Potential energy surfaces of super-heavy nuclei in the 4D Fourier parameter space

Krzysztof Pomorski
Maria Curie Skłodowska University, Lublin, Poland

Seminar given at the IHIC-2018, Tsinghua University, Beijing

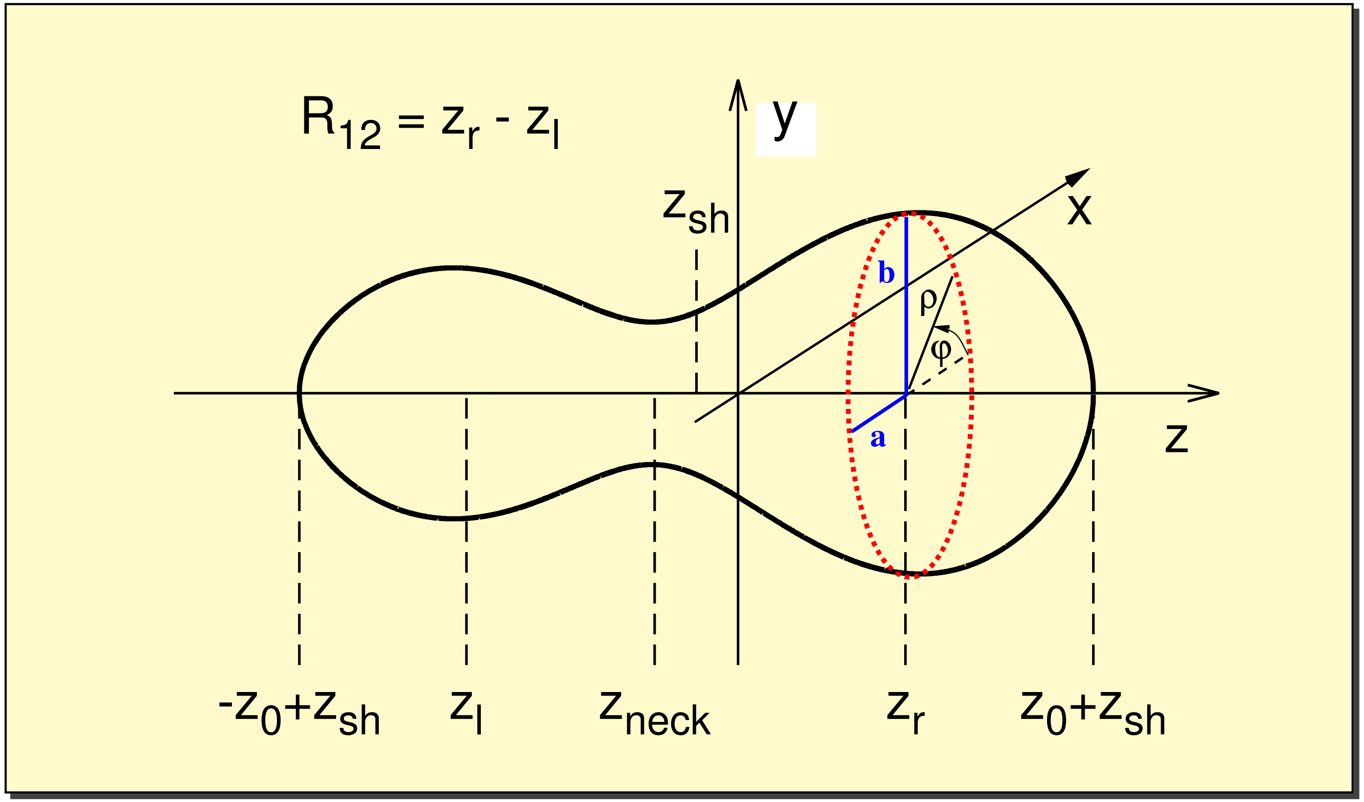
My collaborators:

- **Bożena Nerlo-Pomorska**, Maria Curie Skłodowska University, Lublin,
- **Johann Bartel**, IPHC and University of Strasbourg,
- **Christelle Schmitt**, IPHC, Strasbourg.

Program:

- Fourier parametrization of nuclear shapes,
- Potential energy surfaces of heavy and super-heavy nuclei,
- Systematics of the ground state deformations,
- Fission barrier height and Q_α systematics,
- α-decay and spontaneous fission half-lives in a simple WKB model,
- Summary.

Fourier expansion of nuclear shapes *



$$(x, y, z) \to (\rho, \varphi, z)$$

Nonaxial shapes:

$$\eta = \frac{b-a}{a+b}$$

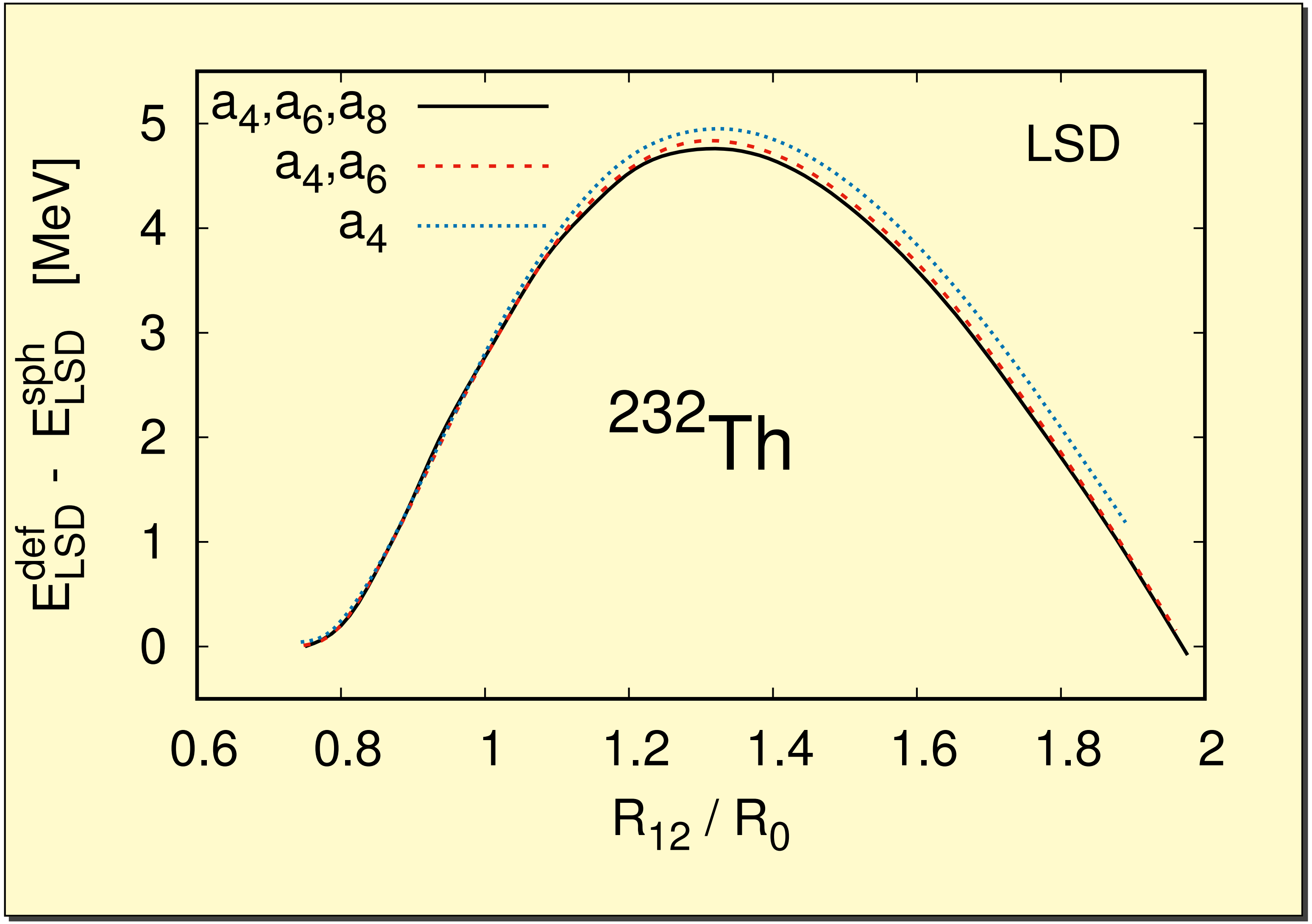
$$\rho_s^2(z) = a(z)b(z)$$

$$\frac{\rho_s^2(z)}{R_0^2} = \sum_{n=1}^{\infty} \left[a_{2n} \cos\left(\frac{(2n-1)\pi}{2} \frac{z - z_{sh}}{z_0} \right) + a_{2n+1} \sin\left(\frac{2n\pi}{2} \frac{z - z_{sh}}{z_0} \right) \right] ,$$

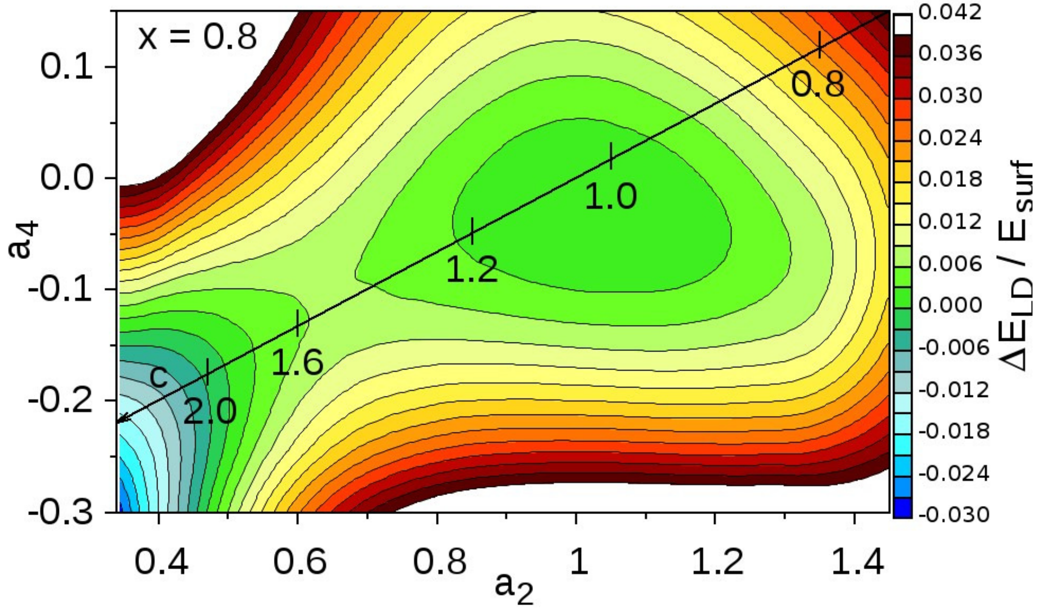
Here R_0 is radius of spherical nucleus, while $2z_0$ is the length of the deformed nucleus.

*K. Pomorski, B. Nerlo-Pomorska, J. Bartel, and C. Schmitt Acta Phys. Pol. B Supl. **8** (2015) 667,
C. Schmitt, K. Pomorski, B. Nerlo-Pomorska, and J. Bartel, Phys. Rev. C **95** (2017) 034612.

Convergence of the Fourier expansion



Potential energy surface in the (a_2, a_4) plane



Optimal coordinates:

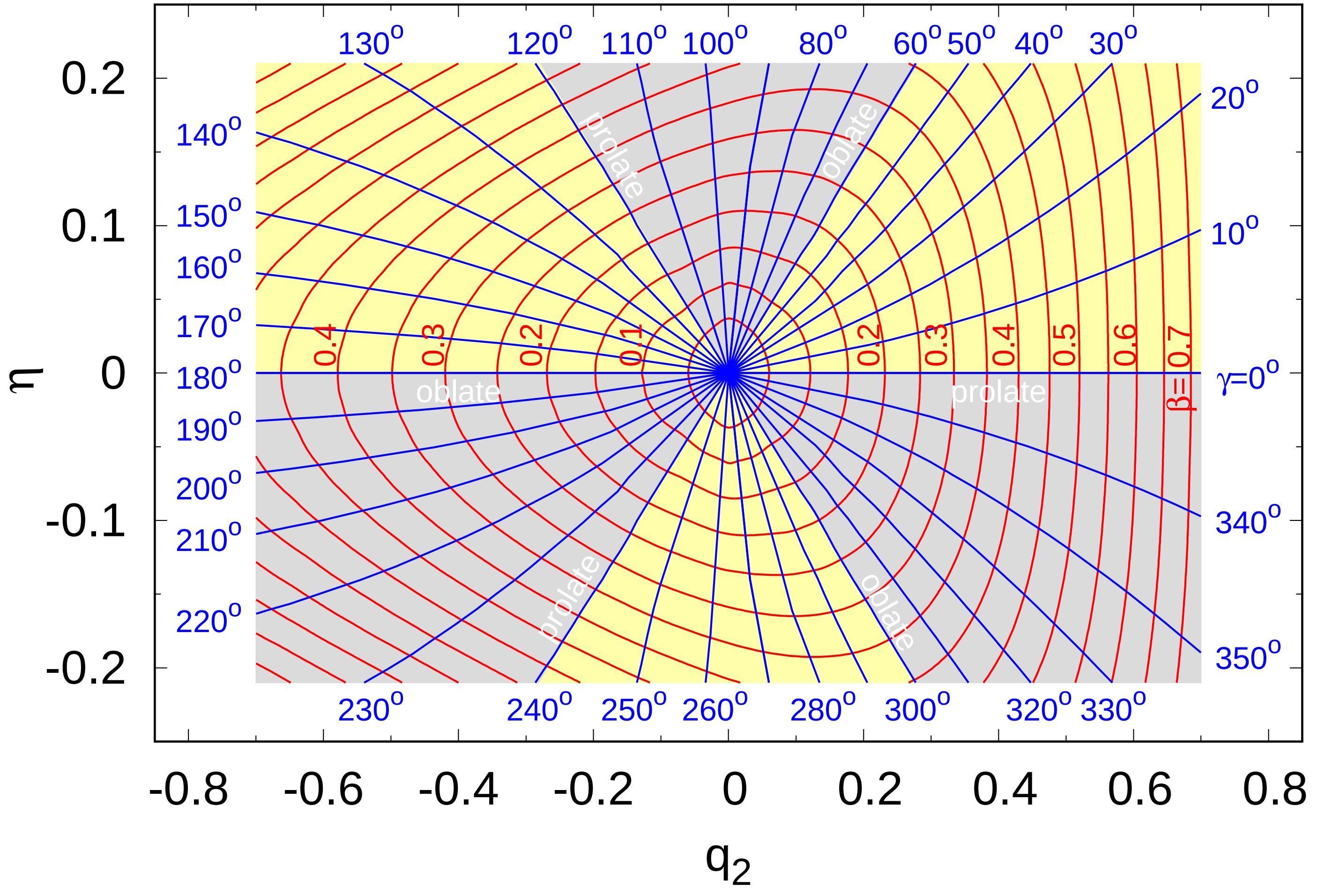
$$q_2 = a_2^{(0)}/a_2 - a_2/a_2^{(0)} \,,$$
$$q_3 = a_3 \,,$$
$$q_4 = a_4 + \sqrt{(q_2/9)^2 + (a_4^{(0)})^2} \,,$$
$$q_5 = a_5 - (q_2 - 2)a_3/10 \,,$$
$$q_6 = a_6 - \sqrt{(q_2/100)^2 + (a_6^{(0)})^2} \,.$$

Here $a_2^{(0)} = 1.03205$, $a_4^{(0)} = -0.03822$, and $a_6^{(0)} = 0.00826$ are the Fourier expansion coefficients of a sphere. q_2 describes the elongation of a nucleus, q_3 its left-right asymmetry, and q_4 formation of the neck.

