



1st Symposium on intermediate-energy Heavy Ion Collisions

Preliminary study of *gravitational wave*
associated with *first-order* transition
in the core of neutron star

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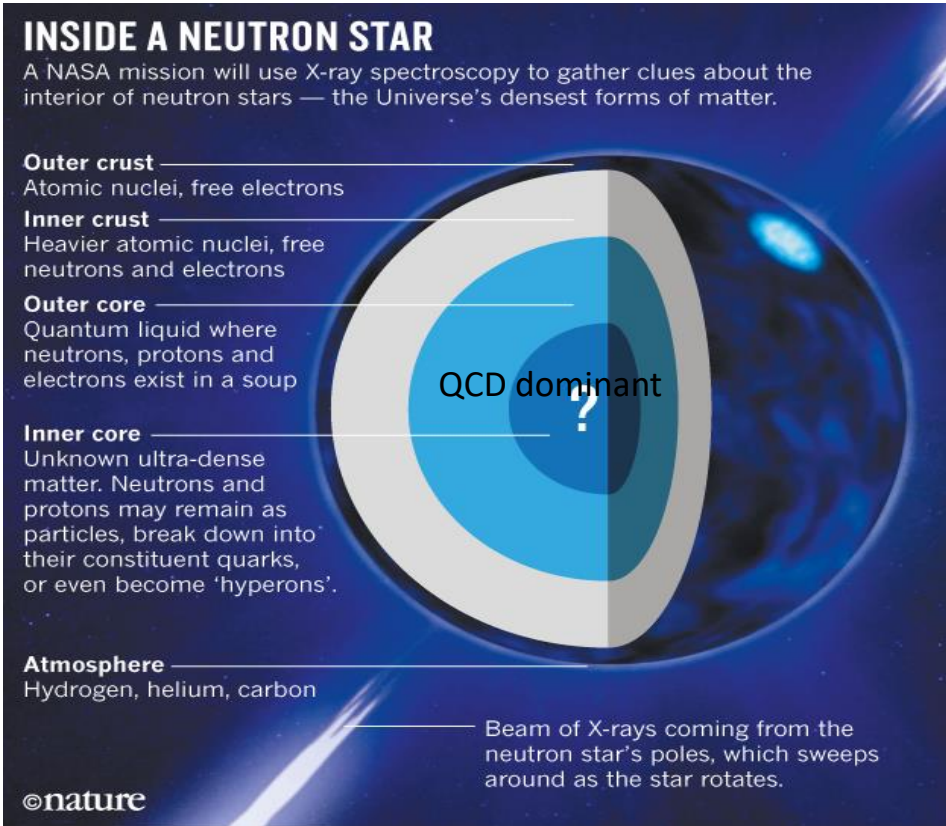


Outline

- ◆ QCD phases in neutron stars
- ◆ Gravitational wave — new probe
- ◆ Bubble dynamics during first-order transition
- ◆ Preliminary calculations
- ◆ Summary and prospective



QCD phases in neutron stars



Possible phases for “?”:

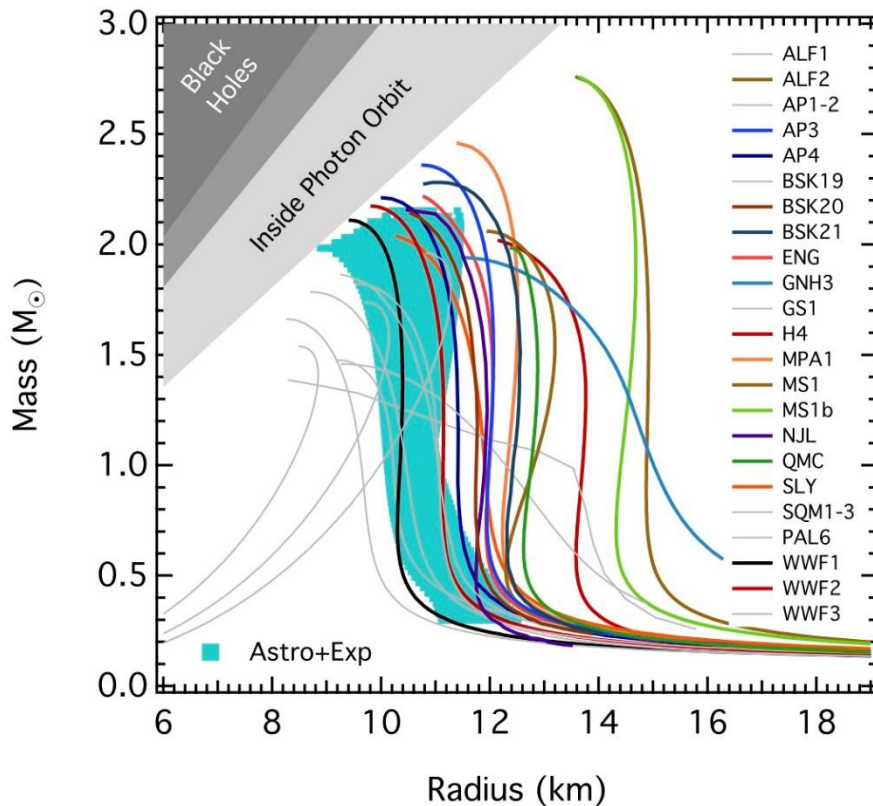
Chiral restoration,
Quarkyonic matter,
Pion superfluidity,
Kaon superfluidity,
Color superconductivity,
Color-flavor-locking.

