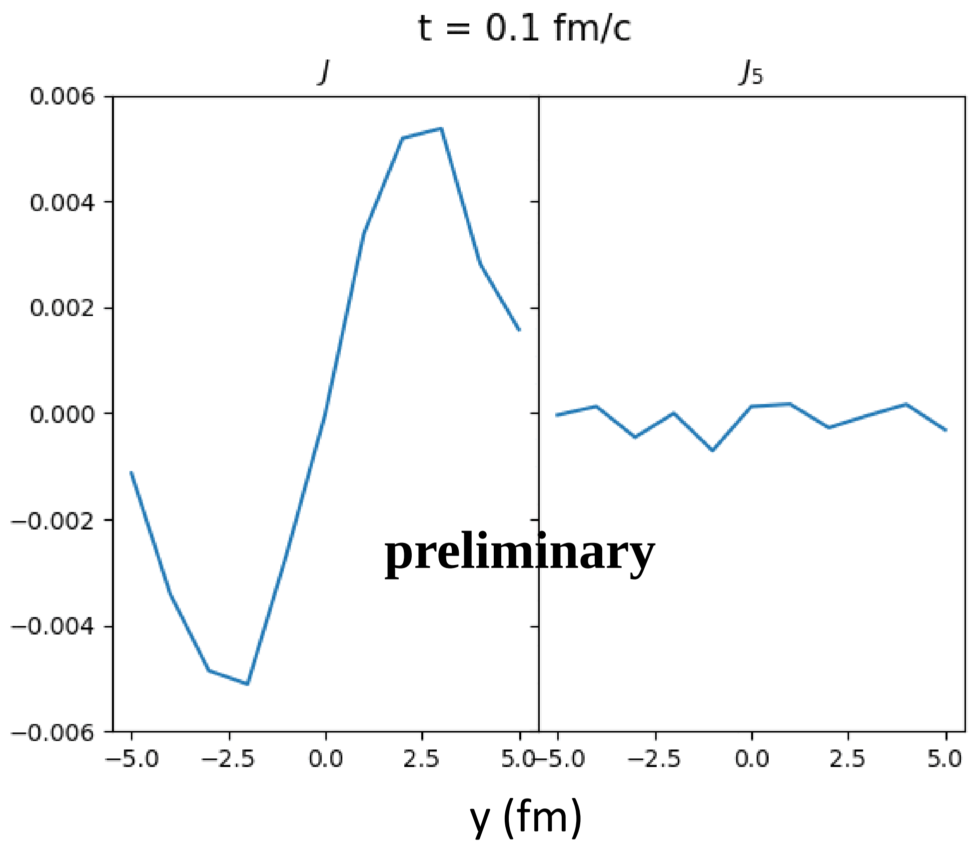
	E	B	ω
J_V	σ Ohm's law	$\frac{N_C e}{2\pi^2}\mu_A$ Chiral magnetic effect	$\frac{N_C}{\pi^2}\mu_V\mu_A$ Vector chiral vortical effect
J_A	$\propto \frac{\mu_V\mu_A}{T^2}\sigma$ Chiral electric separation effect	$\frac{N_C e}{2\pi^2}\mu_V$ Chiral separation effect	$N_C\left(\frac{T^2}{6}+\frac{\mu_V^2+\mu_A^2}{2\pi^2}\right)$ Axial chiral vortical effect

D.E. Kharzeev, Prog. Part. Nucl. Phys., 2014

Chiral magnetic wave:
Chiral magnetic effect+Chiral separation effect

$$j_A = \frac{N_c e}{2\pi^2}\mu_V B \qquad j_V = \frac{N_c e}{2\pi^2}\mu_A B$$

Initialization: $\mu_5(y) = \mu_5' \sin\left(\frac{2\pi y}{L}\right)$



Conclusion

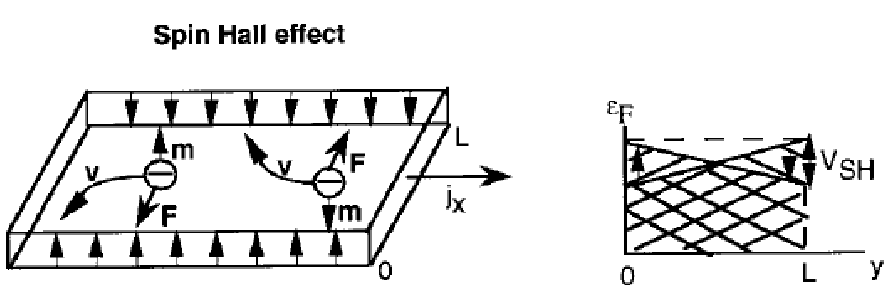
Nuclear Skyrme-type spin-orbit couplings → Low-energy HIC: affect fusion threshold

$$\vec{\sigma}\cdot(\nabla\rho\times\vec{p})$$

→ intermediate-energy HIC: Spin splitting of collective flows

Weyl spin-orbit couplings (massless particle) → Relativistic HIC: Chiral dynamics

$$\pm\vec{\sigma}\cdot(\vec{p}-\vec{A})$$



Acknowledge:

Bao-An Li (TAMUC)

Wen-Qing Shen (SINAP)

Che Ming Ko (TAMU)

Yi-Feng Sun (TAMU)

Lie-Wen Chen (SJTU)

Kai-Jia Sun (SJTU)

Students in SINAP:

Yin Xia, Zhang-Zhu Han, Wen-Hao Zhou

Thank you!

xujun@sinap.ac.cn

Spin-dependent Boltzmann equation

$$\frac{\partial \hat{f}}{\partial t} + \frac{i}{\hbar}\left[\hat{\varepsilon}, \hat{f}\right] + \frac{1}{2}\left(\frac{\partial \hat{\varepsilon}}{\partial \vec{p}} \cdot \frac{\partial \hat{f}}{\partial \vec{r}} + \frac{\partial \hat{f}}{\partial \vec{r}} \cdot \frac{\partial \hat{\varepsilon}}{\partial \vec{p}}\right) - \frac{1}{2}\left(\frac{\partial \hat{\varepsilon}}{\partial \vec{r}} \cdot \frac{\partial \hat{f}}{\partial \vec{p}} + \frac{\partial \hat{f}}{\partial \vec{p}} \cdot \frac{\partial \hat{\varepsilon}}{\partial \vec{r}}\right) = I_c$$

H. Smith and H.H. Jensen, transport phenomena (Oxford University press, Oxford, 1989)

Single-particle energy: $\hat{\varepsilon}(\vec{r}, \vec{p}) = \varepsilon(\vec{r}, \vec{p})\hat{I} + \vec{h}(\vec{r}, \vec{p}) \cdot \vec{\sigma},$

Wigner function: $\hat{f}(\vec{r}, \vec{p}) = f_0(\vec{r}, \vec{p})\hat{I} + \vec{g}(\vec{r}, \vec{p}) \cdot \vec{\sigma}$

Spin-dependent Wigner function:

$$f_{\sigma,\sigma'}(\vec{r}\vec{p}, t) = \int d^3 s e^{-i\vec{p}\cdot\vec{s}/\hbar} \psi^*_{\sigma'}(\vec{r} - \frac{\vec{s}}{2}, t)\psi_\sigma(\vec{r} + \frac{\vec{s}}{2}, t),$$

$$f(\vec{r}\vec{p}, t, 0) = f_{1,1}(\vec{r}\vec{p}, t) + f_{-1,-1}(\vec{r}\vec{p}, t), \quad = 2 f_0(\vec{r}, \vec{p}, t)$$

$$\left.\begin{aligned} \tau(\vec{r}\vec{p}, t, x) &= f_{-1,1}(\vec{r}\vec{p}, t) + f_{1,-1}(\vec{r}\vec{p}, t), \\ \tau(\vec{r}\vec{p}, t, y) &= -i[f_{-1,1}(\vec{r}\vec{p}, t) - f_{1,-1}(\vec{r}\vec{p}, t)], \\ \tau(\vec{r}\vec{p}, t, z) &= f_{1,1}(\vec{r}\vec{p}, t) - f_{-1,-1}(\vec{r}\vec{p}, t), \end{aligned}\right\} = 2\vec{g}(\vec{r}, \vec{p}, t)$$

R. F. O'Connell and E.P. Wigner, Phys. Rev. A 30, 2613 (1984)

Single-particle Hamiltonian from Skyrme interaction

Skyrme spin-orbit interaction:

$$V_{so} = iW_0(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{k} \times \delta(\vec{r}_1 - \vec{r}_2)\vec{k}'$$