See also an alternative expansion around a spheroid:

K. Pomorski, B. Nerlo-Pomorska, J. Bartel, Phys. Scr. **92** (2017) 064006.

Relation between (q_2, η) and (β, γ)



$$\beta = \frac{\sqrt{5\pi}}{3r_0^2 A^{5/3}} \sqrt{Q_{20}^2 + Q_{22}^2}$$

$$\gamma = \arctan\left(\frac{Q_{22}}{Q_{20}}\right)$$

$$Q_{20} = < 2z^2 - x^2 - y^2 >$$

$$Q_{22} = \sqrt{3} < y^2 - x^2 >$$

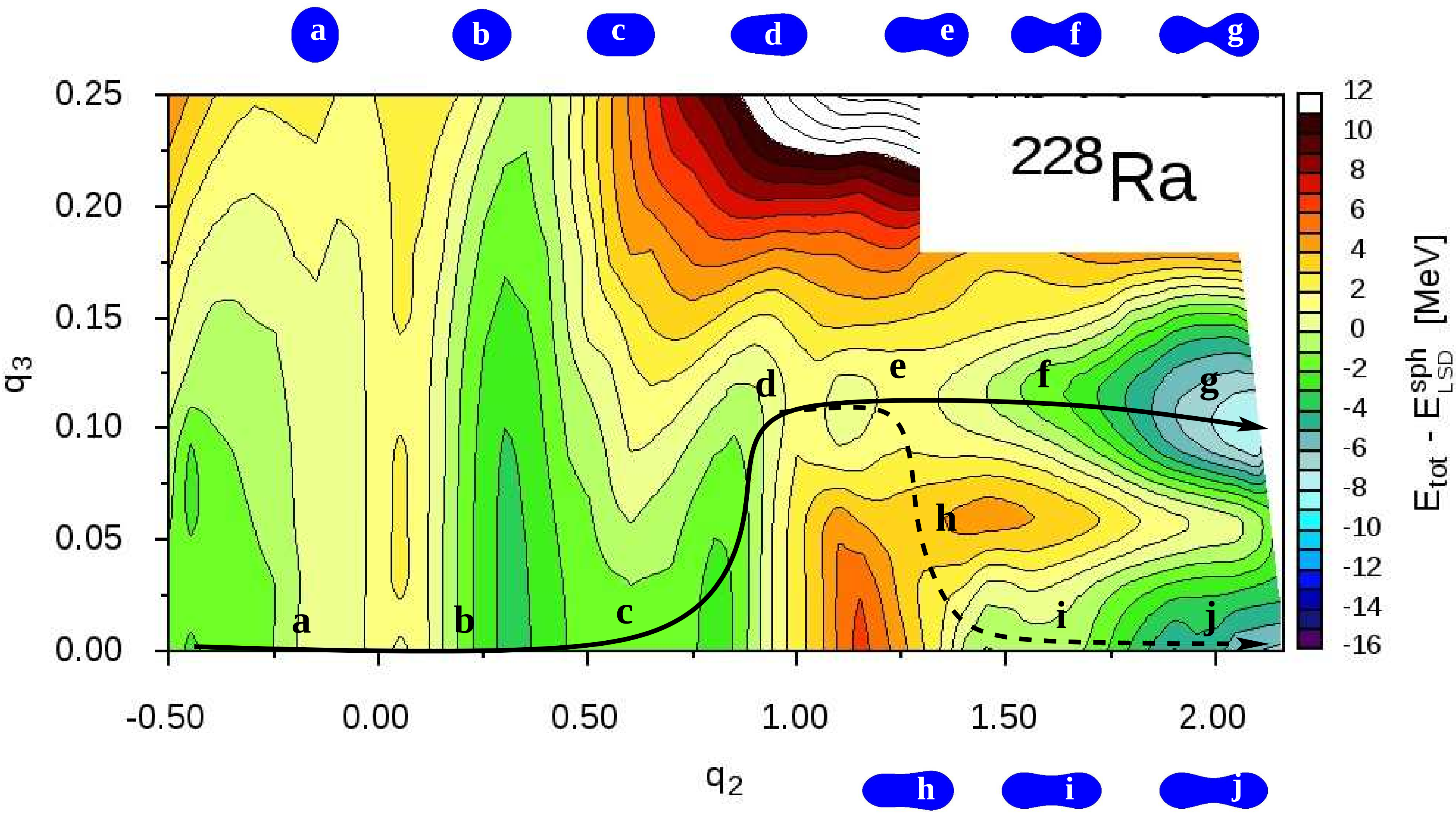
Main ingredients of our model:

- Macroscopic-microscopic approximation of nuclear energy,
- Lublin-Strasbourg-Drop for the macroscopic part of energy,
- Yukawa-folded single-particle potential,
- Strutinsky shell-correction method,
- BCS with the monopole pairing force and the GCM+GOA projection,
- Fourier parametrization of nuclear shapes,

Calculations are already preformed for 324 even-even isotopes from ^{226}U to ^{324}126. The results for odd-Z or/and odd-N nuclei are in preparation.

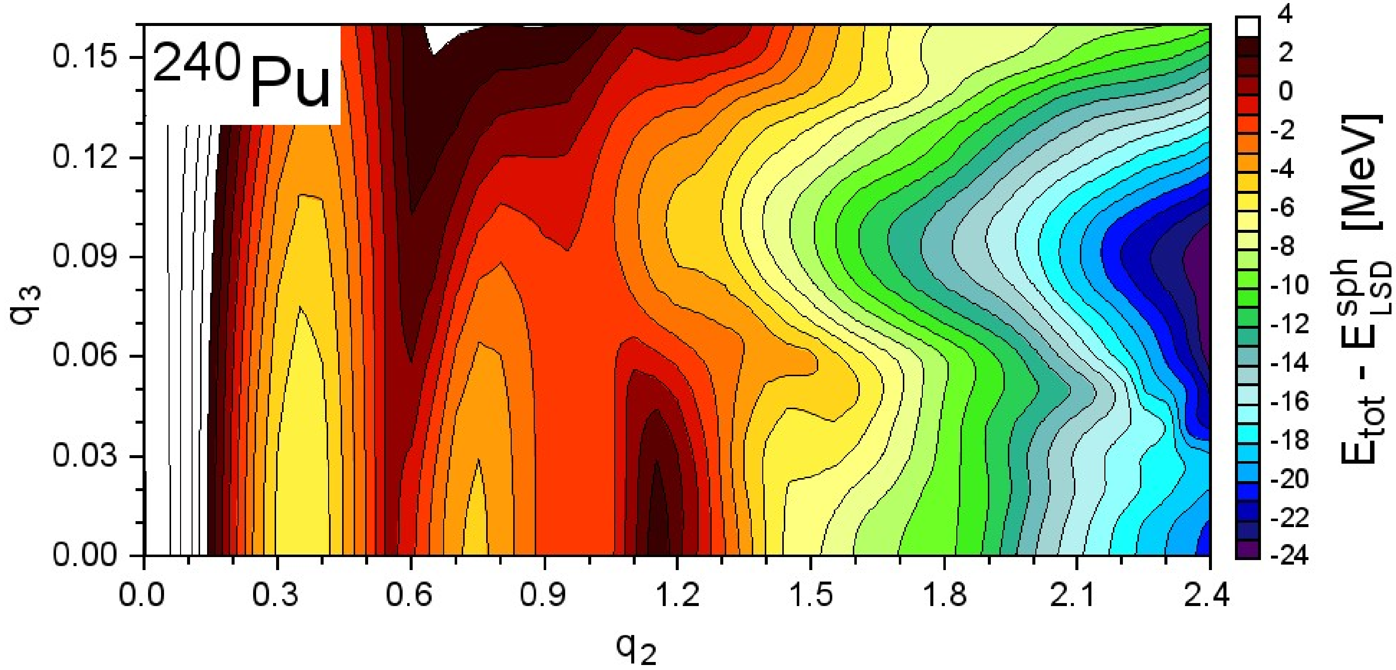
All parameters of the calculation are standard. None of them is specially fitted to the considered nuclei.

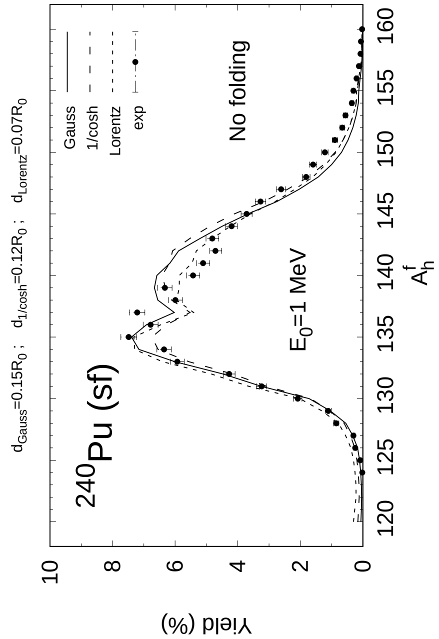
Potential energy surface of ^{228}Ra on the (q_2, q_3) plane*



*C. Schmitt, K. Pomorski, B. Nerlo-Pomorska, and J. Bartel, Phys. Rev. C **95** (2017) 034612.

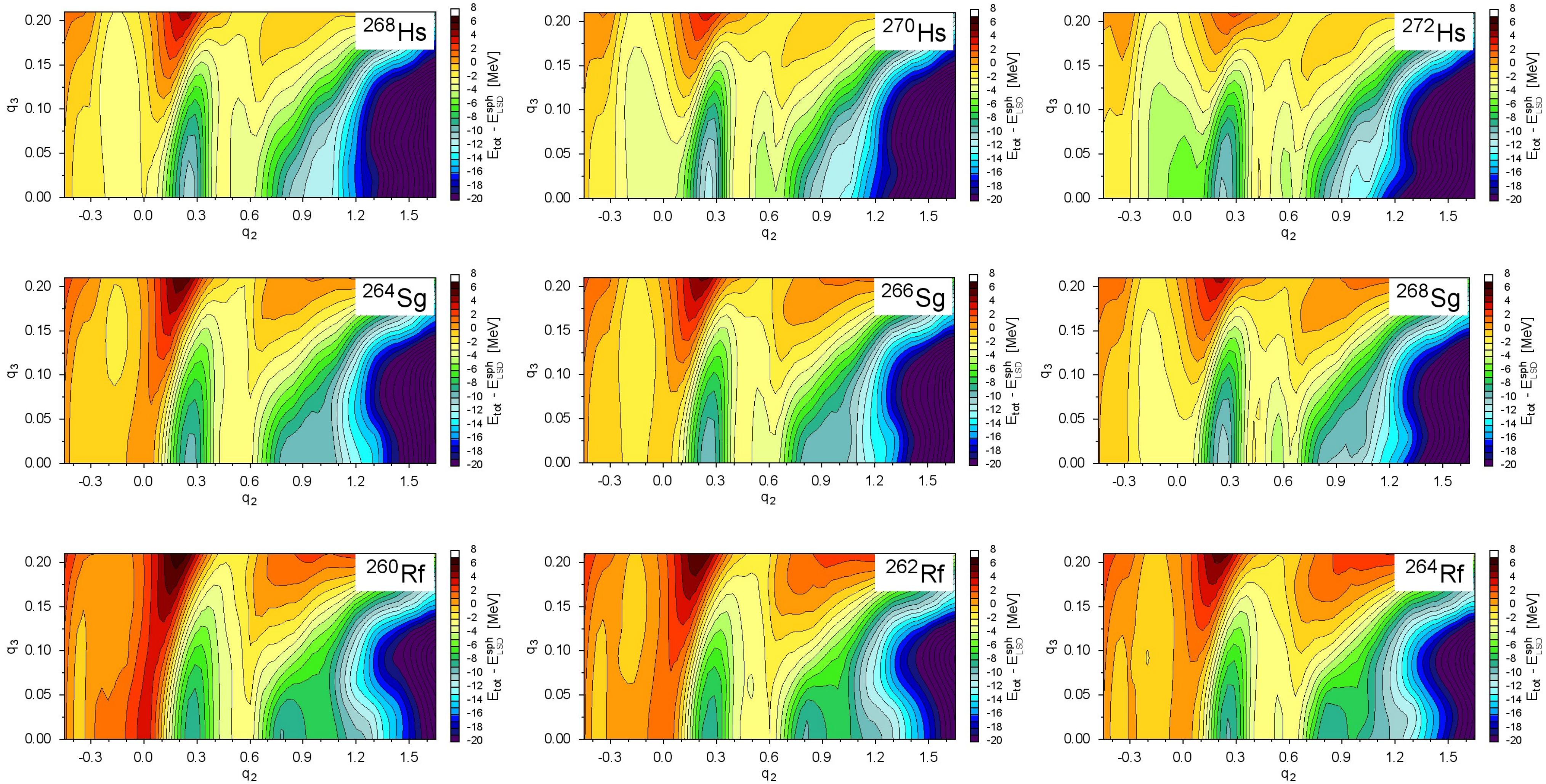
Potential energy surface of ^{236}Pu on the (q_2, q_3) plane*



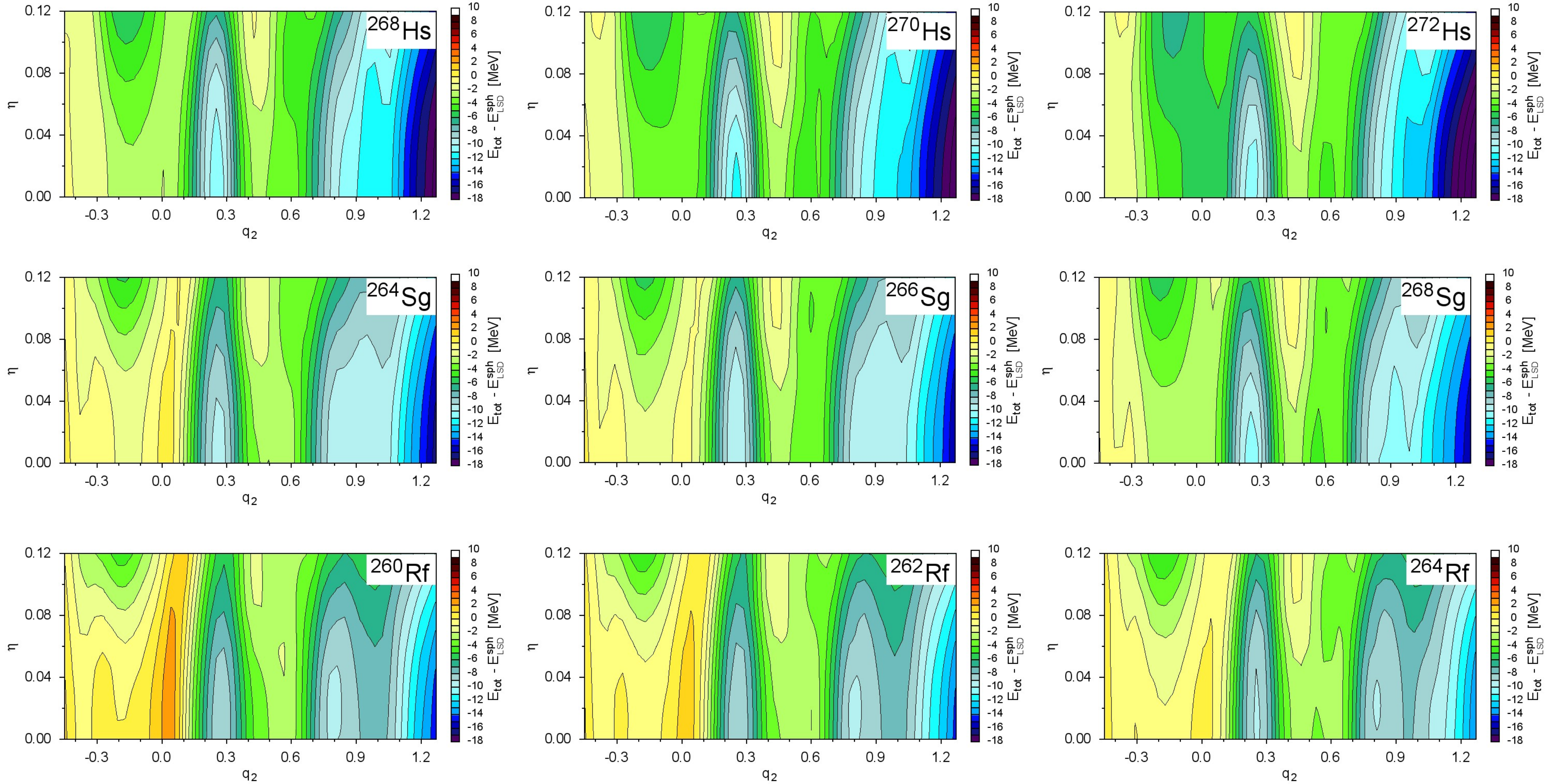


*K. Pomorski, B. Nerlo-Pomorska, J. Bartel, C. Schmitt, Eur. Phys. Journ. Web-of-Conf. **169**, 00016 (2018)