Significance: compensate the “criticality” exploration at HIC which is usually limited at low density and high temperature



Traditional probes

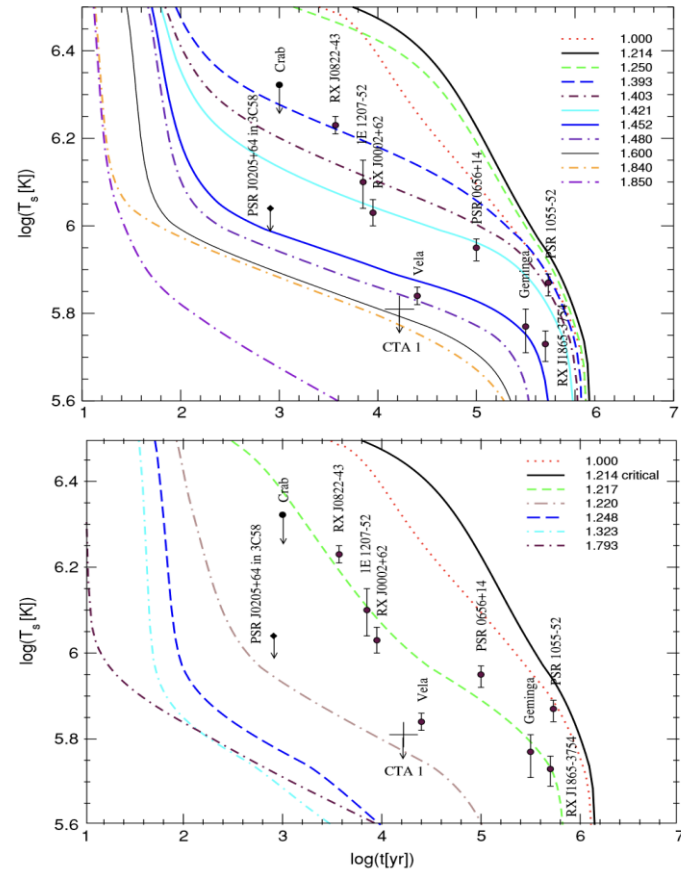
EM field exploration: Mass-radius relationship



[O. Feryal, et al. Astrophys.J. 820 \(2016\) no.1, 28](#)

$2.01 \pm 0.04 M_{\odot}$ from a combination of radio timing and precise spectroscopy of the white dwarf companion for **PSR J0348+0432**

Neutrino emission:



Neutron star

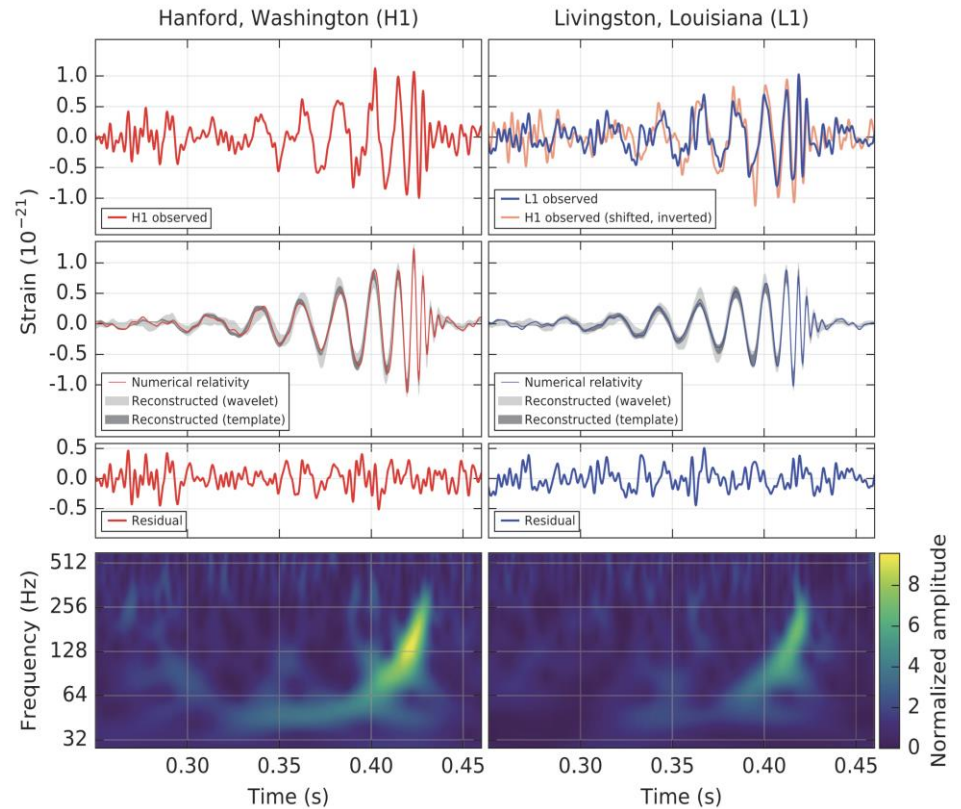
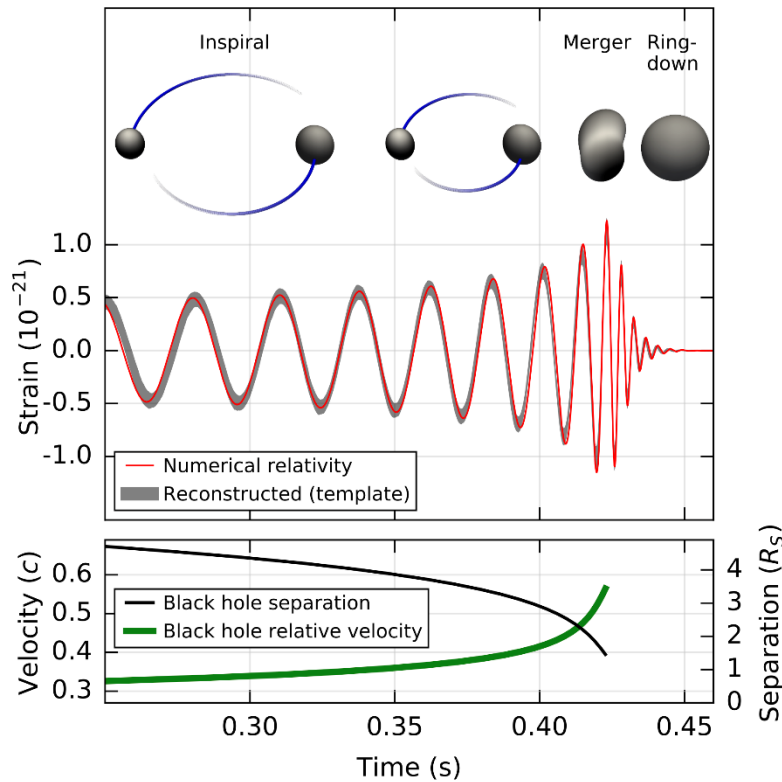
Hybrid star

[Hovik Grigorian, et al., Phys. Rev. C 71, 045801 \(2005\)](#)



Gravitational wave — new probe

GW150914 : Binary Black Hole Merger



[B.P. Abbott et al., PRL 116, 061102 \(2016\)](#)