Single-particle hamiltonian:

$$h_q^{so}(\vec{r}, \vec{p}) = h_1 + h_4 + (\vec{h}_2 + \vec{h}_3) \cdot \vec{\sigma}.$$

Y.M. Engel et al., NPA (1975)

$$h_1 = -\frac{W_0}{2} \nabla_{\vec{r}} \cdot [\vec{J}(\vec{r}) + \vec{J}_q(\vec{r})]$$ time-even

$$\vec{J}(\vec{r}) = \int d^3p \frac{\vec{p}}{\hbar} \times \vec{\tau}(\vec{r}, \vec{p}).$$

$$\vec{h}_2 = -\frac{W_0}{2} \nabla_{\vec{r}} \times [\vec{j}(\vec{r}) + \vec{j}_q(\vec{r})]$$ time-odd

$$\vec{j}(\vec{r}) = \int d^3p \frac{\vec{p}}{\hbar} f(\vec{r}, \vec{p}),$$

$$\vec{h}_3 = \frac{W_0}{2} \nabla_{\vec{r}} [\rho(\vec{r}) + \rho_q(\vec{r})] \times \vec{p}$$ time-even

$$\rho(\vec{r}) = \int d^3p f(\vec{r}, \vec{p}),$$

$$h_4 = -\frac{W_0}{2} \nabla_{\vec{r}} \times [\vec{s}(\vec{r}) + \vec{s}_q(\vec{r})] \cdot \vec{p}$$ time-odd

$$\vec{s}(\vec{r}) = \int d^3p \vec{\tau}(\vec{r}, \vec{p}),$$

Test particle method:

$$f(\vec{r}, \vec{p}) = \frac{1}{N_{TP}} \sum_i \delta(\vec{r} - \vec{r}_i)\delta(\vec{p} - \vec{p}_i),$$

$$\vec{\tau}(\vec{r}, \vec{p}) = \frac{1}{N_{TP}} \sum_i \vec{n}_i \delta(\vec{r} - \vec{r}_i)\delta(\vec{p} - \vec{p}_i),$$

with $\vec{n}_i$ being the spin expectation direction of the ith nucleon

Equation of motion I

Separate BUU equation for scalar and vector part **assuming** $\vec{g}(\vec{r},\vec{p}) = \vec{n} f_1(\vec{r},\vec{p})$.

$\vec{n}$ is the direction (unit vector) of the local spin polarization in 3-dimensional coordinate space

$$\frac{\partial f_0}{\partial t} + \frac{\partial \varepsilon}{\partial \vec{p}} \cdot \frac{\partial f_0}{\partial \vec{r}} - \frac{\partial \varepsilon}{\partial \vec{r}} \cdot \frac{\partial f_0}{\partial \vec{p}} + (\frac{\partial \vec{h}}{\partial \vec{p}} \cdot \vec{n}) \cdot \frac{\partial f_1}{\partial \vec{r}} - (\frac{\partial \vec{h}}{\partial \vec{r}} \cdot \vec{n}) \cdot \frac{\partial f_1}{\partial \vec{p}} \approx 0,$$

$$\boxed{\frac{\partial f_1}{\partial t}\vec{n} + (\frac{\partial \varepsilon}{\partial \vec{p}} \cdot \frac{\partial f_1}{\partial \vec{r}})\vec{n} - (\frac{\partial \varepsilon}{\partial \vec{r}} \cdot \frac{\partial f_1}{\partial \vec{p}})\vec{n}} + \boxed{\frac{\partial f_0}{\partial \vec{r}} \cdot \frac{\partial \vec{h}}{\partial \vec{p}} - \frac{\partial f_0}{\partial \vec{p}} \cdot \frac{\partial \vec{h}}{\partial \vec{r}}} + \boxed{(\frac{2\vec{n} \times \vec{h}}{\hbar} + \frac{\partial \vec{n}}{\partial t}) f_1} \approx 0.$$

$// \vec{n}$ **small** $\perp \vec{n}$

$$\frac{\partial \vec{n}}{\partial t} \approx \frac{2\vec{h} \times \vec{n}}{\hbar}$$

define $f^{\pm} = f_0 \pm f_1$

$$\frac{\partial f^+}{\partial t} + \left(\frac{\partial \epsilon}{\partial \vec{p}} + \frac{\partial V_{hn}}{\partial \vec{p}}\right) \cdot \frac{\partial f^+}{\partial \vec{r}} - \left(\frac{\partial \epsilon}{\partial \vec{r}} + \frac{\partial V_{hn}}{\partial \vec{r}}\right) \cdot \frac{\partial f^+}{\partial \vec{p}} = 0,$$

$$\frac{\partial f^-}{\partial t} + \left(\frac{\partial \epsilon}{\partial \vec{p}} - \frac{\partial V_{hn}}{\partial \vec{p}}\right) \cdot \frac{\partial f^-}{\partial \vec{r}} - \left(\frac{\partial \epsilon}{\partial \vec{r}} - \frac{\partial V_{hn}}{\partial \vec{r}}\right) \cdot \frac{\partial f^-}{\partial \vec{p}} = 0,$$

f^+ and f^- are the eigenfunctions of $\hat{f}$, representing the phase-space distributions of particles with their spin in $+\vec{n}$ and $-\vec{n}$ directions, respectively, i.e., spin-up and spin-down particles.

Equation of motion II

$$f^{\pm}(\vec{r},\vec{p},t) = \int \frac{d^3r_0 d^3p_0 d^3s}{(2\pi\hbar)^3} \exp\{i\vec{s}\cdot[\vec{p}-\vec{P}(\vec{r}_0\vec{p}_0\vec{s},t)]/\hbar\} \times \delta[\vec{r}-\vec{R}(\vec{r}_0\vec{p}_0\vec{s},t)] f^{\pm}(\vec{r}_0,\vec{p}_0,t_0),$$

Following the method by (C. Y. Wong, PRC 25, 1460 (1982))

with the initial conditions $\vec{R}(\vec{r}_0\vec{p}_0\vec{s},t_0) = \vec{r}_0$ and $\vec{P}(\vec{r}_0\vec{p}_0\vec{s},t_0) = \vec{p}_0$

find the new phase space coordinates $R(\vec{r}_0\vec{p}_0\vec{s},t)$ and $\vec{P}(\vec{r}_0\vec{p}_0\vec{s},t)$ at $t = t_0 + \delta_t$

Substitute into the spin-dependent Boltzmann equation

$$\left[-\frac{\partial\vec{R}(\vec{r}_0\vec{p}_0\vec{s},t)}{\partial t} + \frac{\partial\varepsilon}{\partial\vec{p}}\right]\cdot\frac{\partial f^{\pm}(\vec{r},\vec{p},t)}{\partial\vec{r}} \pm \frac{\partial V_{hn}}{\partial\vec{p}}\cdot\frac{\partial f^{\pm}(\vec{r},\vec{p},t)}{\partial\vec{r}} = 0,$$

$$\Rightarrow \quad \frac{\partial\vec{R}(\vec{r}_0\vec{p}_0\vec{s},t)}{\partial t} = \frac{\partial\varepsilon}{\partial\vec{p}} \pm \frac{\partial V_{hn}}{\partial\vec{p}},$$

$$f^{\pm}(\vec{r}_0,\vec{p}_0,t_0)\left\{\frac{-i\vec{s}}{\hbar}\cdot\frac{\partial\vec{P}(\vec{r}_0\vec{p}_0\vec{s},t)}{\partial t} - \frac{[\varepsilon(\vec{r}-\frac{\vec{s}}{2},t)-\varepsilon(\vec{r}+\frac{\vec{s}}{2},t)]}{i\hbar}\right\} \mp f^{\pm}(\vec{r}_0,\vec{p}_0,t_0)$$

$$\times\left\{\frac{V_{hn}(\vec{r}-\frac{\vec{s}}{2},t)-V_{hn}(\vec{r}+\frac{\vec{s}}{2},t)}{i\hbar}\right\} = 0.$$