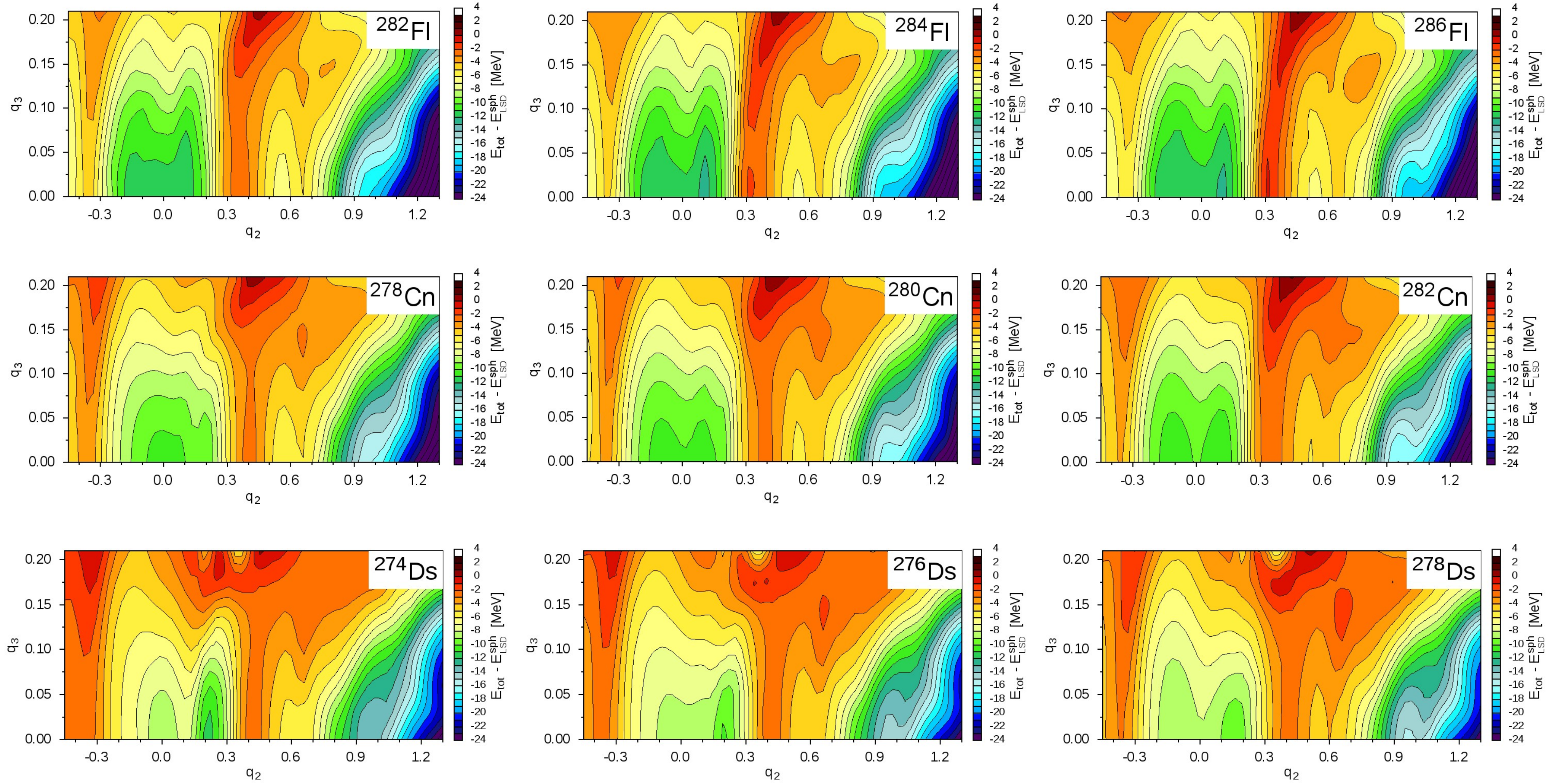
PES of ^{266}Sg and neighboring nuclei on the (q_2, q_3) plane



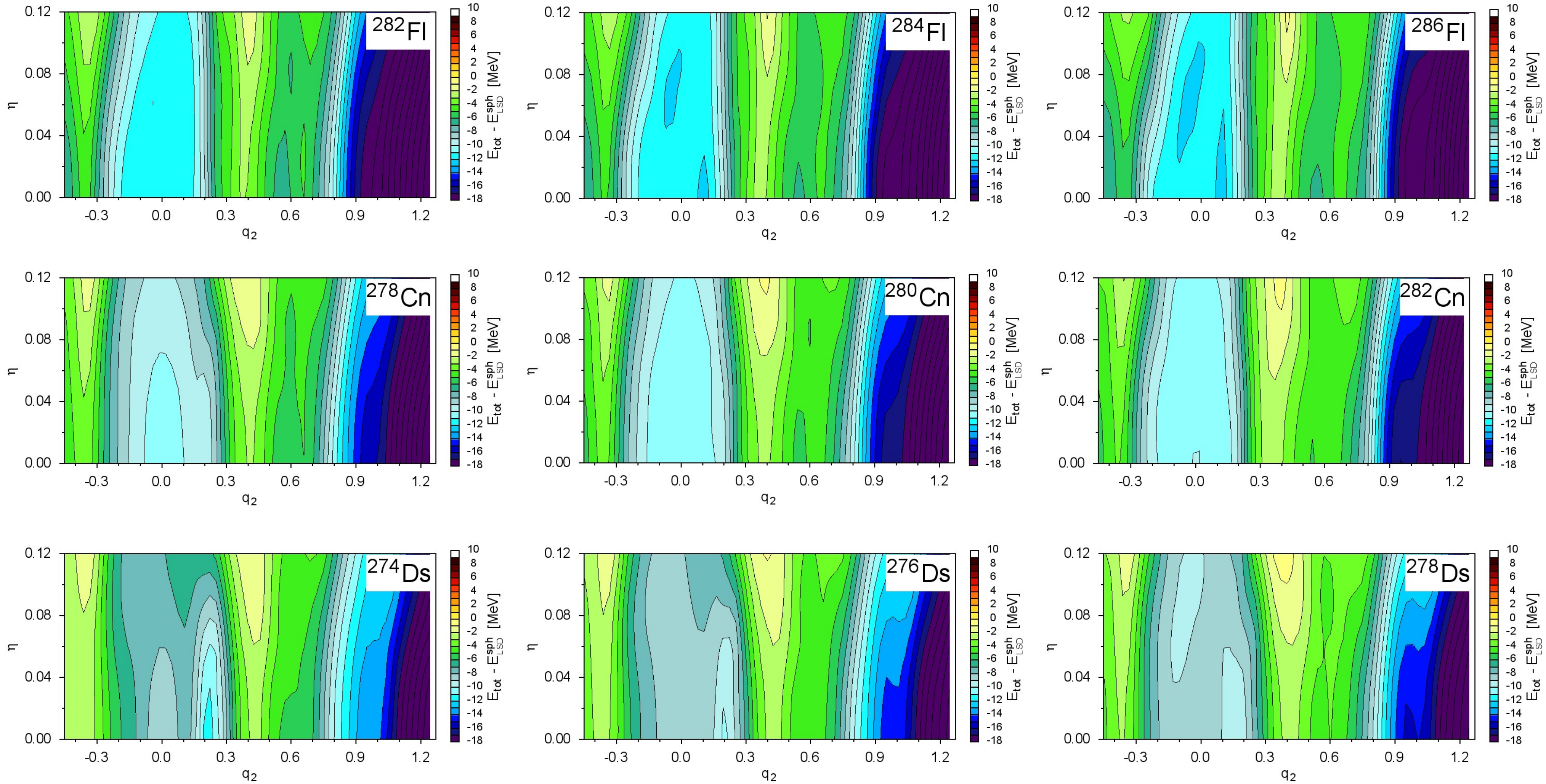
PES of ^{266}Sg and neighboring nuclei on the (q_2, η) plane



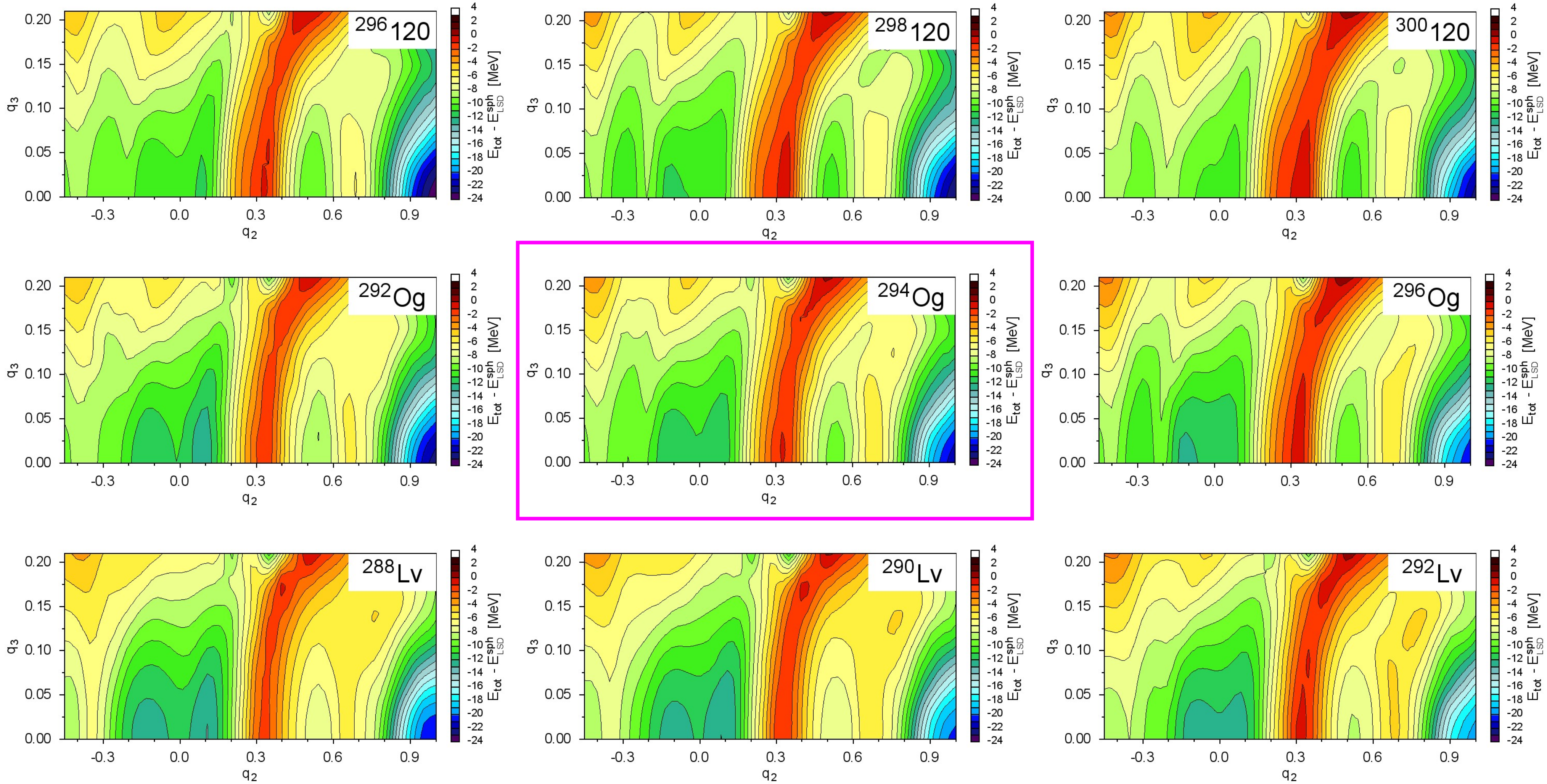
PES of ^{280}Cn and neighboring nuclei on the (q_2, q_3) plane



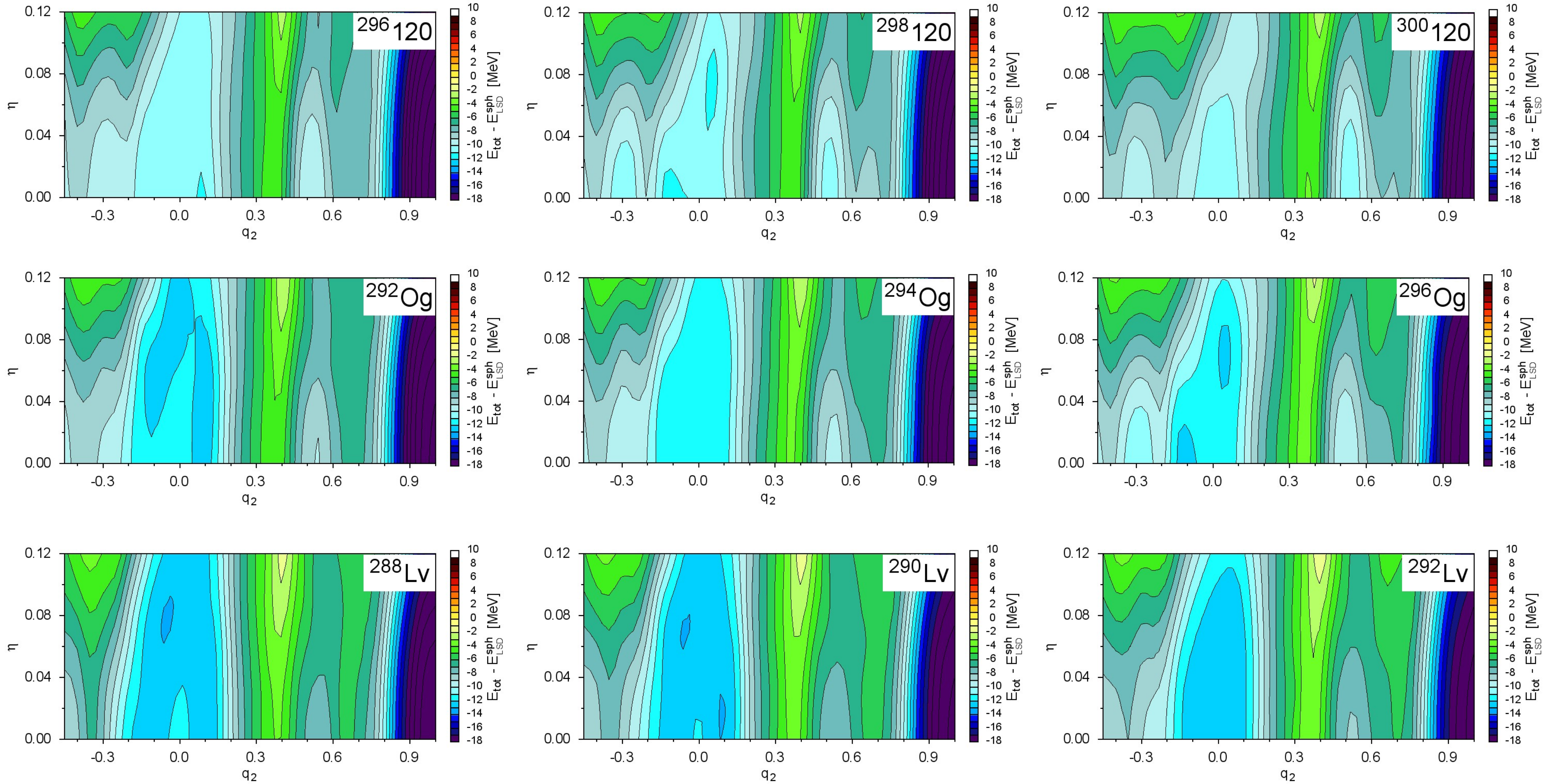
PES of ^{280}Cn and neighboring nuclei on the (q_2, η) plane