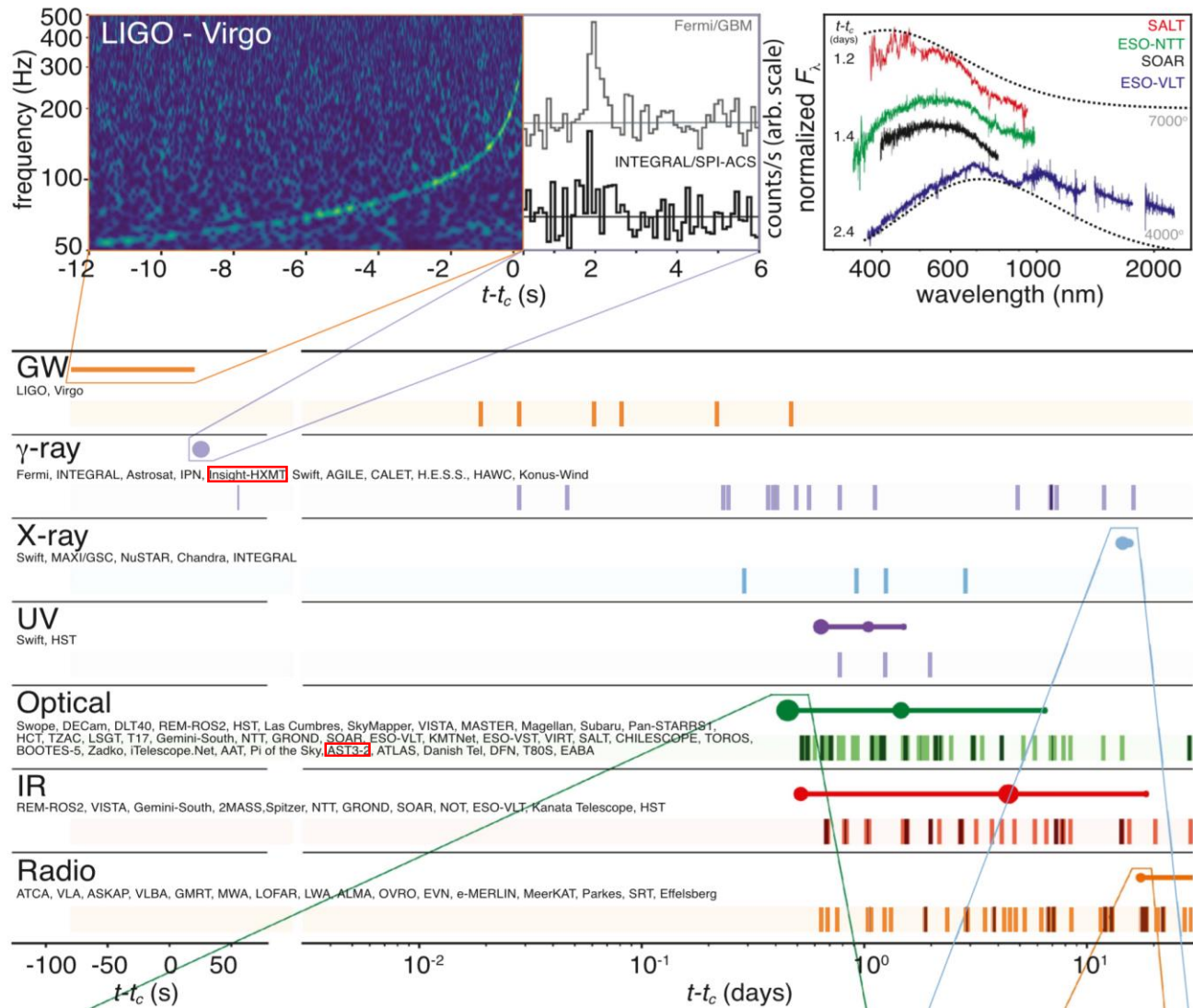
Multi-messenger Observations

GW170817,
GRB 170817A,
SSS17a/AT 2017gfo:

Binary Neutron
Star Merger

Refer to C. Horowitz,
Liewen Chen,
and Ang Li's talks

[B. P. Abbott, et al., ApJL, 848:L12, 2017](#)





Four Steps to GWD

**Step-3:
TianQin
2016-2035**

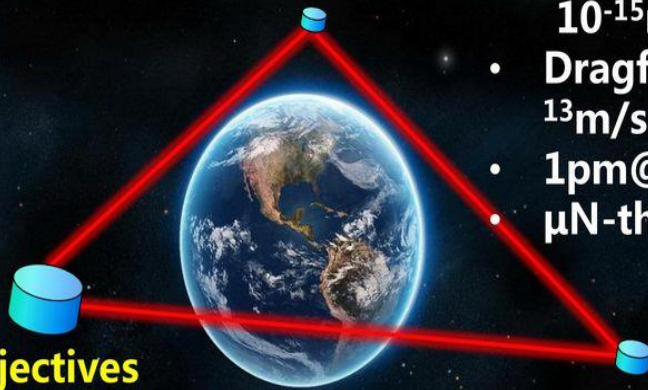
Technology objectives

- Inertial sensor 10^{-15}m/s^2
- Dragfree 10^{-13}m/s^2
- $1 \text{pm}@10^7 \text{m}$
- μN -thruster