Cut higher-order terms as in C.Y. Wong's paper

$$\Rightarrow \quad \frac{\partial\vec{P}(\vec{r}_0\vec{p}_0\vec{s},t)}{\partial t} = -\frac{\partial\varepsilon}{\partial\vec{r}} \mp \frac{\partial V_{hn}}{\partial\vec{r}}.$$

with $V_{hn} = \vec{n}\cdot\vec{h}$

Y. Xia, JX, B.A. Li, and W.Q. Shen, Phys. Lett. B (2016)

Equations of motion III

$$\frac{\partial \vec{R}}{\partial t} = \frac{\vec{p}}{m} + \nabla_{\vec{p}}(h_1 + h_4) \pm \nabla_{\vec{p}}(\vec{h}_2 \cdot \vec{n} + \vec{h}_3 \cdot \vec{n}),$$

$$\frac{\partial \vec{P}}{\partial t} = -\nabla_{\vec{r}} U_q - \nabla_{\vec{r}}(h_1 + h_4) \mp \nabla_{\vec{r}}(\vec{h}_2 \cdot \vec{n} + \vec{h}_3 \cdot \vec{n}),$$

$$\frac{\partial \vec{n}}{\partial t} = \frac{2(\vec{h}_2 + \vec{h}_3) \times \vec{n}}{\hbar},$$

upper sign for f^+ and lower sign for f^-

precession

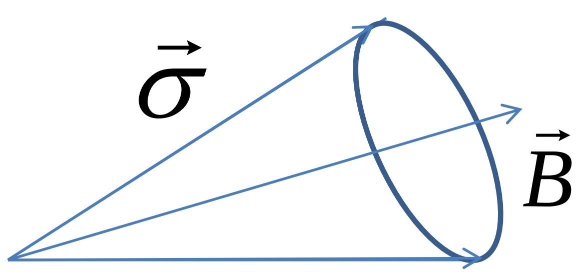
$$\varepsilon \sim -\vec{\sigma} \cdot \vec{B}$$

Similar to the canonical equation and Heisenburg picture of quantum mechanics

$$\frac{d\vec{r}}{dt} = \nabla_p \varepsilon$$

$$\frac{d\vec{p}}{dt} = -\nabla_r \varepsilon$$

$$\frac{d\vec{\sigma}}{dt} = \frac{1}{i}[\vec{\sigma}, \varepsilon]$$

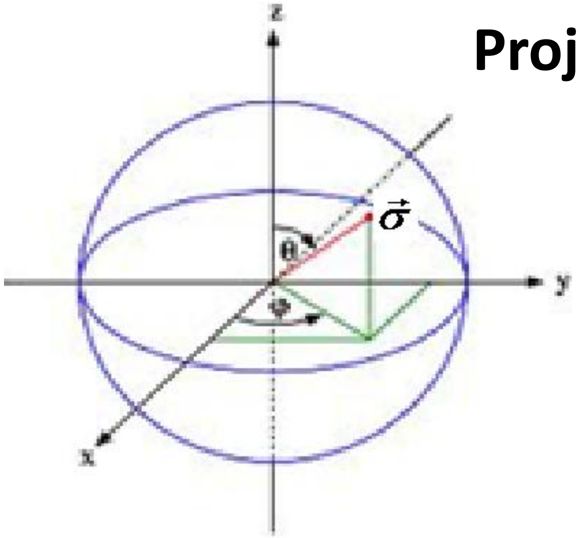


$$\frac{d\vec{\sigma}}{dt} \sim \vec{\sigma} \times \vec{B}$$

$$\vec{n} \sim \vec{\sigma}$$

$\vec{\sigma}$ a unit vector for each nucleon

expectation value of the spin $\vec{s} = \frac{\hbar}{2}\vec{\sigma}$



Projection in y direction (total angular momentum)

$$\sigma_y = \sin\theta \sin\varphi$$

$(1+\sigma_y)/2$ **probability** $s_y = \frac{\hbar}{2}$ **Spin-up**

$(1-\sigma_y)/2$ **probability** $s_y = -\frac{\hbar}{2}$ **Spin-down**

Spin- and isospin-dependent phase space distribution function

$$\tilde{f}_{\sigma\tau}(ix, iy, iz, ipx, ipy, ipz)$$

spin- and isospin-dependent Pauli blocking

$$n_{occup} = \frac{h^3}{d * dx * dy * dz * dpx * dpy * dpz}\tilde{f}_{\sigma\tau}(ix, iy, iz, ipx, ipy, ipz), d = 1$$

Nucleon spin may flip after nucleon-nucleon scattering (randomized?)

Other approaches for spin transport

1) Adiabatic approximation for spin

$$\vec{n} \approx -\vec{h}_0 - \frac{\hbar}{2|\vec{h}|}\vec{h}_0 \times \frac{d\vec{h}_0}{dt} \quad \text{with} \quad \vec{h}_0 = \vec{h}/|\vec{h}|$$

Solve spin up to the first order

Equations of motion:

$$\dot{\vec{r}} = \frac{\vec{p}}{m} + \nabla_p(\varepsilon + |\vec{h}|) + \hbar\Omega_{pr} \cdot \dot{\vec{r}} + \hbar\Omega_{pp} \cdot \dot{\vec{p}}$$

$$\dot{\vec{p}} = -\nabla_r(\varepsilon + |\vec{h}|) - \hbar\Omega_{rr} \cdot \dot{\vec{r}} - \hbar\Omega_{rp} \cdot \dot{\vec{p}}$$

Berry curvature

$$\Omega_{AB}^{ij} = \frac{1}{2}\left(\frac{\partial \vec{h}_0}{\partial A_i} \times \frac{\partial \vec{h}_0}{\partial B_j}\right) \cdot \vec{h}_0$$

G. Sundaram and Q. Niu, Phys. Rev. B, 1999;
D. Xiao, M.C. Chang, and Q. Niu, Rev. Mod. Phys., 2010;
X.G. Huang, Sci. Rep., 2016

2) Relaxation time approach

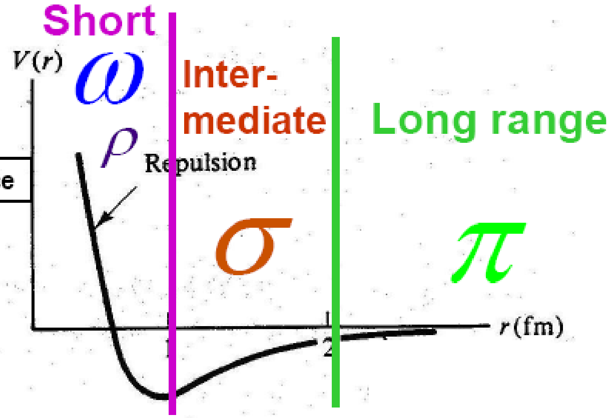
$$I_c = -\frac{f_0\hat{I} - \langle f_0\hat{I}\rangle}{\tau_0} - \frac{g - \langle g\rangle}{\tau_{sf}}$$

τ_0 relaxation time between scatterings

τ_{sf} relaxation time for spin flipping

G. Stirnati et al., Phys. Rev. B, 1989; T. Valet and A. Fert, Phys. Rev. B, 1993;
J.W. Zhang et al., Phys. Rev. Lett., 2004; K. Morawetz, Phys. Rev. B, 2015

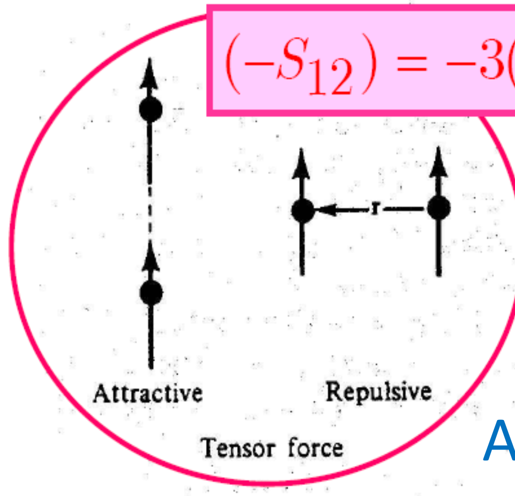
Tensor force



Tensor force π ρ

Spin-orbit force ω σ

$$(-S_{12}) = -3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) + \vec{\sigma}_1 \cdot \vec{\sigma}_2$$