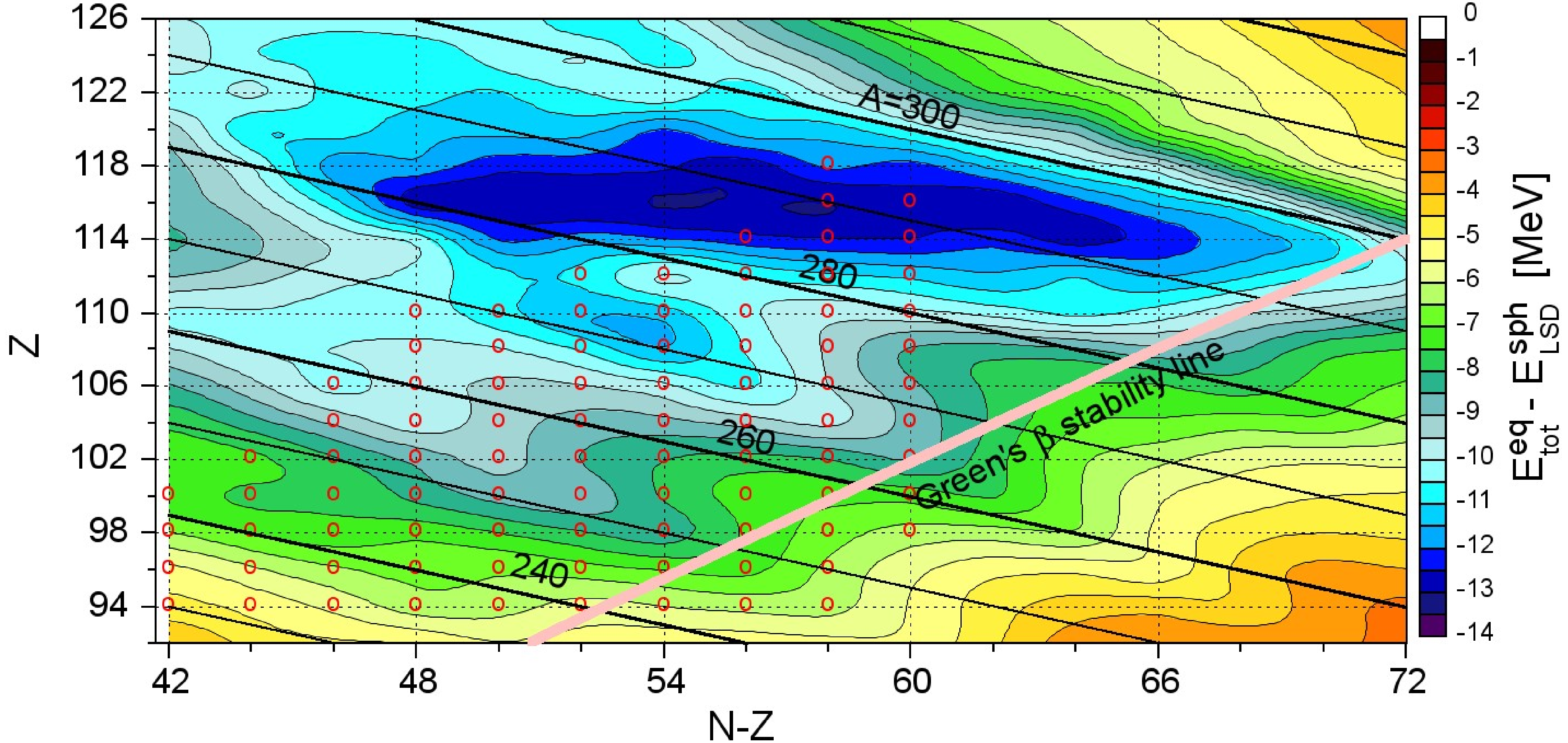
PES of ^{294}Og and neighboring nuclei on the (q_2, q_3) plane



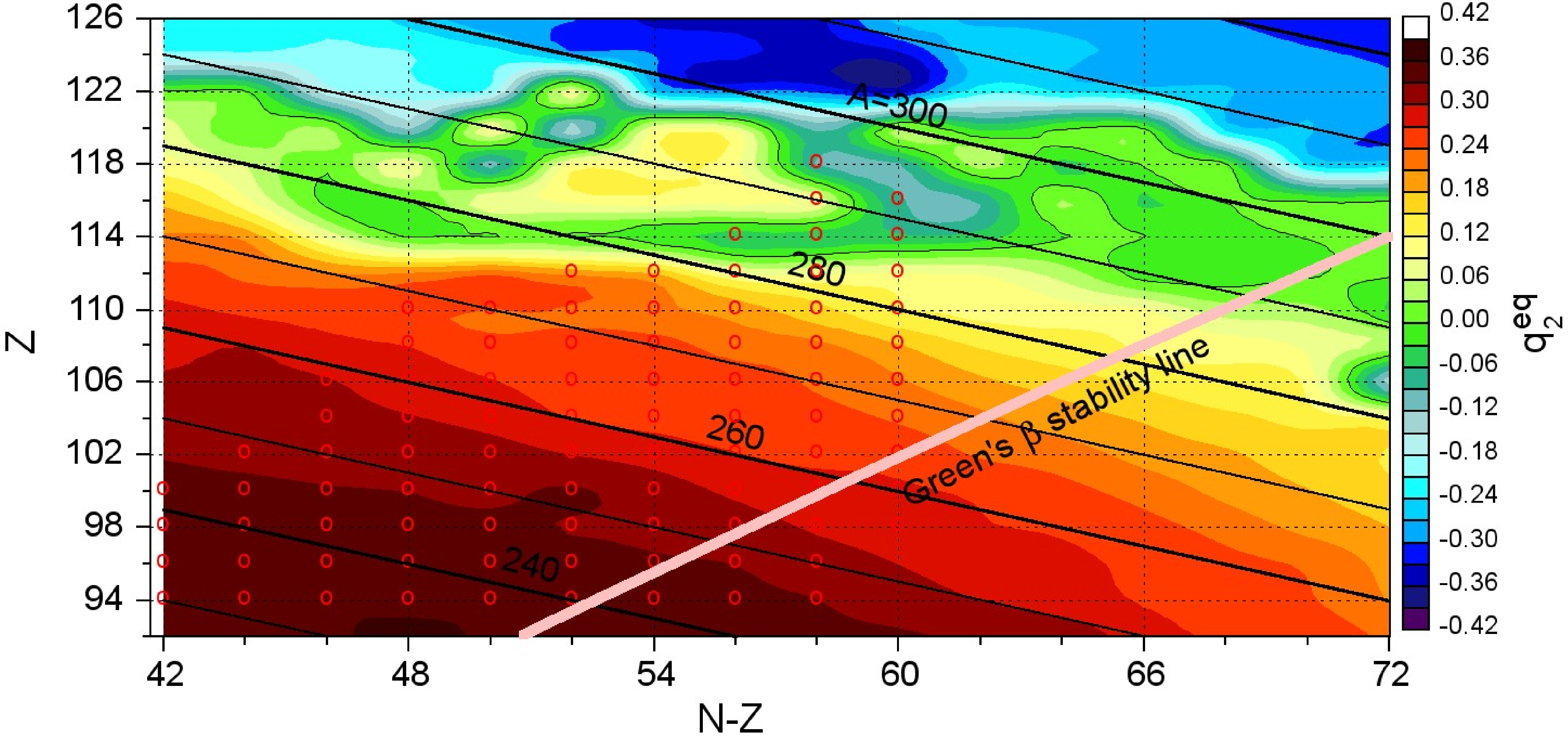
PES of ^{294}Og and neighboring nuclei on the (q_2, η) plane



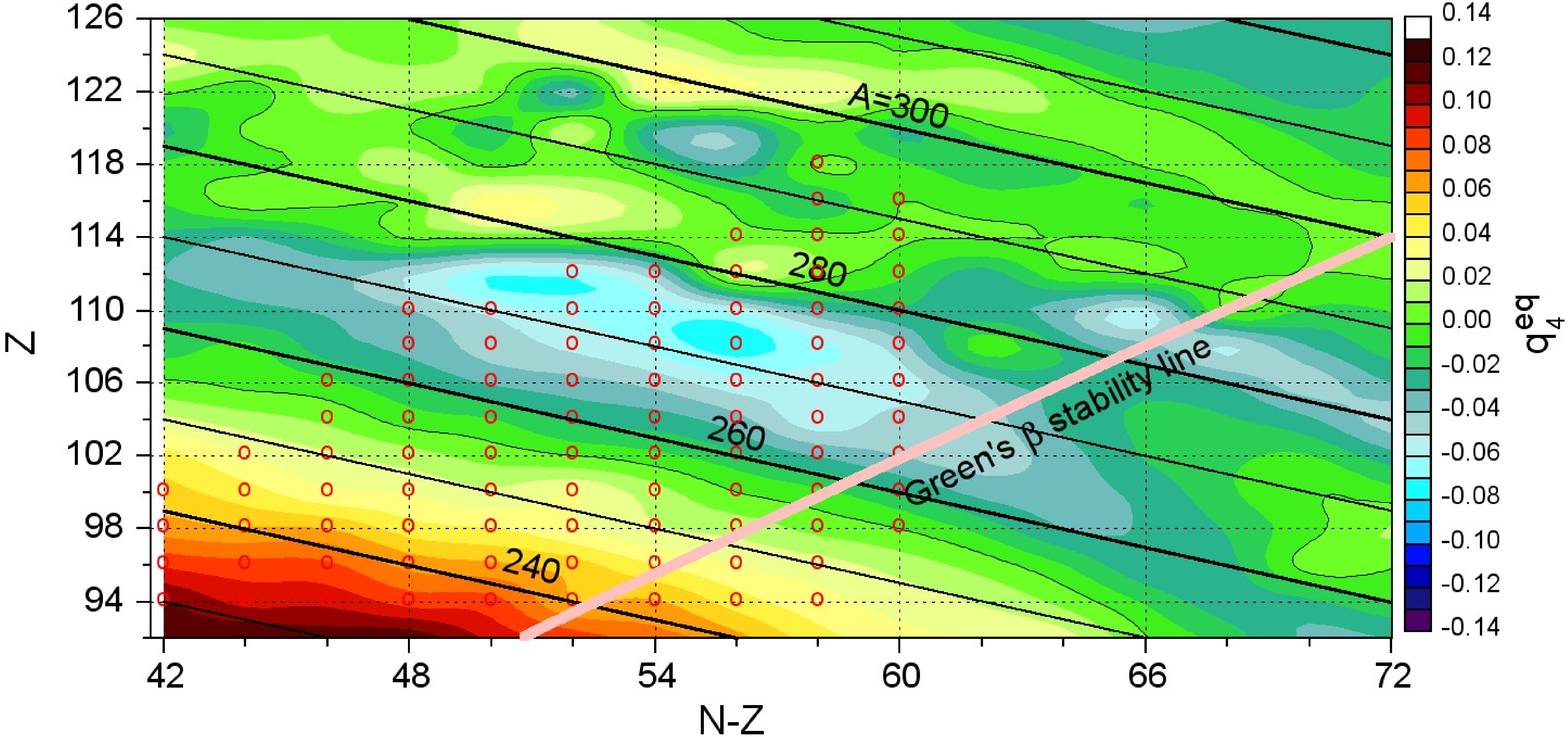
Microscopic energy correction: $E^{\mathrm{gs}}_{\mathrm{micr}} = E^{\mathrm{gs}}_{\mathrm{tot}} - E^{\mathrm{sph}}_{\mathrm{LSD}}$