Another interaction related to nucleon spin

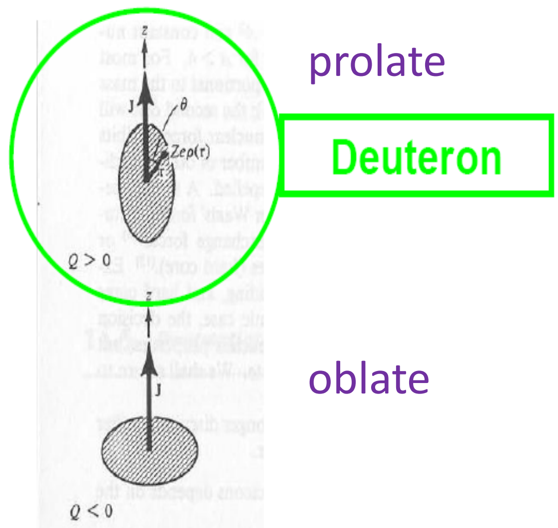
$\pi(138)$ $V_\pi = \frac{f_{\pi NN}^2}{3m_\pi^2}\frac{\vec{q}^2}{\vec{q}^2+m_\pi^2}\left[-\vec{\sigma}_1\cdot\vec{\sigma}_2 - S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$ — Long-ranged tensor force

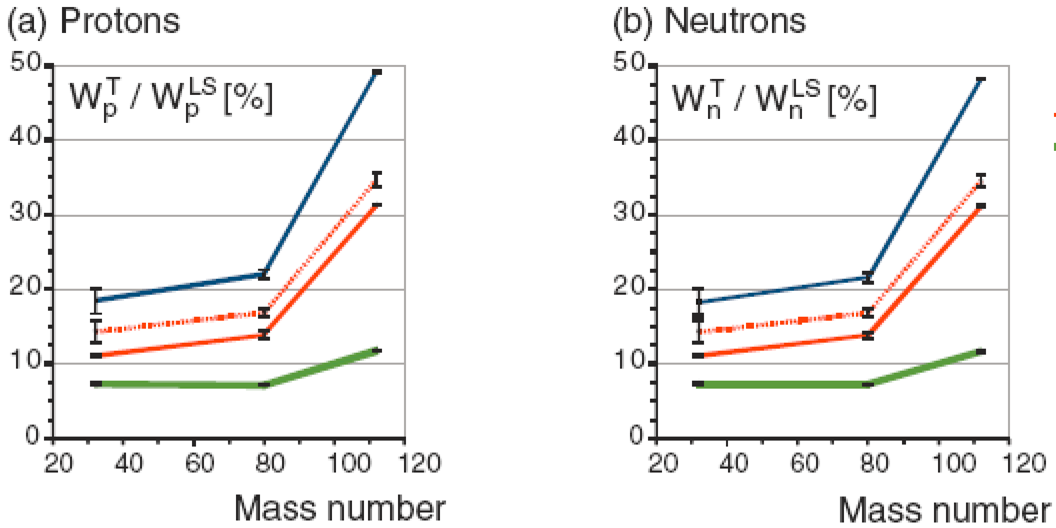
$\sigma(600)$ $V_\sigma \approx \frac{g_\sigma^2}{\vec{q}^2+m_\sigma^2}\left[-1-\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$ — intermediate-ranged, attractive central force plus LS force

$\omega(782)$ $V_\omega \approx \frac{g_\omega^2}{\vec{q}^2+m_\omega^2}\left[+1-3\frac{\vec{L}\cdot\vec{S}}{2M^2}\right]$ — short-ranged, repulsive central force plus strong LS force

$\rho(770)$ $V_\rho = \frac{f_\rho^2}{12M^2}\frac{\vec{q}^2}{\vec{q}^2+m_\rho^2}\left[-2\vec{\sigma}_1\cdot\vec{\sigma}_2 + S_{12}(\hat{q})\right]\vec{\tau}_1\cdot\vec{\tau}_2$ — short-ranged tensor force, opposite to pion

Tensor Force: First evidence from the deuteron





Y. Iwata and J.A. Maruhn, Phys. Rev. C, 2011

Force	Threshold (MeV)
SkM* (basic)	77
SkM* (inc. J^2)	71
SkM* (full)	73
SLy5 (full)	68
SLy5t	65
$T12$	61
$T14$	69
$T22$	64
$T24$	71
$T26$	82
$T42$	69
$T44$	79
$T46$	87

TABLE I. Upper fusion threshold energies for the ^{16}O + ^{16}O collision using various parameterizations of the Skyrme interaction.

P.D. Stevenson et al., arXiv: 1507.00645 [nucl-th]

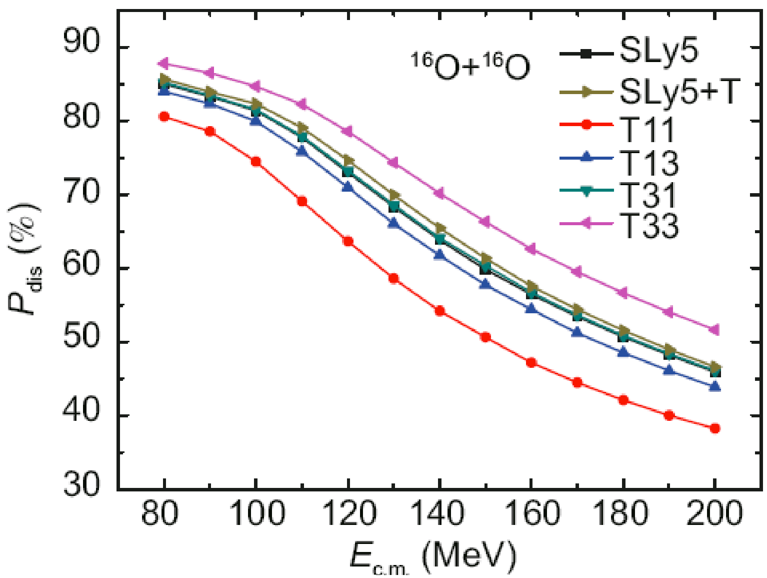


Figure 2 (Color online) Percentage of energy dissipation as a function of initial c.m. energy for head-on collisions of $^{16}O+^{16}O$ with the six Skyrme parameter sets.

G.F. Dai, L. Guo, E.G. Zhao, and S.G. Zhou, Sci China-Phys Mech Astron, 2014

Add tensor force to IBUU?

Skyrme-type tensor force:

$$v_T = \frac{t_e}{2}\{\left[3\left(\vec{\sigma}_1 \cdot \vec{k}'\right)\left(\vec{\sigma}_2 \cdot \vec{k}'\right) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)k'^2\right]\delta(\vec{r})$$

$$+\delta(\vec{r})\left[3\left(\vec{\sigma}_1 \cdot \vec{k}\right)\left(\vec{\sigma}_2 \cdot \vec{k}\right) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)k^2\right]\}$$

$$+t_0\left[3\left(\vec{\sigma}_1 \cdot \vec{k}'\right)\delta(\vec{r})\left(\vec{\sigma}_2 \cdot \vec{k}\right) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\vec{k}' \cdot \delta(\vec{r})\vec{k}\right]$$

Hartree-Fock framework:

$$E_T = \frac{1}{2}\sum_{i,j}\langle ij \,|\, v_T(1 - P_r P_\sigma P_\tau) \,|\, ij\rangle = \int H_T(\vec{r})d^3r$$

$$\frac{\delta H_T}{\delta \varphi_i^*}\varphi_i \sim h_T \varphi_i$$

$$s_\mu = \sum_i \varphi_i^* \sigma_\mu \varphi_i$$ **Spin density**

$$T_\mu = \sum_i \nabla \varphi_i^* \cdot \nabla \varphi_i \sigma_\mu$$ **Spin kinetic density**

$$J_{\mu\nu} = \frac{1}{2i} \sum_i \sigma_\nu \left(\varphi_i^* \nabla_\mu \varphi_i - \nabla_\mu \varphi_i^* \varphi_i \right)$$ **Spin current density**

$$F_\mu = \frac{1}{2} \sum_i \sigma_\nu \left(\nabla_\nu \varphi_i^* \nabla_\nu \varphi_i + \nabla_\mu \varphi_i^* \nabla_\nu \varphi_i \right)$$ **Pseudovector tensor kinetic density**

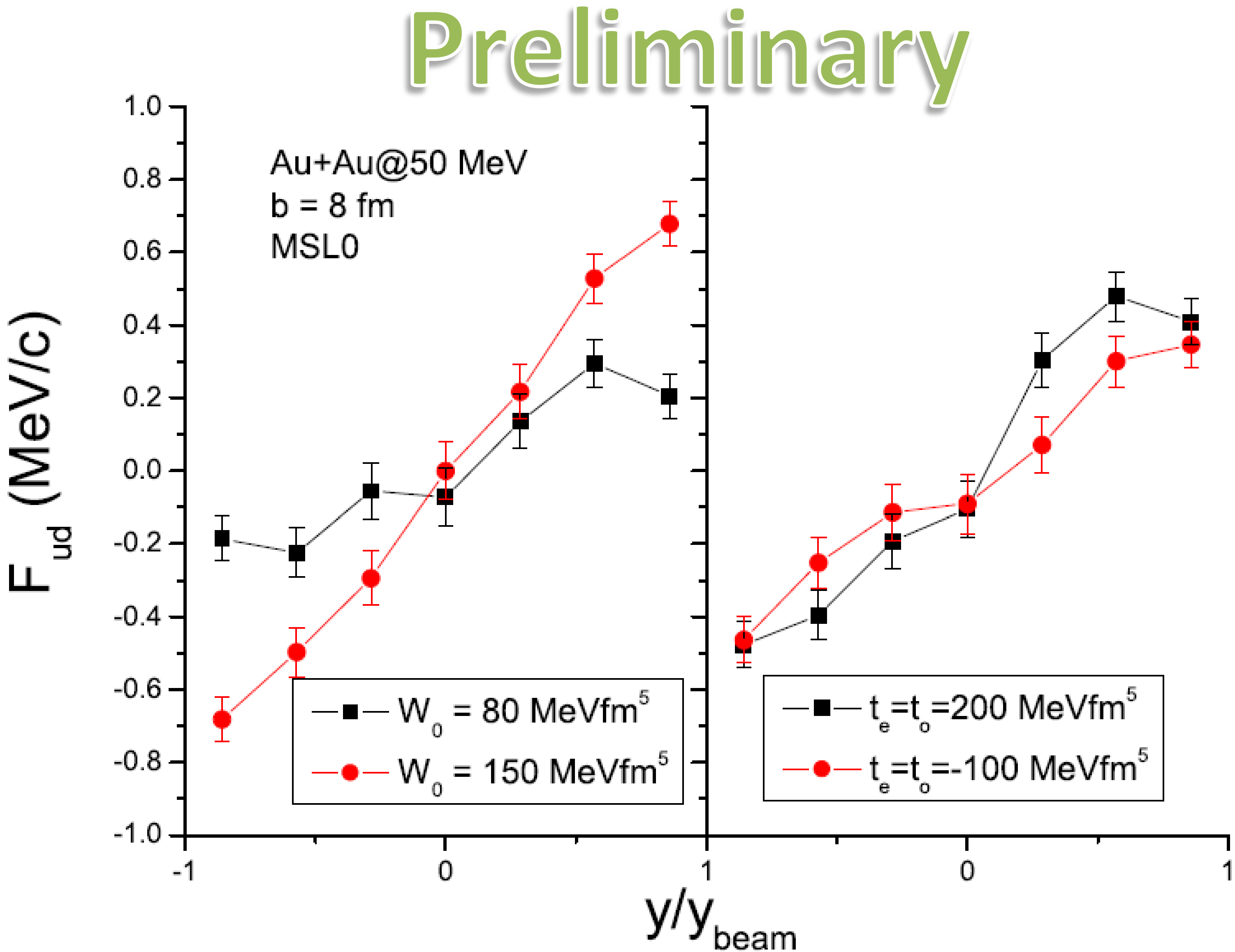
Only consider vector component of $J_{\mu\nu}$ $\quad J_{\mu\mu}^2 = 0, J_{\mu\nu} J_{\mu\nu} = \frac{1}{2} J^2, J_{\mu\nu} J_{\nu\mu} = -\frac{1}{2} J^2$

Potential energy density

$$\begin{aligned} H_T = {} & \frac{3}{16}(3t_e - t_o)(\nabla \cdot \vec{s})^2 - \frac{3}{16}(3t_e + t_o)\sum_q (\nabla \cdot \vec{s}_q)^2 \\ & - \frac{1}{4}(t_e + t_o)\left(\vec{s} \cdot \vec{T} - \frac{1}{2} J^2 \right) + \frac{1}{4}(t_e - t_o)\sum_q \left(\vec{s}_q \cdot \vec{T}_q - \frac{1}{2} J_q^2 \right) \\ & + \frac{3}{4}(t_e + t_o)\left(\vec{s} \cdot \vec{F} + \frac{1}{4} J^2 \right) - \frac{3}{4}(t_e - t_o)\sum_q \left(\vec{s}_q \cdot \vec{F}_q + \frac{1}{4} J_q^2 \right) \\ & + \frac{1}{16}(3t_e - t_o)\vec{s} \cdot \nabla^2 \vec{s} - \frac{1}{16}(3t_e + t_o)\sum_q \vec{s}_q \nabla^2 \vec{s}_q \rightarrow h_T \rightarrow \text{Equation of motion} \end{aligned}$$

$$q = n, p$$

Spin-orbit interaction+tensor interaction

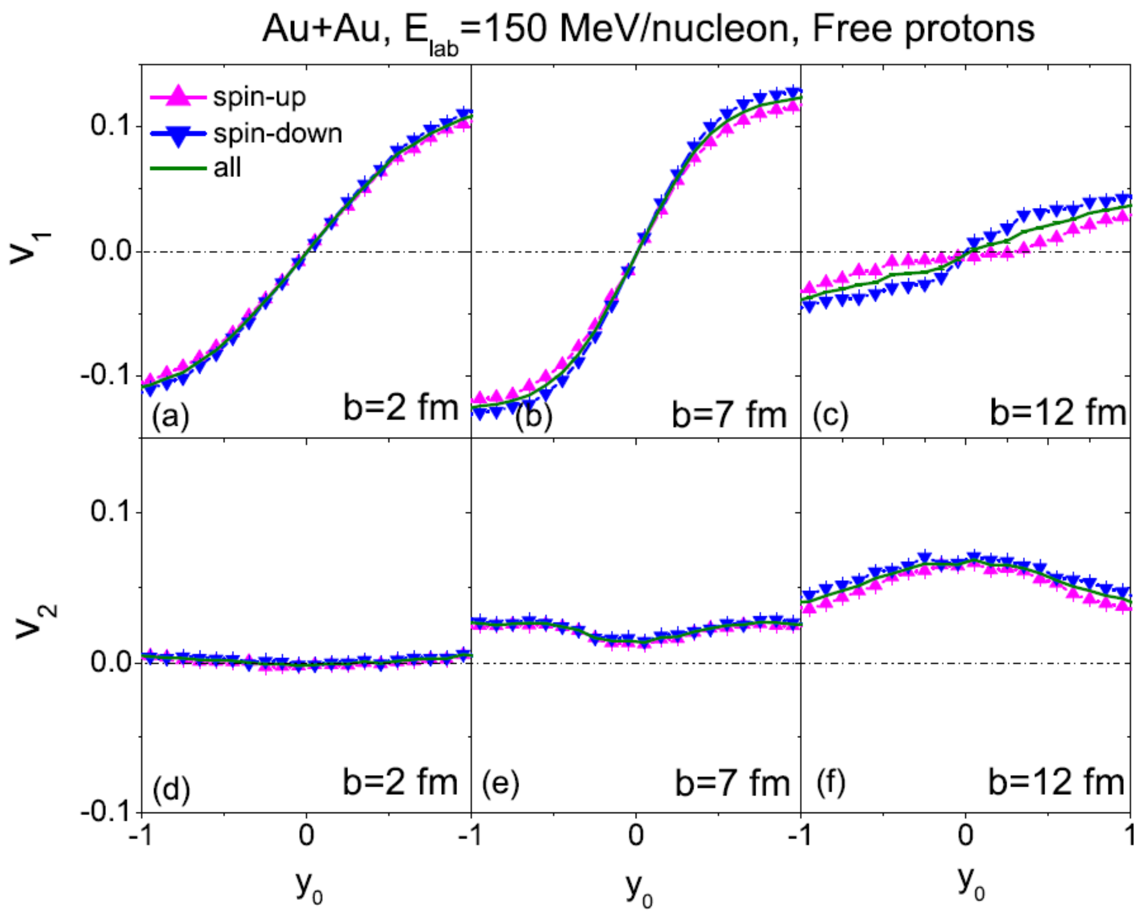


Spin dynamics from QMD model

$$u_{so}^{even} = -\frac{1}{2}W_0(\rho\nabla\cdot\vec{J} + \rho_n\nabla\cdot\vec{J}_n + \rho_p\nabla\cdot\vec{J}_p)$$

$$u_{so}^{odd} = -\frac{1}{2}W_0[\vec{s}\cdot(\nabla\times\vec{j}) + \vec{s}_n\cdot(\nabla\times\vec{j}_n) + \vec{s}_p\cdot(\nabla\times\vec{j}_p)]$$