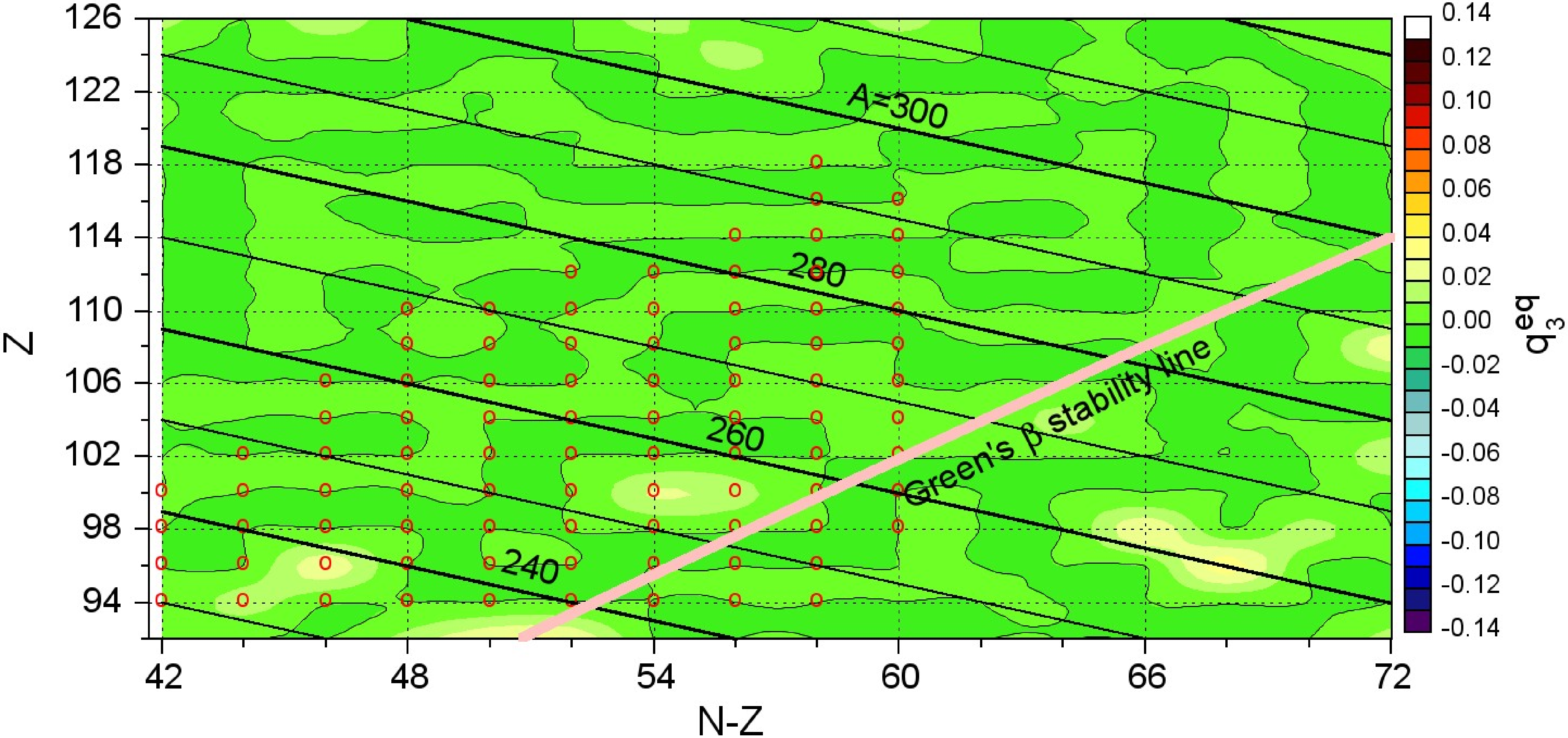
Ground state *quadrupole* deformation



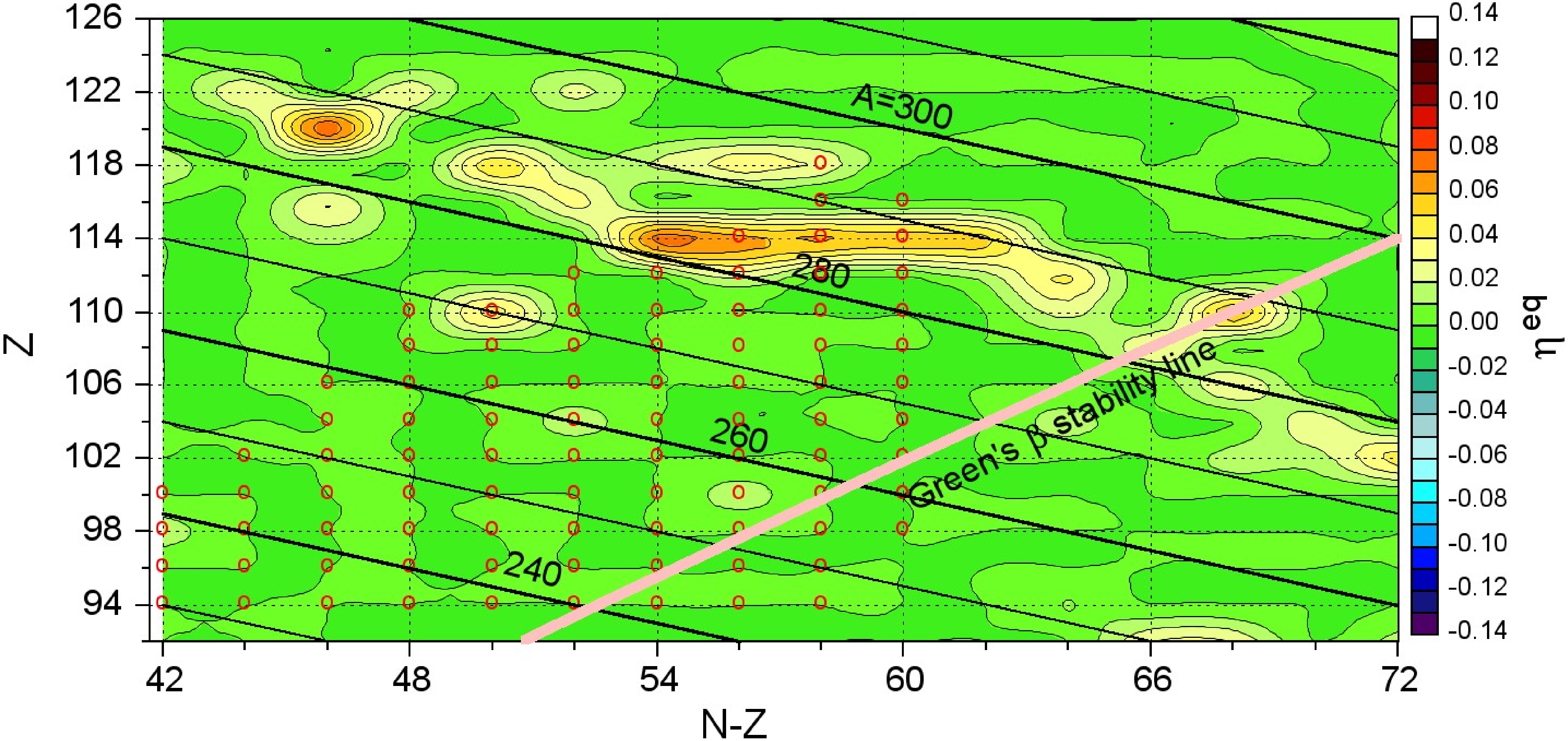
Ground state *hexadecapole* deformation



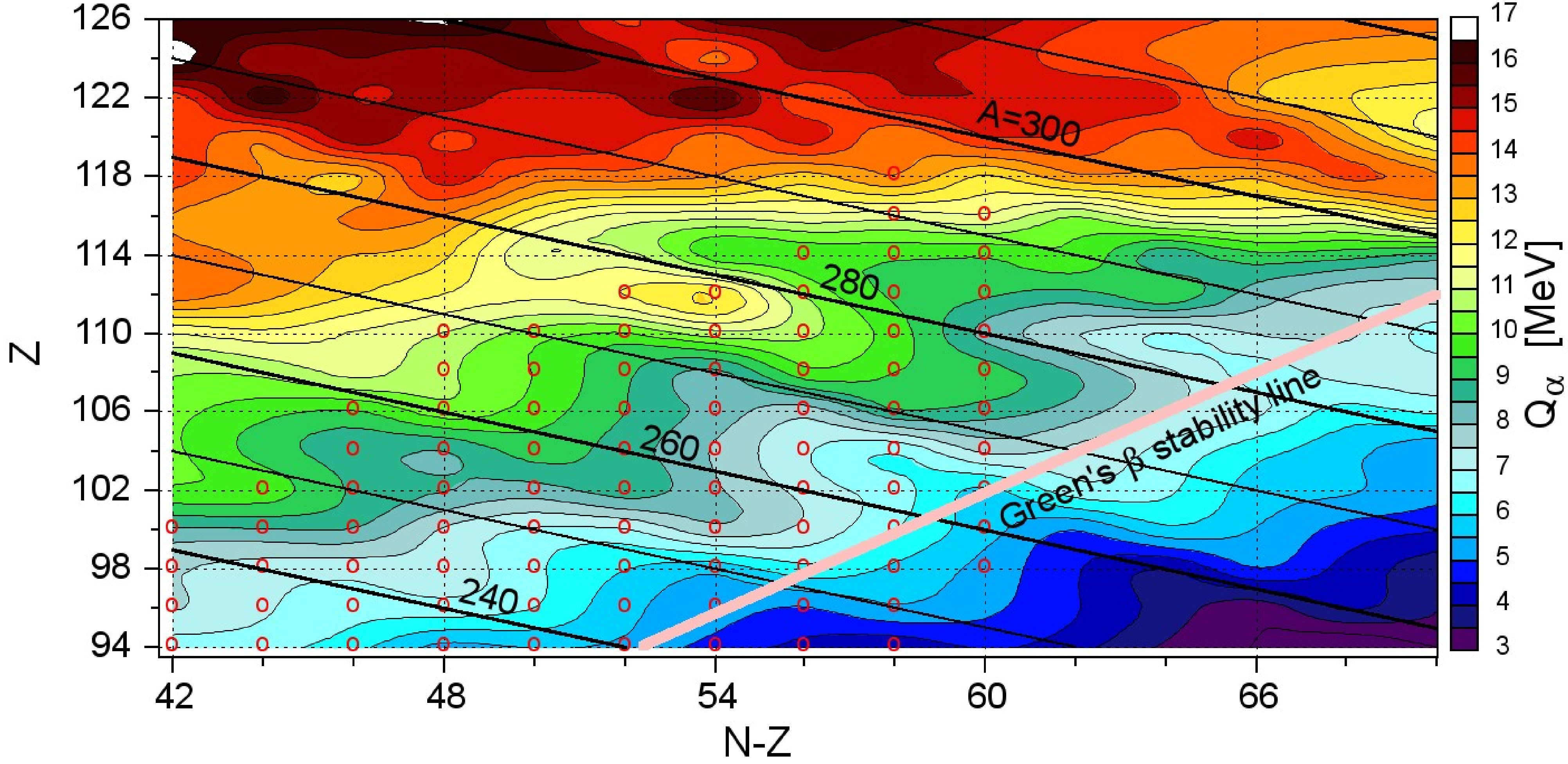
Ground state *octupole* deformation



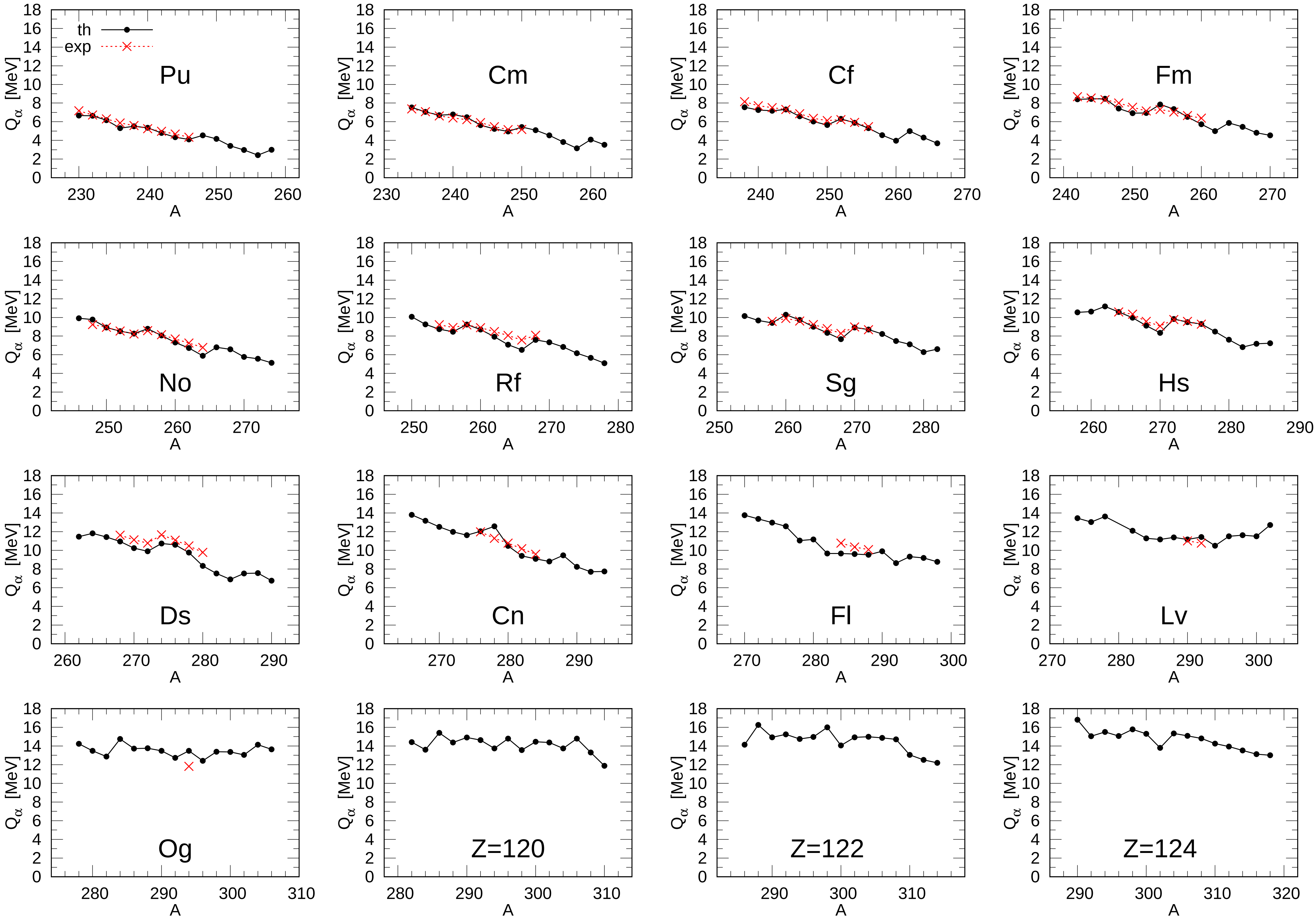
Ground state nonaxial deformation



Alpha particle energy Q_α

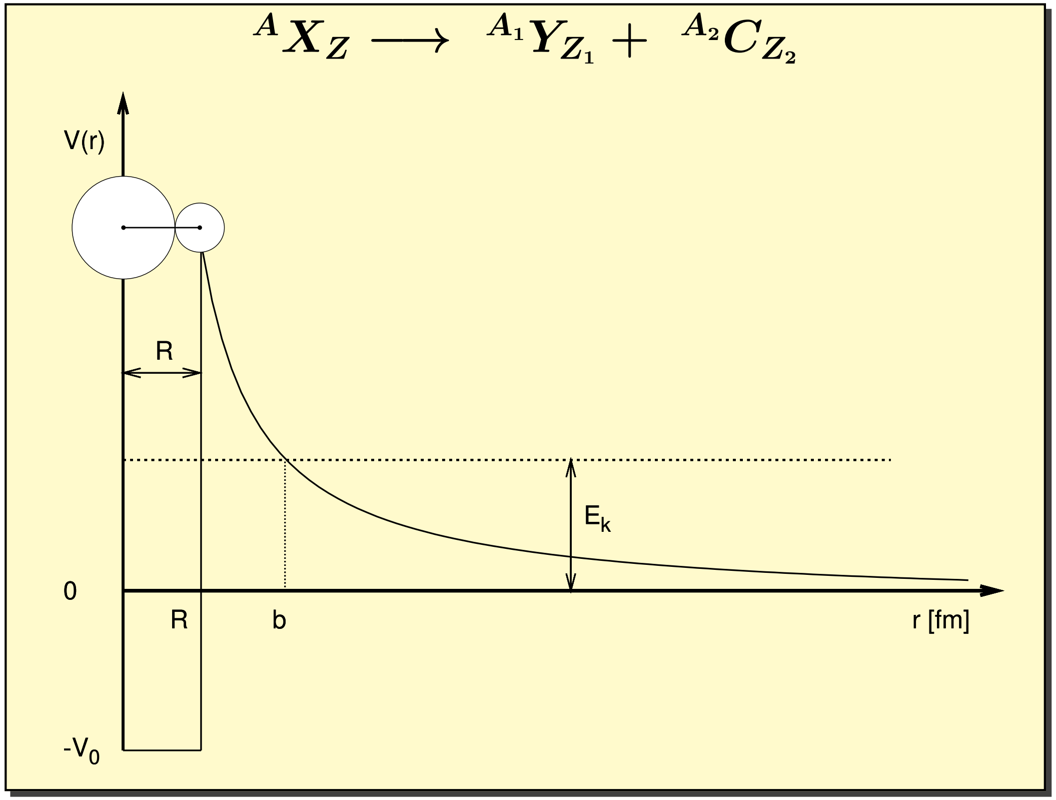


The r.m.s. deviation of Q_α for Z$\geq$ 94 is 0.51 MeV only.



th
exp
Pu
Cm
Cf
Fm
No
Rf
Sg
Hs
Ds
Cn
Fl
Lv
Og
Z=120
Z=122
Z=124
Qα [MeV]
A

Gamow like model of the alpha decay and cluster radioactivity*



The half-life of a decaying nuclei is given by:

$$T_{1/2} = \frac{\ln 2}{\lambda} \cdot 10^h$$

Here h is a hindrance factor for o-o or o-e nuclei ($h = 0$ for e-e) and $\lambda = \nu P$, where ν is the number of assaults against the barrier.
Within the WKB theory the probability of the barrier penetration P is equal to:

$$P = \exp\left[-\frac{2}{\hbar}\int_R^b \sqrt{2\mu(V(r) - E_\mathrm{k})}\, dr\right] .$$

Here μ is the reduced mass of emitted particle.
The exit point from the barrier b corresponds to point, where the Coulomb potential is equal to the kinetic energy (E_k).