C.C. Guo, Y.J. Wang, Q.F. Li, and F.S. Zhang, Phys. Rev. C 90, 034606 (2014)



$$\rho(\vec{r}) = \sum_i \rho_i(\vec{r}) = \sum_i \frac{1}{(2\pi L)^{3/2}} e^{[-(\vec{r}-\vec{r}_i)^2/(2L)]}$$

$$\vec{s}(\vec{r}) = \sum_i \rho_i(\vec{r})\vec{\sigma}_i,$$

$$\vec{j}(\vec{r}) = \sum_i \rho_i(\vec{r})\vec{p}_i,$$

$$\vec{J}(\vec{r}) = \sum_i \rho_i(\vec{r})\vec{p}_i \times \vec{\sigma}_i,$$

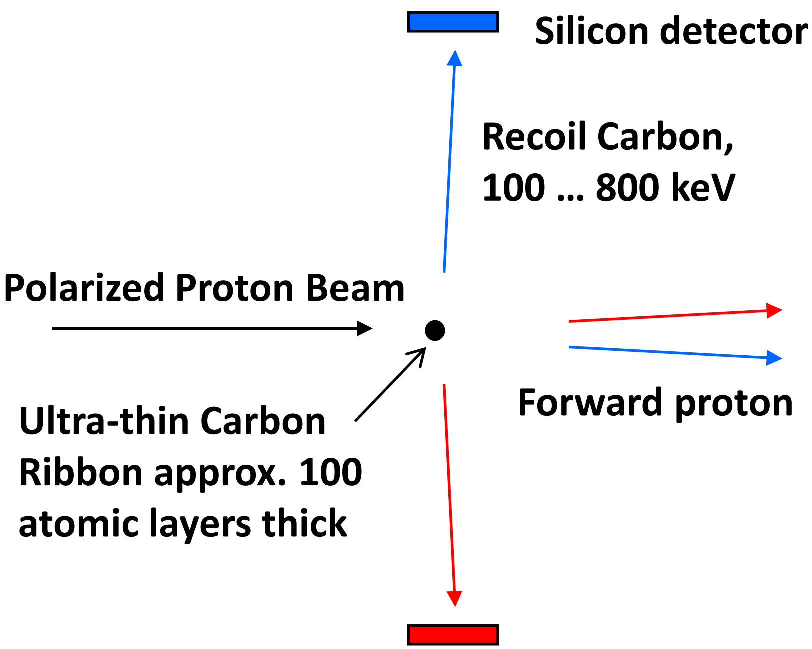
Effects of SO coupling on spin dynamics are robust and model independent.

Analyzing power measurement at AGS and RHIC

$$A = \frac{\sqrt{LU}\sqrt{RD} - \sqrt{LD}\sqrt{RU}}{\sqrt{LU}\sqrt{RD} + \sqrt{LD}\sqrt{RU}}$$

The analyzing power can be as large as 100% at certain angles and energies

Providing a possible way of identifying nucleon spin



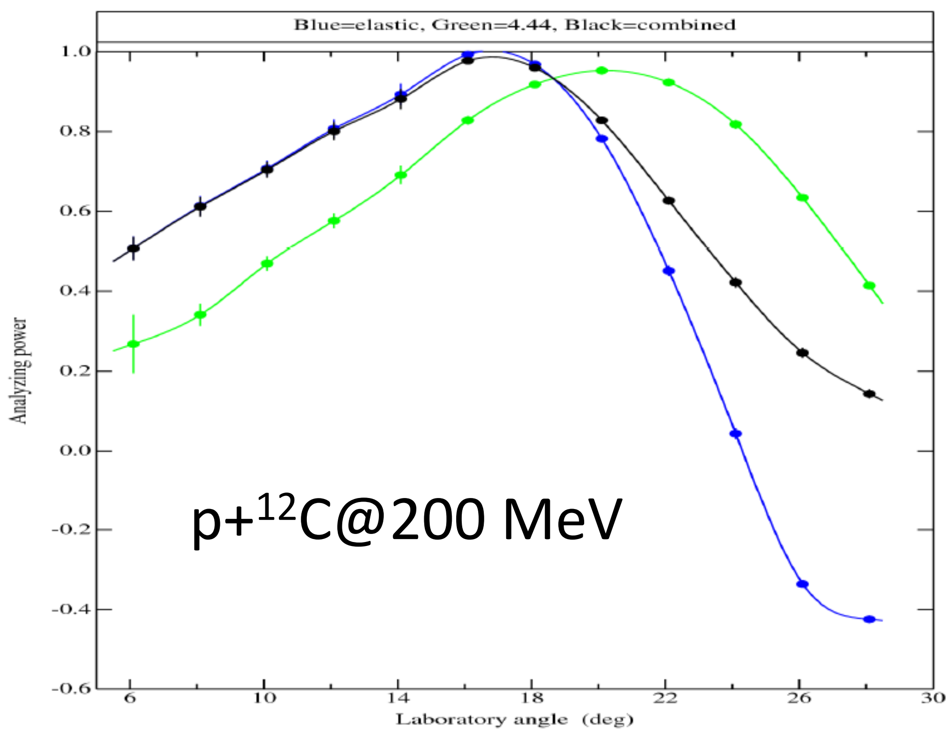


Figure 1: Measurements of the analyzing power for proton scattering from ^{12}C at 200 MeV. The blue (green) curves correspond to protons exiting from the ground (4.44-MeV) state. The black curve represents the sum of the two data sets / 7/.

Code	Evaporation	User	Author	Ref.
Statistical Multifragmentati				
ISMM-c	MSU-decay	Tsang	Das Gupta	[2]
ISMM-m	MSU-decay	Souza	Souza	[13, 14]
SMM95	own code	Bougault	Botvina	[4, 9]
MMM1	own code	AH Raduta	AH Raduta	[15]
MMM2	own code	AR Raduta	AR Raduta	[15]
MMMC	own code	Le Fèvre	Gross	[5, 16]
LGM	N/A	Regnard	Gulminelli	[17]
QSM	own code	Trautmann	Stöcker	[18]
EES	EES	Friedman	Friedman	[7, 8]
BNV-box	N/A	Colonna	Colonna	[24]
Evaporation code				
Gemini		Charity	Charity	[25]
Gemini-w		Wada	Wada	[25–28]
SIMON		Durand	Durand	[29]
EES		Friedman	Friedman	[7, 8]
MSU-decay		Tsang	Tan *et al.*	[14]

Different statistical multifragmentation models and evaporation codes

Different approaches for multifragmentation and cluster deexictation

Taken from M.B. Tsang et al., EPJA (2006)

The multiplicity of a M-nucleon cluster

$$\frac{dN_M}{d^3K} = G \binom{A}{M} \binom{M}{Z} \frac{1}{A^M} \int \left[\prod_{i=1}^{Z} f_p(\mathbf{r}_i, \mathbf{k}_i)\right] \left[\prod_{i=Z+1}^{M} f_n(\mathbf{r}_i, \mathbf{k}_i)\right]$$
$$\times \rho^W(\mathbf{r}_{i_1}, \mathbf{k}_{i_1}, \ldots, \mathbf{r}_{i_{M-1}}, \mathbf{k}_{i_{M-1}}) \, \delta\big(\mathbf{K} - (\mathbf{k}_1 + \cdots + \mathbf{k}_M)\big) \, d\mathbf{r}_1 \, d\mathbf{k}_1 \cdots d\mathbf{r}_M \, d\mathbf{k}_M$$

R. Mattiello et al., Phys. Rev. Lett 1995 Phys. Rev. C 1997.

ρ^W **the Wigner phase-space density of the M-nucleon cluster**

Spatial wave function: s-wave assumption

Statistical factor
Only asymmetric spin-isospin allowed

G : coalescence with a given isospin
G': coalescence with a given spin and isospin

3/8 ,d
1/12, t
1/12, ^{3}He

$^2_1H(S=1)$ G'	$^3_1H(S=1/2)$ G'	$^3_2He(S=1/2)$ G'
$p\uparrow \& n\uparrow \longrightarrow 1/2\ (S_z=1)$	$p\uparrow \& n\uparrow \& n\downarrow \longrightarrow 1/6(S_z=+1/2)$	$n\uparrow \& p\uparrow \& p\downarrow \longrightarrow 1/6(S_z=+1/2)$
$p\uparrow \& n\downarrow \longrightarrow 1/4\ (S_z=0)$		
$p\downarrow \& n\uparrow \longrightarrow 1/4\ (S_z=0)$	$p\downarrow \& n\uparrow \& n\downarrow \longrightarrow 1/6(S_z=-1/2)$	$n\downarrow \& p\uparrow \& p\downarrow \longrightarrow 1/6(S_z=-1/2)$
$p\downarrow \& n\downarrow \longrightarrow 1/2\ (S_z=-1)$		

Wigner phase-space density

deuteron

$$\rho_d^W(\mathbf{r},\mathbf{k}) = \int \phi\left(\mathbf{r}+\frac{\mathbf{R}}{2}\right)\phi^*\left(\mathbf{r}-\frac{\mathbf{R}}{2}\right)\exp(-i\mathbf{k}\cdot\mathbf{R})\,d\mathbf{R},$$

$$\mathbf{k} = (\mathbf{k}_1 - \mathbf{k}_2)/2 \qquad \mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)$$

Internal wave function $\phi(r)$ ⇨ root-mean-square radius of 1.96 fm

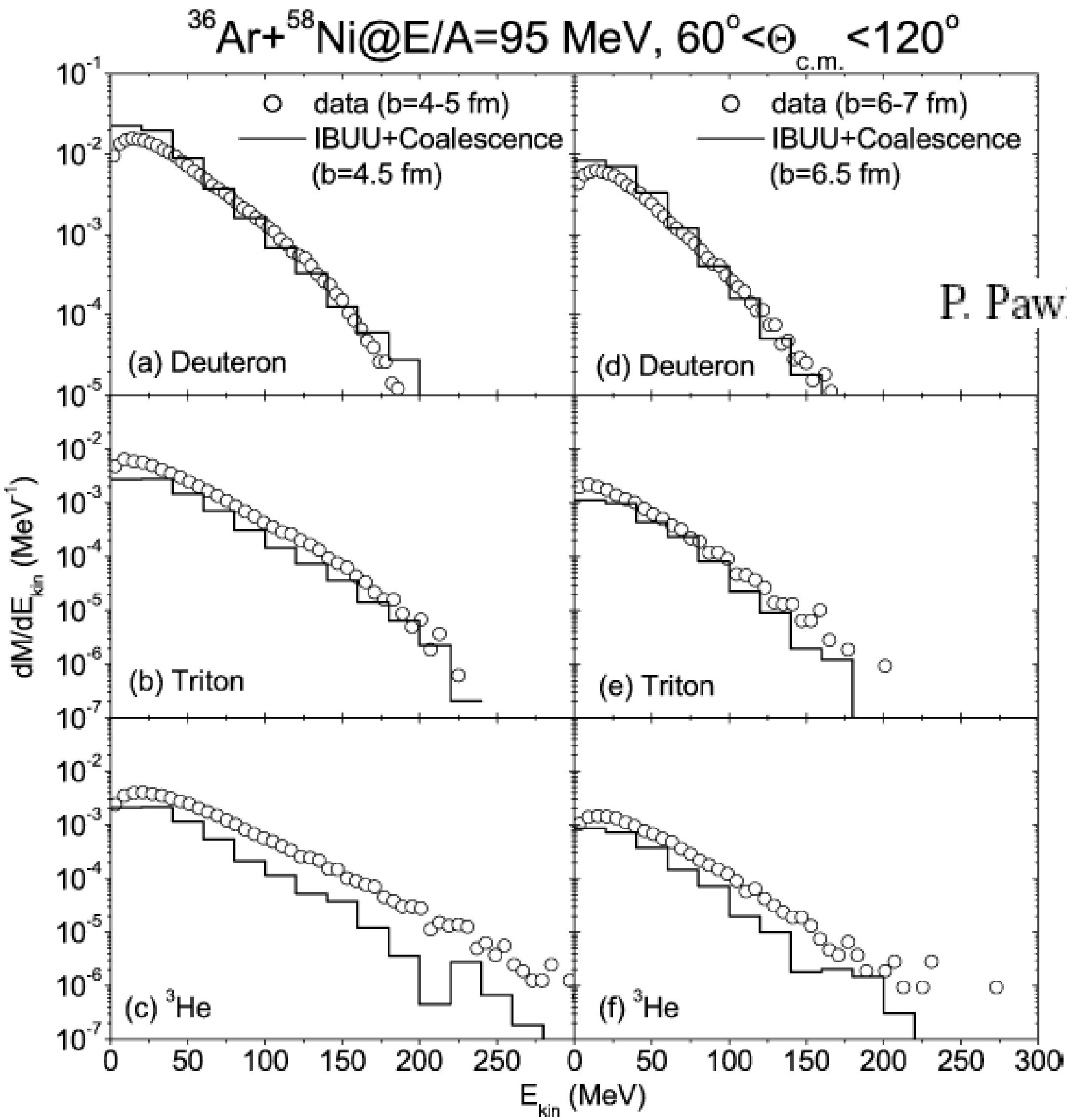
Triton or Helium3

$$\rho^W_{t(^3\mathrm{He})}(\rho,\lambda,\mathbf{k}_\rho,\mathbf{k}_\lambda) = \int \psi\left(\rho+\frac{\mathbf{R}_1}{2},\lambda+\frac{\mathbf{R}_2}{2}\right)\psi^*\left(\rho-\frac{\mathbf{R}_1}{2},\lambda-\frac{\mathbf{R}_2}{2}\right) \times \exp(-i\mathbf{k}_\rho\cdot\mathbf{R}_1)\exp(-i\mathbf{k}_\lambda\cdot\mathbf{R}_2)3^{3/2}\,d\mathbf{R}_1\,d\mathbf{R}_2$$

$$\begin{pmatrix}\mathbf{R}\\ \rho\\ \lambda\end{pmatrix} = J\begin{pmatrix}\mathbf{r}_1\\ \mathbf{r}_2\\ \mathbf{r}_3\end{pmatrix} \quad J = \begin{pmatrix}\frac{1}{3} & \frac{1}{3} & \frac{1}{3}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}}\end{pmatrix} \quad \begin{pmatrix}\mathbf{K}\\ \mathbf{k}_\rho\\ \mathbf{k}_\lambda\end{pmatrix} = J^{-,+}\begin{pmatrix}\mathbf{k}_1\\ \mathbf{k}_2\\ \mathbf{k}_3\end{pmatrix} \quad J^{-,+} = \begin{pmatrix}1 & 1 & 1\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}}\end{pmatrix}$$

Internal wave function $\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ ⇨ RMS radius 1.61 and 1.74 fm for triton and ^{3}He.

Light cluster production from coalescence using Wigner function method



reproduce experimental data from
P. Pawlowski, et al., Eur. Phys. J. A 9 (2000) 371
reasonably well

Suitable for loosely bound clusters

Suitable for rare particles:
Perturbative treatment

L.W. Chen, B.A. Li, and C.M. Ko, NPA (2003)

^2_1H wave function

$$\left|{}^2_1H\right\rangle \sim \underbrace{\left|spin\right\rangle}_{S=1} \underbrace{\left|isospin\right\rangle}_{T=0}$$

	S	T	
$S=1$	$\uparrow\uparrow$	pp	$T=1$
	$\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)$	$\frac{1}{\sqrt{2}}(pn+np)$	
	$\downarrow\downarrow$	nn	
$S=0$	$\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)$	$\frac{1}{\sqrt{2}}(pn-np)$	$T=0$

Assign all many-nucleon states which are allowed from the Pauli principle the same weight.

8 wave function (considering the spin-isospin and antisymmetrization), 3 of 8 are feasible.