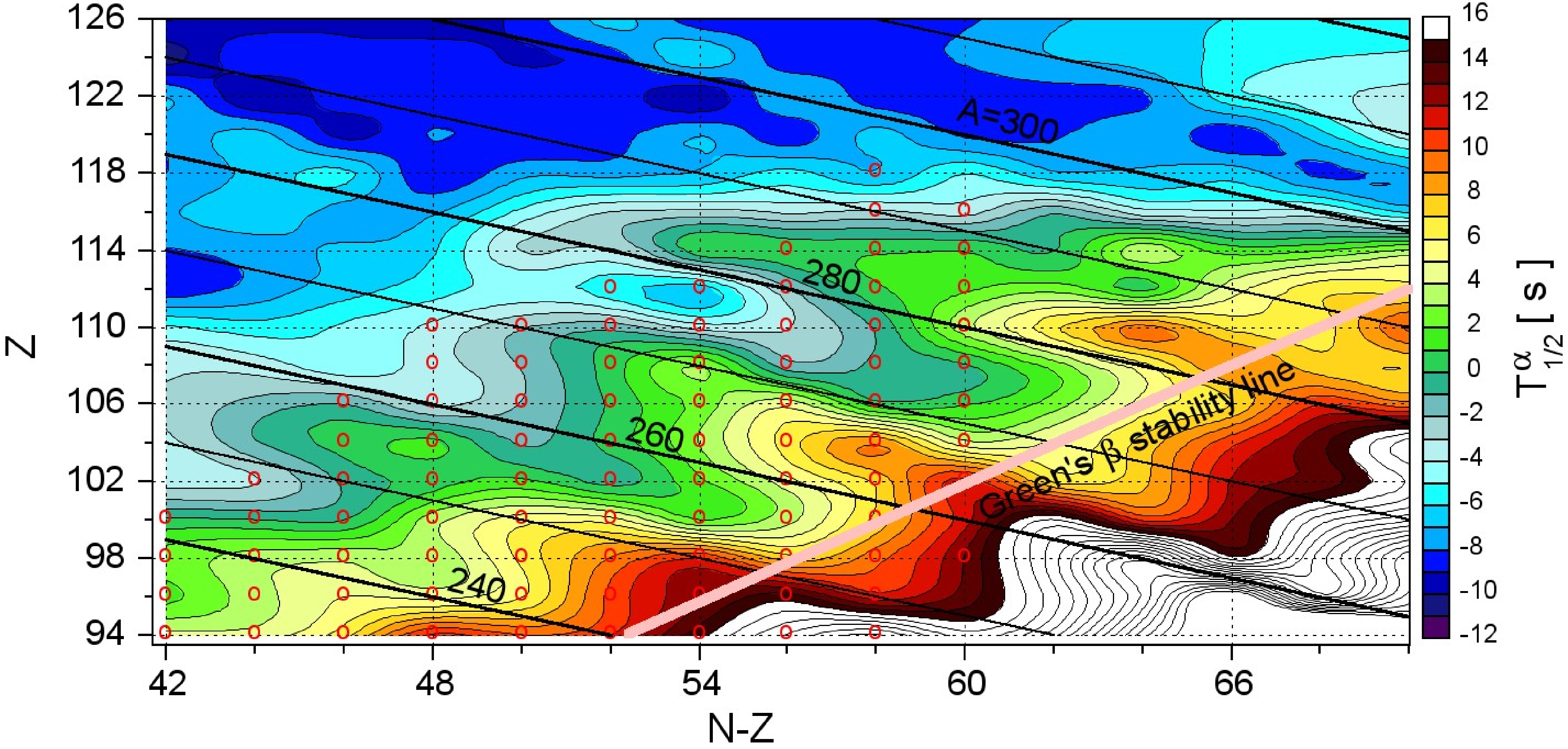
The probability of tunneling of the above barrier is given by:

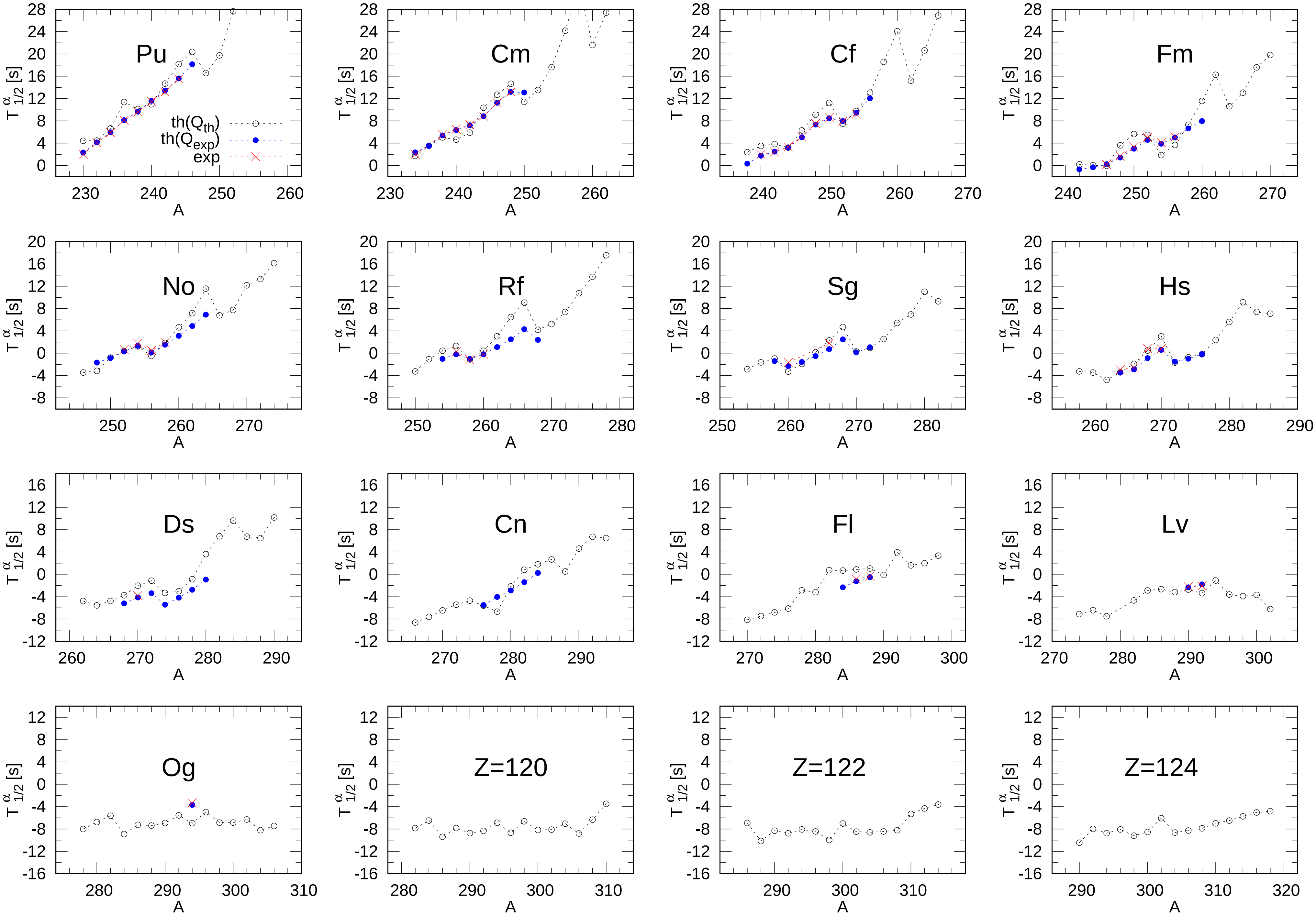
$$P = \exp\left\{-\frac{2}{\hbar}\sqrt{2\mu Z_1 Z_2 e^2 b}\left[\arccos\sqrt{\frac{R}{b}} - \sqrt{\frac{R}{b} - \left(\frac{R}{b}\right)^2}\right]\right\}$$

Here $R = r_0(A_1^{1/3} + A_2^{1/3})$. In the g.s. of a square well potential one has $\nu = \frac{\pi\hbar}{2\mu R^2}$.

* A. Zdeb, M. Warda, and K. Pomorski, Phys. Rev. C **87**, 024308 (2013).

Prediction of the α-decay half-lives





Pu
Cm
Cf
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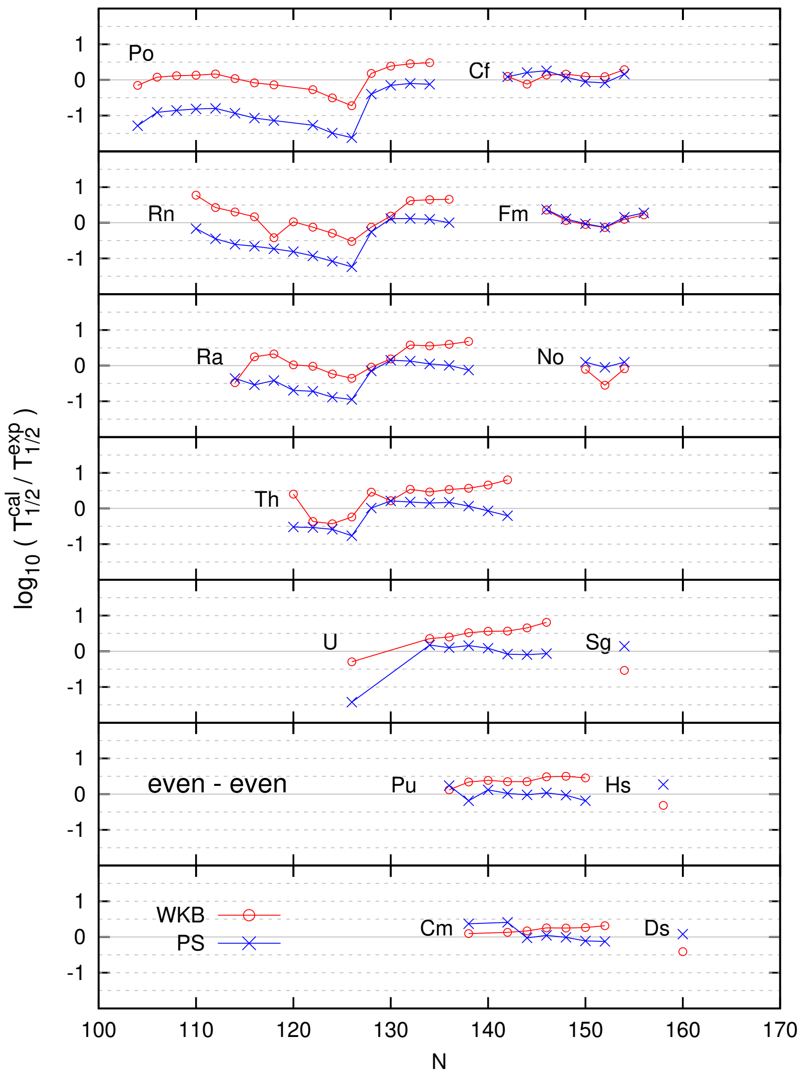
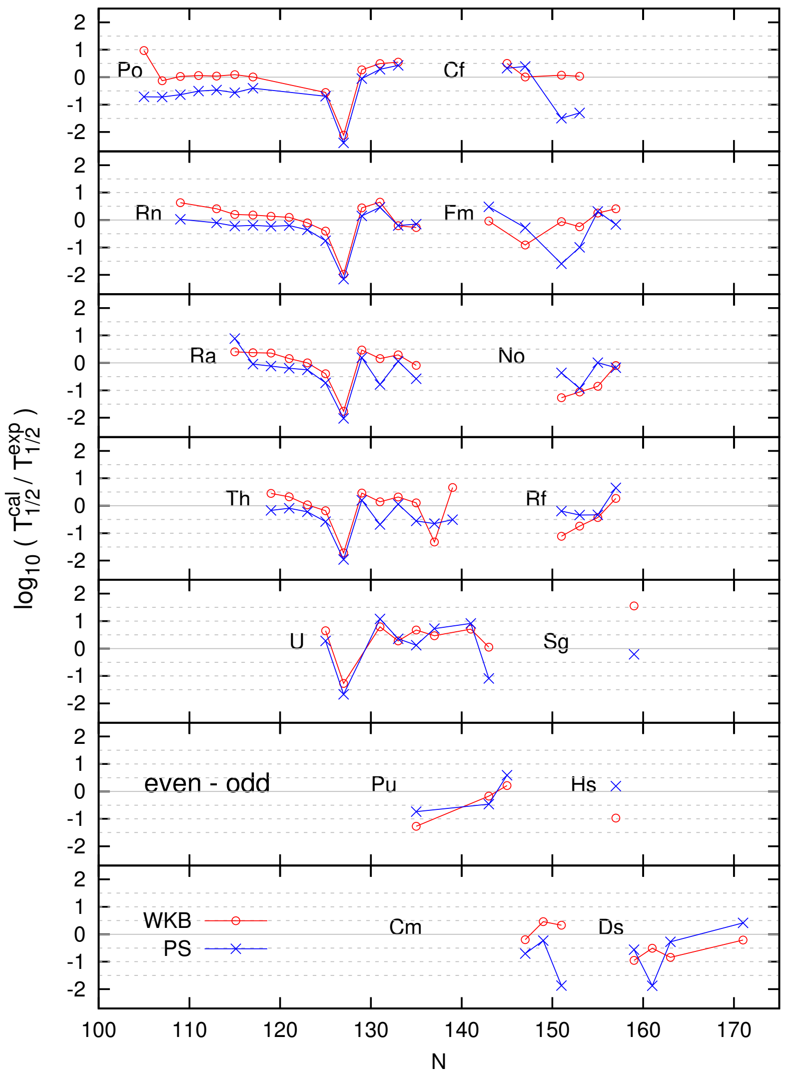
No
Rf
Sg
Hs
Ds
Cn
Fl
Lv
Og
Z=120
Z=122
Z=124
th(Q_th)
th(Q_exp)
exp
T^α_1/2 [s]
A