G= 3/8 (no information about spin)

S_z = +1: $\psi_1 \sim \frac{1}{\sqrt{2}}(p\uparrow n\uparrow - n\uparrow p\uparrow)$

S_z = 0: $\psi_2 \sim \frac{1}{2}(p\uparrow n\downarrow + p\downarrow n\uparrow - n\uparrow p\downarrow - n\downarrow p\uparrow)$

S_z = -1: $\psi_3 \sim \frac{1}{\sqrt{2}}(p\downarrow n\downarrow - n\downarrow p\downarrow)$

$$\psi_4 \sim \frac{1}{2}(p\uparrow n\downarrow - p\downarrow n\uparrow - n\uparrow p\downarrow + n\downarrow p\uparrow)$$

$$\psi_5 \sim \frac{1}{\sqrt{2}}(p\uparrow n\uparrow + n\uparrow p\uparrow)$$

$$\psi_6 \sim \frac{1}{2}(p\uparrow n\downarrow + p\downarrow n\uparrow + n\uparrow p\downarrow + n\downarrow p\uparrow)$$

$$\psi_7 \sim \frac{1}{\sqrt{2}}(p\downarrow n\downarrow + n\downarrow p\downarrow)$$

$$\psi_8 \sim \frac{1}{2}(p\uparrow n\downarrow - p\downarrow n\uparrow + n\uparrow p\downarrow - n\downarrow p\uparrow)$$

$p\uparrow \& n\uparrow \longrightarrow G' = 1/2 (S_z = +1)$

$p\uparrow \& n\downarrow \longrightarrow G' = 1/4 (S_z = 0)$

$p\downarrow \& n\uparrow \longrightarrow G' = 1/4 (S_z = 0)$

$p\downarrow \& n\downarrow \longrightarrow G' = 1/2 (S_z = -1)$

$^{3}_{1}H$ & $^{3}_{2}He$ **wave function**

S=1/2 **T=1/2**

$$\left|{}^{3}_{1}H / {}^{3}_{2}He\right\rangle \sim \left|spin\right\rangle \left|isospin\right\rangle \qquad S_\rho T_\lambda - S_\lambda T_\rho$$

	S	**T**	
$S=3/2$	$\uparrow\uparrow\uparrow$	ppp	$T=3/2$
	$\frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow+\downarrow\uparrow\uparrow+\uparrow\downarrow\uparrow)$	$\frac{1}{\sqrt{3}}(ppn+npp+pnp)$	
	$\frac{1}{\sqrt{3}}(\downarrow\downarrow\uparrow+\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow)$	$\frac{1}{\sqrt{3}}(nnp+pnn+npn)$	
	$\downarrow\downarrow\downarrow$	nnn	
$S=1/2$ ρ	$\frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow-\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)$	$\frac{1}{\sqrt{6}}(2ppn-pnp-npp)$	$T=1/2$
	$\frac{1}{\sqrt{6}}(\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow-2\downarrow\downarrow\uparrow)$	$\frac{1}{\sqrt{6}}(pnn+npn-2nnp)$	
$S=1/2$ λ	$\frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow)$	$\frac{1}{\sqrt{2}}(pnp-npp)$	$T=1/2$
	$\frac{1}{\sqrt{2}}(\uparrow\downarrow\downarrow-\downarrow\uparrow\downarrow)$	$\frac{1}{\sqrt{2}}(pnn-npn)$	

$\{S({}^{3}_{1}H)=1/2\ \&\ S({}^{3}_{2}He)=1/2\}$

24 wave function (considering the spin- isospin and antisymmetrization), 2 of 24 are feasible.

G= 1/12 (no information of spin)

$(S_z=+1/2)$

$$\Psi_1({}^3_2\mathrm{He}) \sim \frac{1}{\sqrt{6}}(p^\uparrow n^\uparrow p^\downarrow - p^\downarrow n^\uparrow p^\uparrow - n^\uparrow p^\uparrow p^\downarrow + n^\uparrow p^\uparrow p^\downarrow - p^\uparrow p^\downarrow n^\uparrow + p^\downarrow p^\uparrow n^\uparrow),$$

$(S_z=-1/2)$

$$\Psi_2({}^3_2\mathrm{He}) \sim \frac{1}{\sqrt{6}}(p^\uparrow n^\downarrow p^\downarrow - p^\downarrow n^\downarrow p^\uparrow - n^\downarrow p^\uparrow p^\downarrow + n^\downarrow p^\uparrow p^\downarrow - p^\uparrow p^\downarrow n\downarrow + p^\downarrow p^\uparrow n^\downarrow).$$

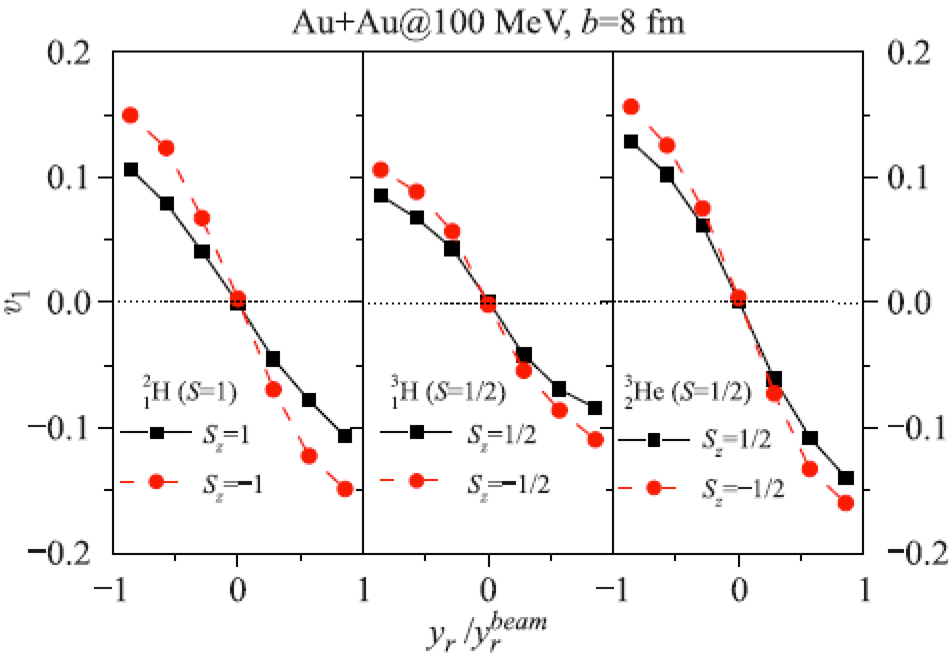
Another 5 states with same spin-isospin states but do not satisfy wave function antisymmetrization

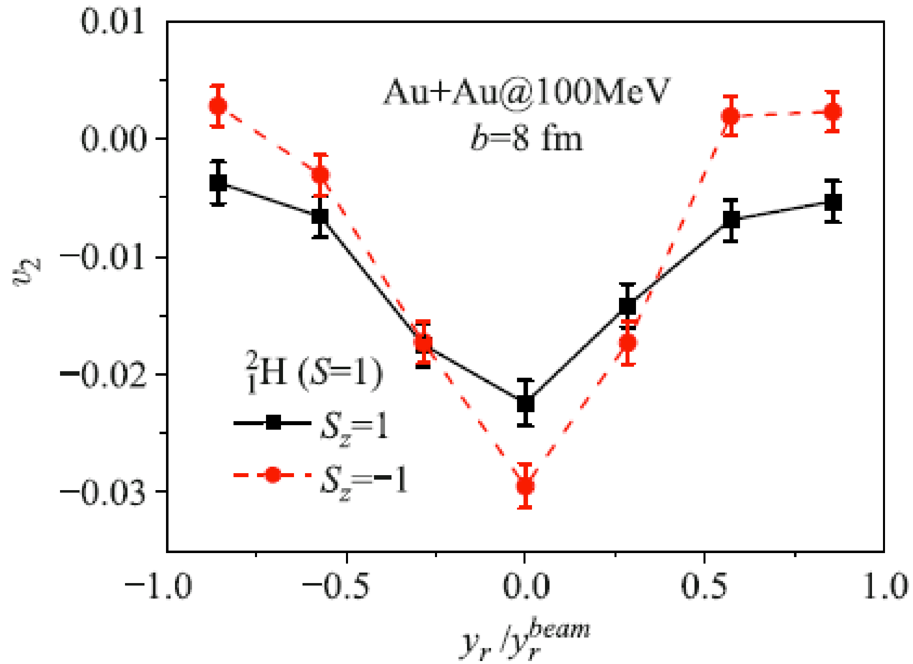
$^{3}_{2}He$ $\qquad G'$

$n\uparrow \& p\uparrow \& p\downarrow \longrightarrow 1/6(S_z=+1/2)$

$n\downarrow \& p\uparrow \& p\downarrow \longrightarrow 1/6(S_z=-1/2)$

Similar for $^{3}_{1}He$





Spin splitting of light clusters collective flows observed

Useful probe of SO coupling