Deviations from the data of $\log(T_{1/2})$ for even-even nuclei

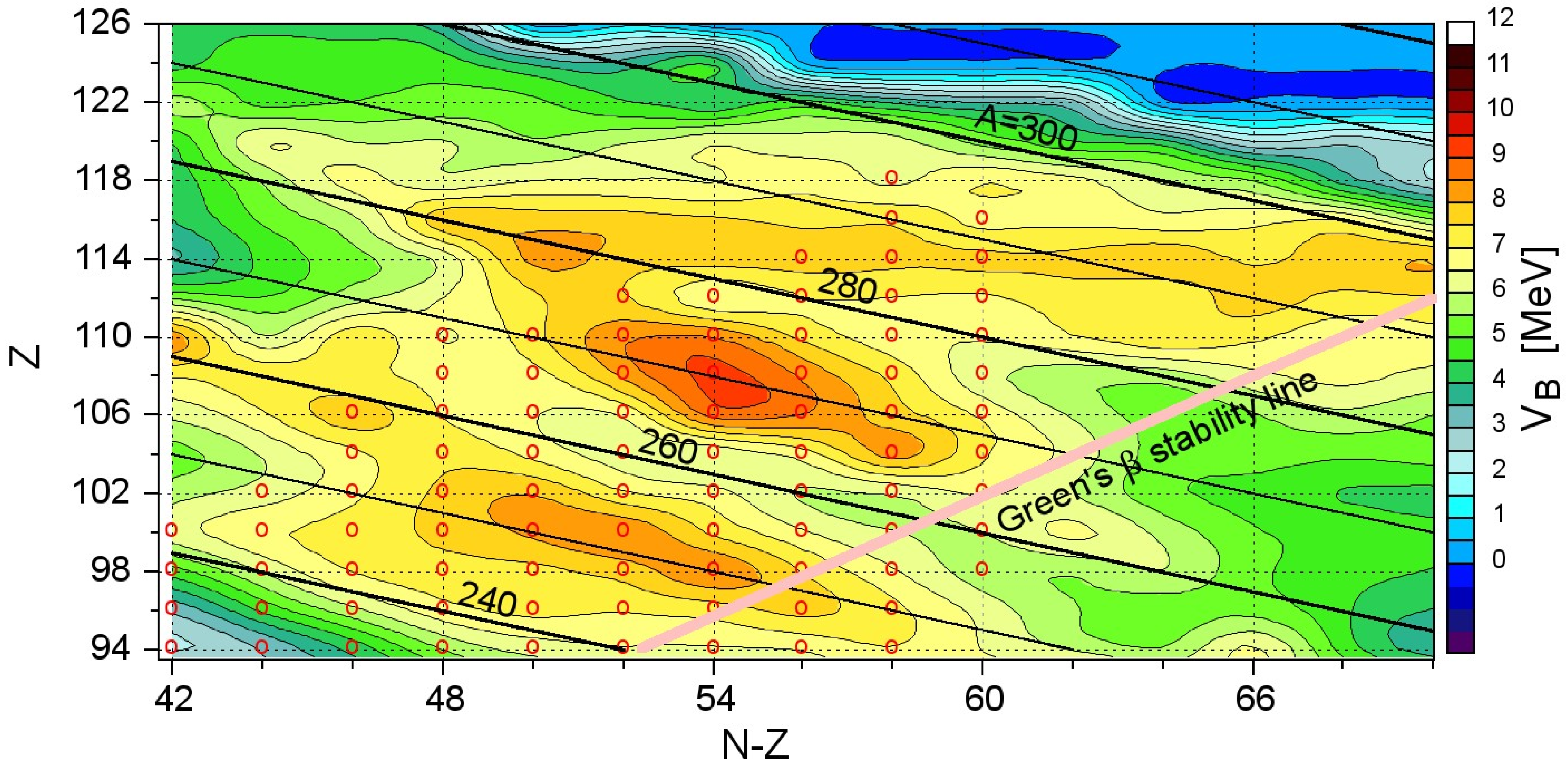


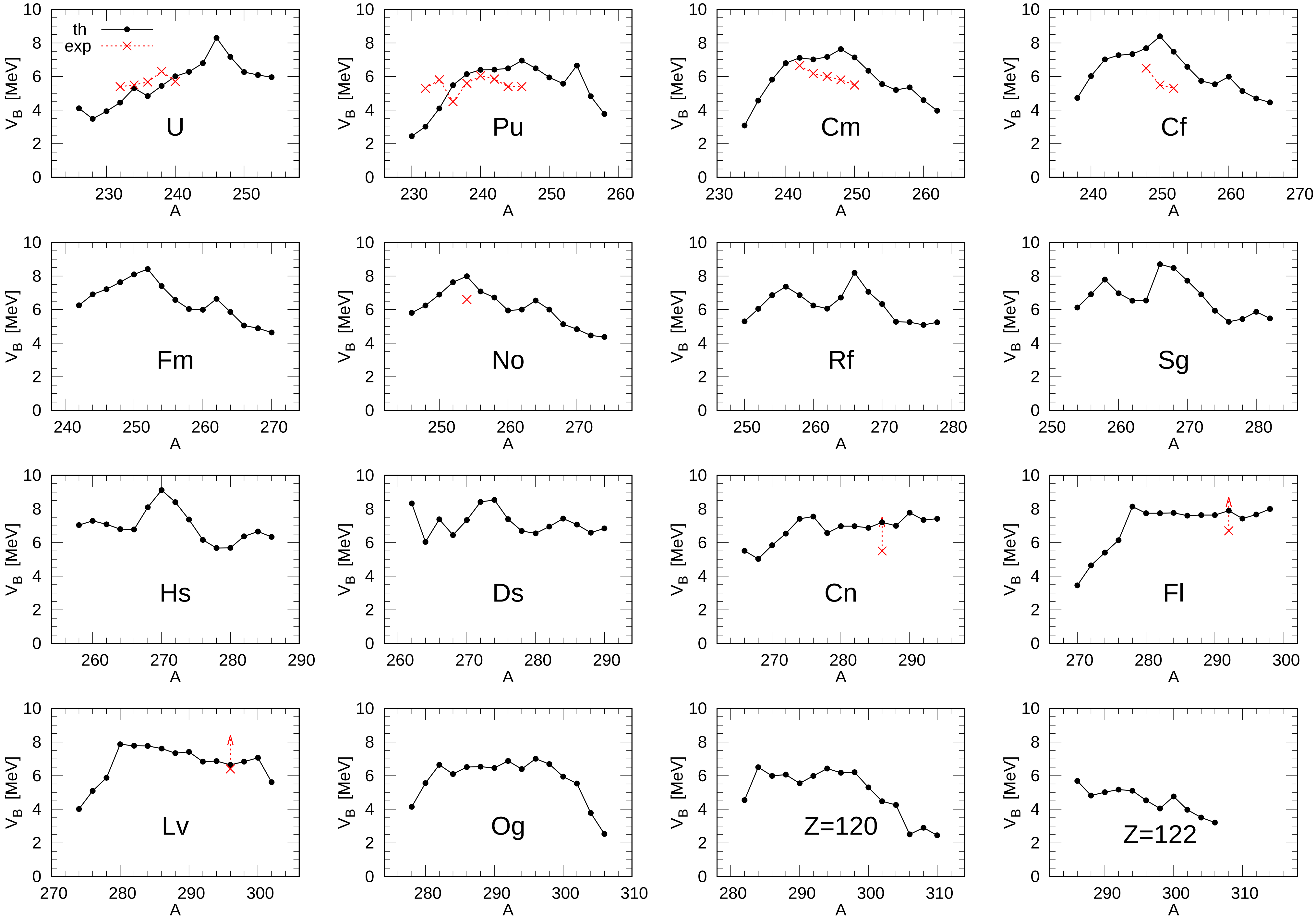


PS: A. Parkhomenko and A. Sobiczewski, Acta Phys. Polon. B **36**, 3095 (2005) ⟶ 4 parameters,

WKB: A. Zdeb, M. Warda, K.P., Phys. Rev. C **87**, 024308 (2013) ⟶ 1 parameter: r_0=1.21 fm.

Fission barrier heights





th
exp
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Cm
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Fm
No
Rf
Sg
Hs
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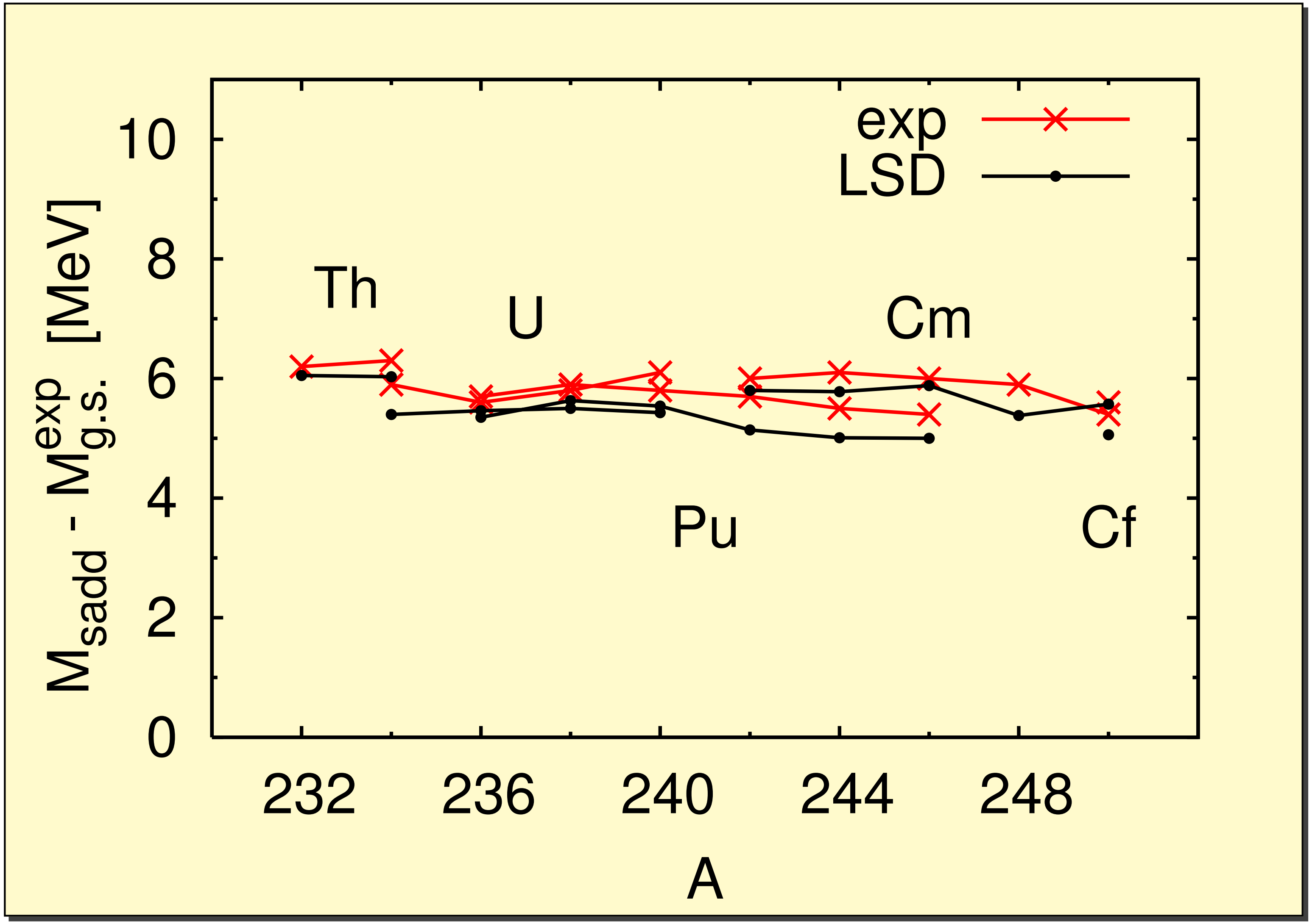
Fl
Lv
Og
Z=120
Z=122
V_B [MeV]
A

Nuclear fission and Świątecki topographical theorem



$$V_B = M^{\rm exp}_{\rm sadd} - M^{\rm exp}_{gs}$$

$$M^{\rm exp}_{\rm sadd} \approx M^{\rm mac}_{\rm sadd}$$

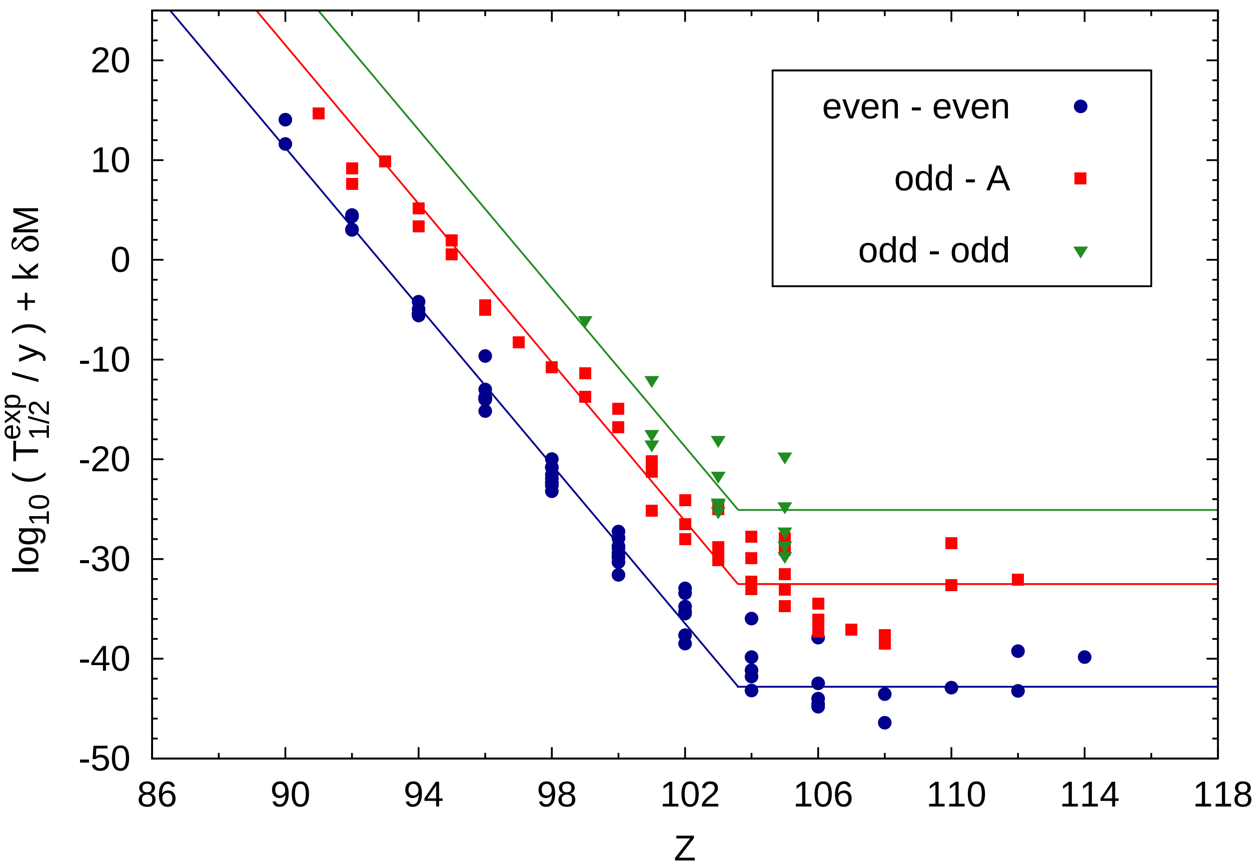
The agreement of the macroscopic saddle point masses with the data is striking. The average difference is only 310 keV ....

Władek Świątecki email from 9 Nov. 2006.

W. D. Myers, W.J. Świątecki, Nucl.Phys. **A612** (1997) 249. ← Topographical theorem

A. Dobrowolski, B. Nerlo-Pomorska, K. Pomorski, Acta Phys. Pol. **B40**, 705 (2009).

Spontaneous fission half-lives systematics *

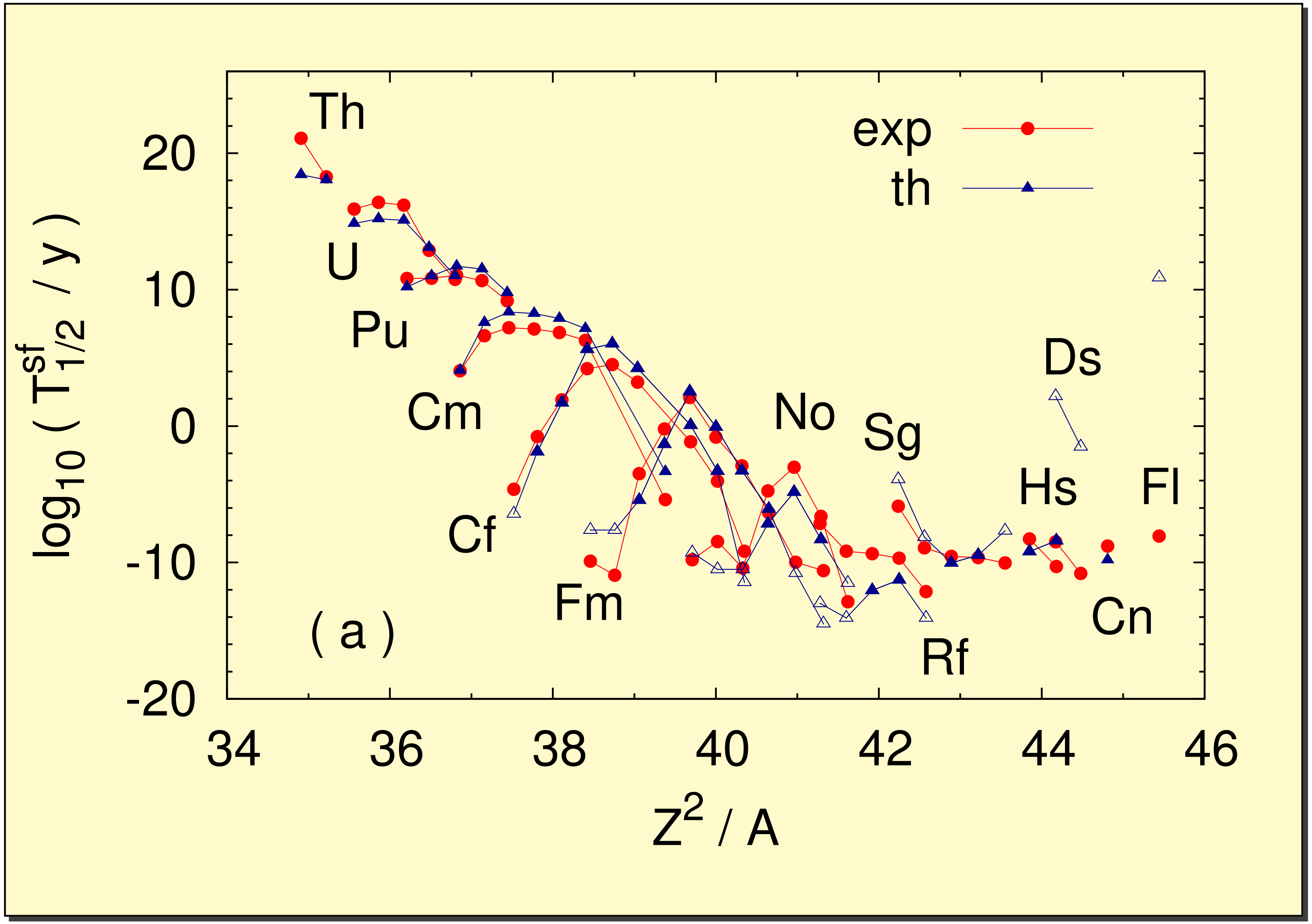


$\log T^{\rm sf}_{1/2} + 7.7 * \delta M(Z,A) = -4.1 \cdot \min(Z, 103) + 308.2$, where $\delta M = M_{\rm exp} - M_{\rm LSD}$.

Confer also: W.J. Świątecki, Phys. Rev. **100**, 937 (1955). $\rightarrow \log T^{\rm sf}_{1/2} = f(Z^2/A) - k\delta M(Z,A)$

*K. Pomorski, M. Warda and A. Zdeb, Acta Phys. Polon. **46**(2015) 4233.

Spontaneous fission half-lives of e-e isotopes*

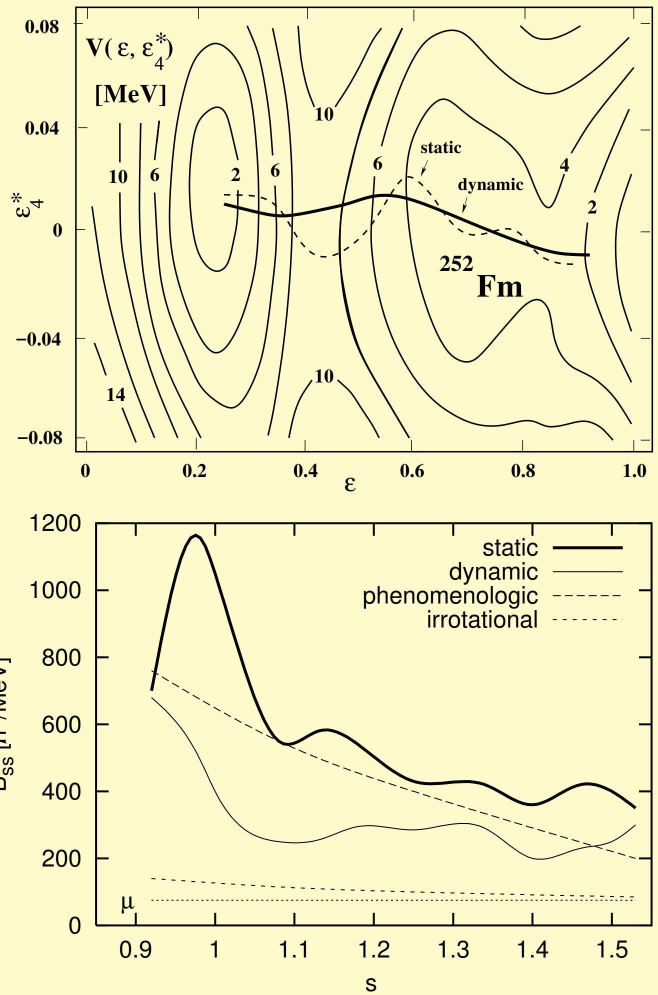


$$\log T^{\mathrm{sf}}_{1/2} = -4.1 \cdot \min(Z, 103) + 308.2 - 7.7 * (M_{\mathrm{exp}} - M_{\mathrm{LSD}}).$$

*K. Pomorski, M. Warda and A. Zdeb, Phys. Scr. **90** (2015) 114013.

Simple model for spontaneous fission probability *

A. Baran, K. Pomorski, A. Łukasiak, and A. Sobiczewski, Nucl. Phys. **A361**, 83 (1981).

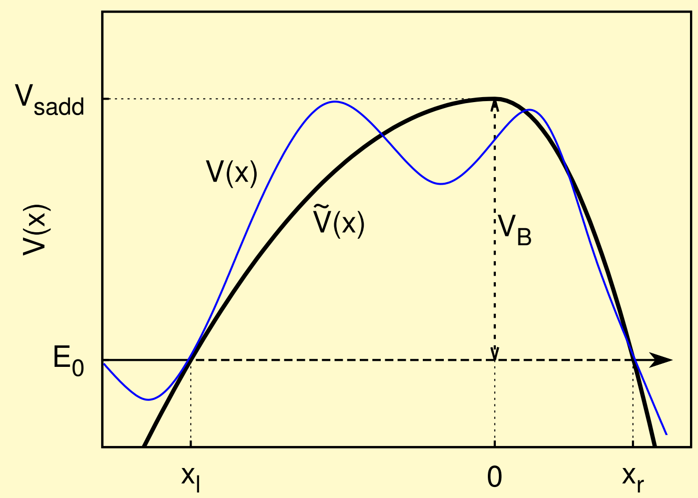


The transformation:

$$x(s) = \int_{s_{\rm sadd}}^{s} \sqrt{\frac{B_{ss}(s')}{m}}\, ds' \,,$$

ensures that $B_{xx} = m$=const.
The potential $V[s(x)]$ in the new coordinate x can be approximated by two (or more) parabolas:

$$\tilde{V}(x) = \begin{cases} V_{\rm sadd} - \frac{1}{2} C_l\, x^2 & \text{for } x < 0 \,, \\ V_{\rm sadd} - \frac{1}{2} C_r\, x^2 & \text{for } x > 0 \,, \end{cases}$$



*K. Pomorski, M. Warda and A. Zdeb, Phys. Scr. **90**, 114013 (2015).

Barrier penetration in the WKB approximation

The spontaneous fission half–life is given by:

$$T^{\rm sf}_{1/2} = \frac{\ln 2}{nP}\ , \quad \text{where} \quad P = \frac{1}{1+\exp\{2S(L)\}.}$$

The WKB action-integral along the fission path $L(x)$ is given by:

$$S(L) = \int_{s_l}^{s_r} \sqrt{\frac{2}{\hbar^2} B_{ss}[V(s) - E_0]}ds \approx \int_{-x_l}^{x_r} \sqrt{\frac{2m}{\hbar^2}[\tilde{V}(x) - E_0]}\, dx$$

For the penetration of the two-inverted parabola barrier of the V_B heights it is equal to:

$$S = \frac{\pi}{2\hbar} V_B \left(\sqrt{\frac{m}{C_l}} + \sqrt{\frac{m}{C_r}} \right) = \frac{\pi}{\hbar} V_B \frac{\omega_l + \omega_r}{2\,\omega_l\,\omega_r} \equiv \frac{\pi}{\hbar} V_B\, \tilde{\omega}^{-1}\ ,$$

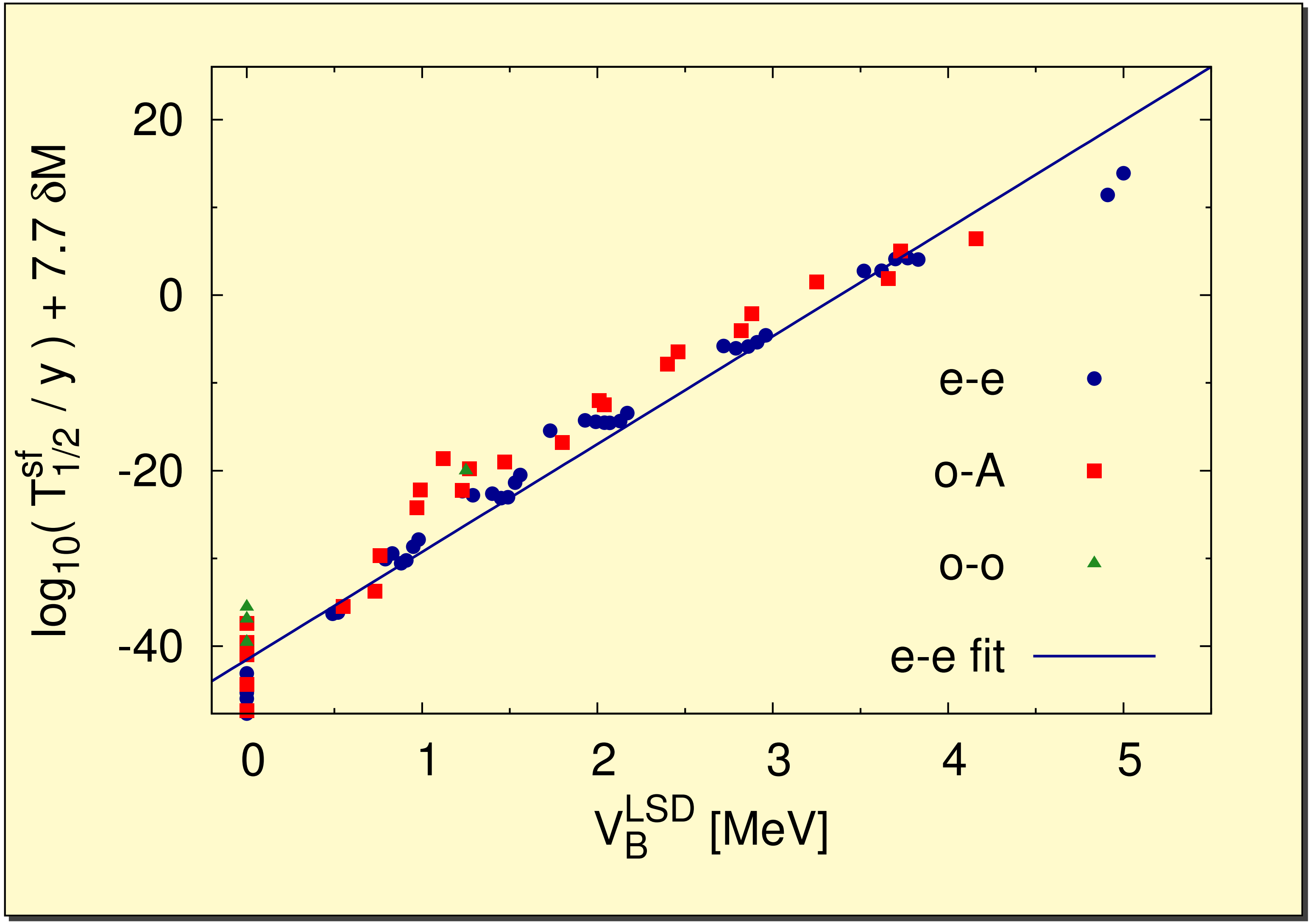
where $\omega_l = \sqrt{C_l/m}$ and $\omega_r = \sqrt{C_r/m}$ are the inverted H.O. frequencies.

For $S \gg 1$ the logarithm of the s.f. half-lives takes the form:

$$\log(T^{\rm sf}_{1/2}) = \frac{2\pi}{\hbar\tilde{\omega}} V_B - \log[n \ln 2] \approx \frac{2\pi}{\hbar\tilde{\omega}} (M^{\rm LSD}_{\rm sadd} - M^{\rm exp}_{gs}) - \log[n \ln 2]\ ,$$

where n is the frequency of assaults against the fission barrier.

$$\log(T^{\rm sf}_{1/2}/{\rm y}) + \frac{2\pi\,\delta{\rm M}}{\hbar\tilde{\omega}} = \frac{2\pi {\rm V}^{\rm LSD}_{\rm B}}{\hbar\tilde{\omega}} - \log({\rm n}\ln 2)\ ; \qquad \delta{\rm M} = {\rm M_{exp}} - {\rm M^{sph}_{LSD}}$$



K. Pomorski, M. Warda and A. Zdeb, Phys. Scr. **90** (2015) 114013.

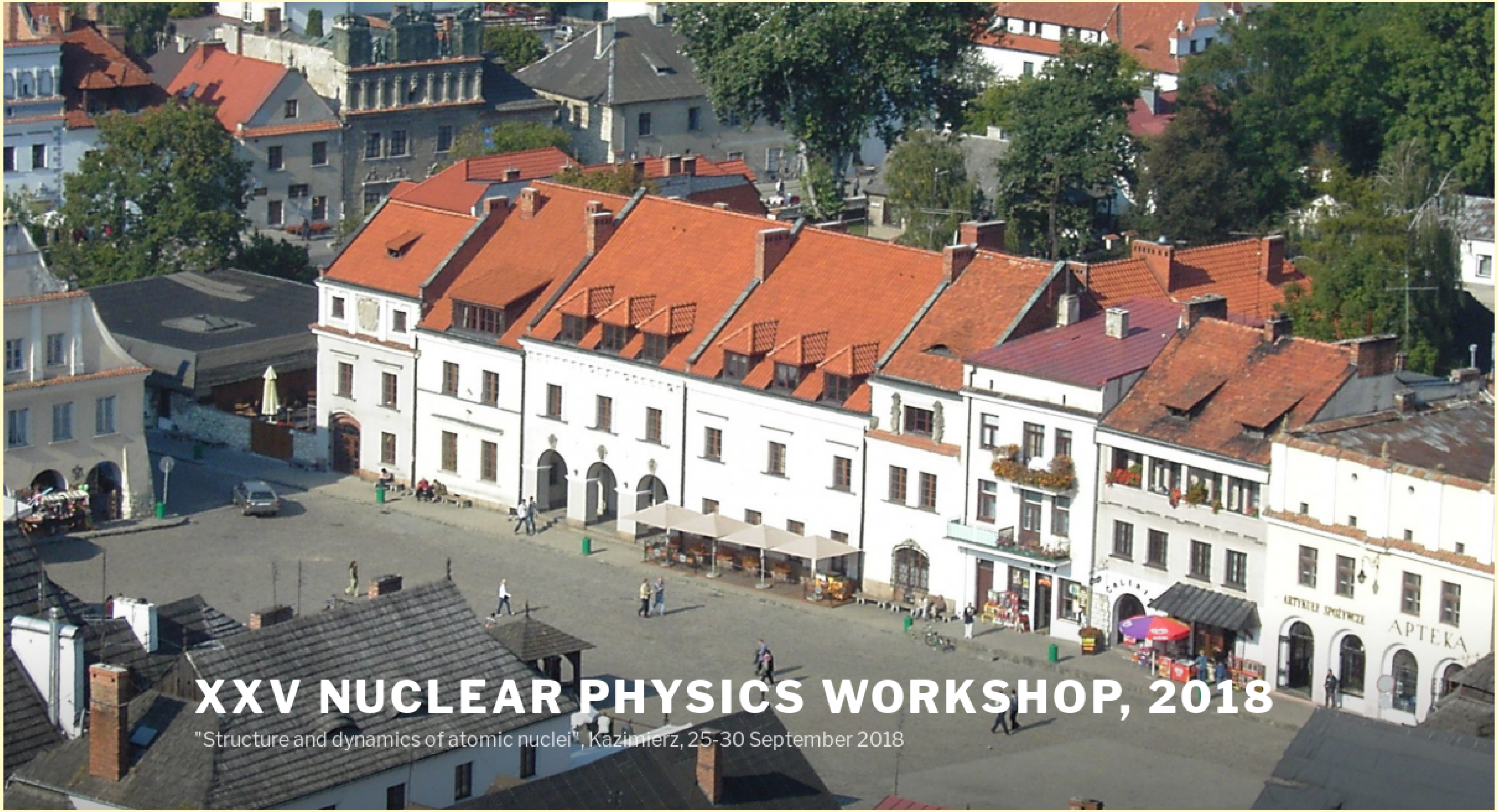
Conclusions:

- Fourier expansion offers a very effective way of describing the shapes of fissioning nuclei.
- Macroscopic-microscopic model basing on the LSD energy and the Yukawa-Folded s.p. potential describes well global properties of the heavy and super-heavy nuclei.
- Simple WKB model with only one adjustable parameter describes well the probabilities of proton, alpha and cluster emission.
- The ground-state shell and pairing effects determine the fission barrier heights. The role of those at the saddle is almost negligible.
- The spontaneous fission life-times of nuclei are mostly determined by the microscopic energy correction in the ground-state and the macroscopic fission barrier.

Thank you for your attention

谢谢!

Dziękuję Państwu za uwagę



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