Science objectives

- General Relativity
- GW astronomy

Class. Quantum Grav. 33,
035010 (2016)



Detailed objectives:

Stochastic GW background,
White draft binaries,
Intermedium-mass BH,
Tidal deformation effect...

**Our proposal:
QCD transition signal**

Collapses of rapidly rotating NS:

[L.-M. Lin, et al., ApJ 639:
382–396, 2006](#)

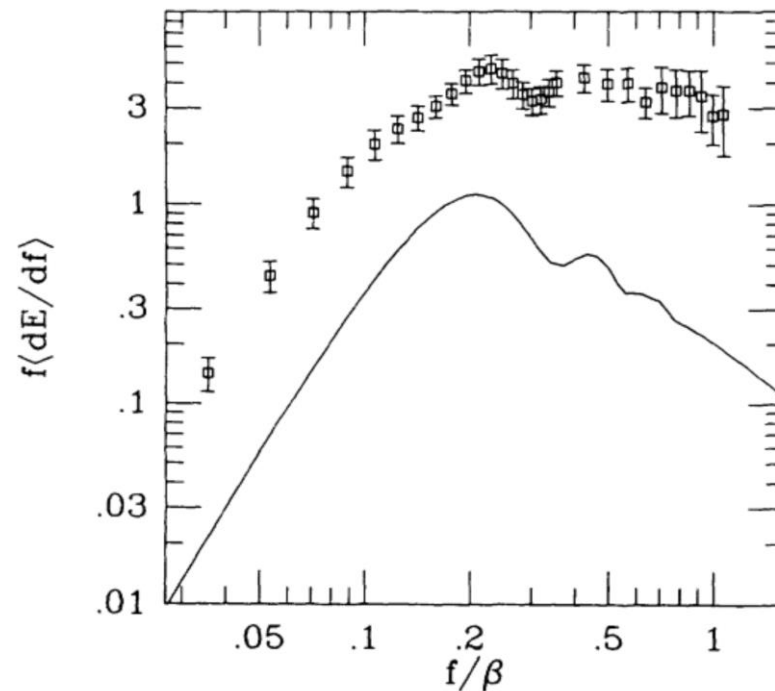
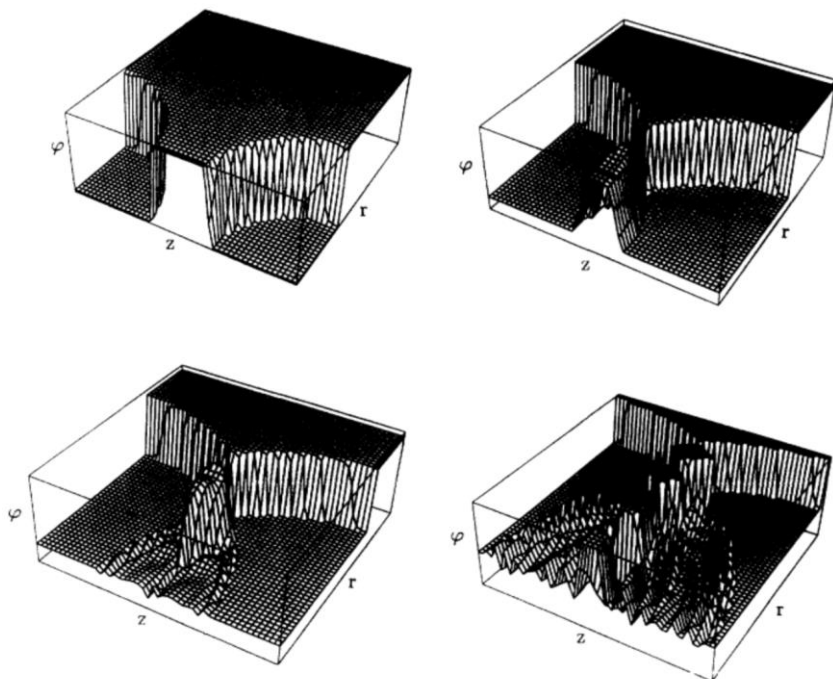
Early universe cooling:

[Y. Chen, M. Huang and Q. Yan,
arXiv:1712.03470](#)



GW from first-order transition

Supercooling for first-order transition



Bubble collisions produce quadrupole moment

Peaks show in the differential spectrum at $\frac{f}{\beta} \sim 0.2$ for early universe phase transition

[A. Kosowsky, M.S. Turner, and R. Watkins, Phys. Rev. D45, 4514 \(1992\); Phys. Rev. Lett. 69, 2026 \(1992\).](#)



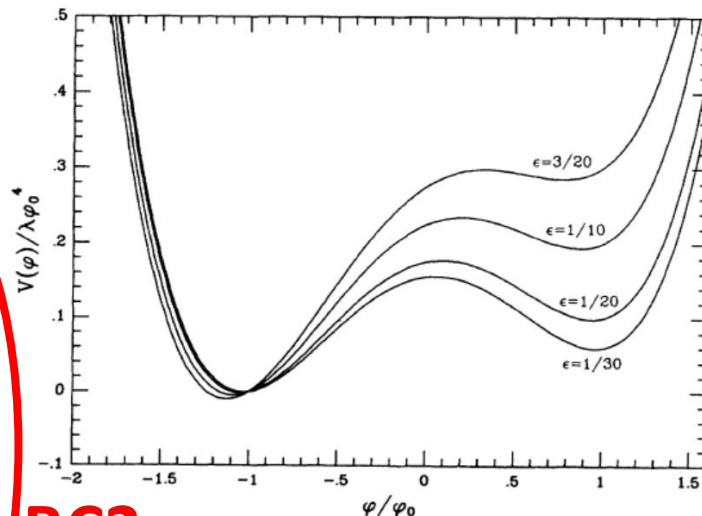
Bubble dynamics

One bubble - $O(4)$ symmetry $\varphi = \varphi(\rho)$, $\rho = (t_E^2 + \mathbf{x}^2)^{1/2}$:

$$\mathcal{L} = \frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi - V(\varphi)$$

EOM1:
$$\frac{d^2 \varphi}{d\rho^2} + \frac{3}{\rho} \frac{d\varphi}{d\rho} = \frac{\partial V}{\partial \varphi}$$

BC1:
$$\lim_{\rho \rightarrow \infty} \varphi(\rho) = \varphi_+, \quad \left. \frac{d\varphi}{d\rho} \right|_{\rho=0} = 0$$



Two bubble - $O(3)$ symmetry :

$$\mathcal{L} = \frac{1}{2} \left[\frac{\partial \varphi}{\partial s} \right]^2 - \frac{1}{2} \left[\frac{\partial \varphi}{\partial z} \right]^2 - V(\varphi), \quad \text{BC2}$$

$$V_0(\varphi) = \frac{\lambda}{8} (\varphi^2 - \varphi_0^2)^2,$$

$$V(\varphi) = V_0(\varphi) + \epsilon \lambda \varphi_0^3 (\varphi + \varphi_0),$$

EOM2:
$$\frac{\partial^2 \varphi}{\partial s^2} + \frac{2}{s} \frac{\partial \varphi}{\partial s} - \frac{\partial^2 \varphi}{\partial z^2} = - \frac{\partial V}{\partial \varphi} \quad \varphi(\mathbf{x}, t) = \varphi(t^2 - r^2, z)$$

Based on [S. Coleman, Phys. Rev. D 15, 2929 \(1977\)](#); [C. Callan and S. Coleman, ibid. 16, 1762 \(1977\)](#).



Effective potential in quark-meson model

Lagrangian density (Renormalizable):

$$\begin{aligned}\mathcal{L}_{QM} = & \bar{q} \left[i \not{\partial} + (\mu_B / N_c + \mu_I \tau_3) \gamma^0 - g_q (\sigma + i \gamma^5 \boldsymbol{\tau} \cdot \boldsymbol{\pi}) \right] q \\ & + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi}) - \frac{\lambda}{4} (\sigma^2 + \boldsymbol{\pi} \cdot \boldsymbol{\pi} - v^2)^2 + c \sigma.\end{aligned}$$

Integrate over quark degrees of freedom and keep only σ dynamics:

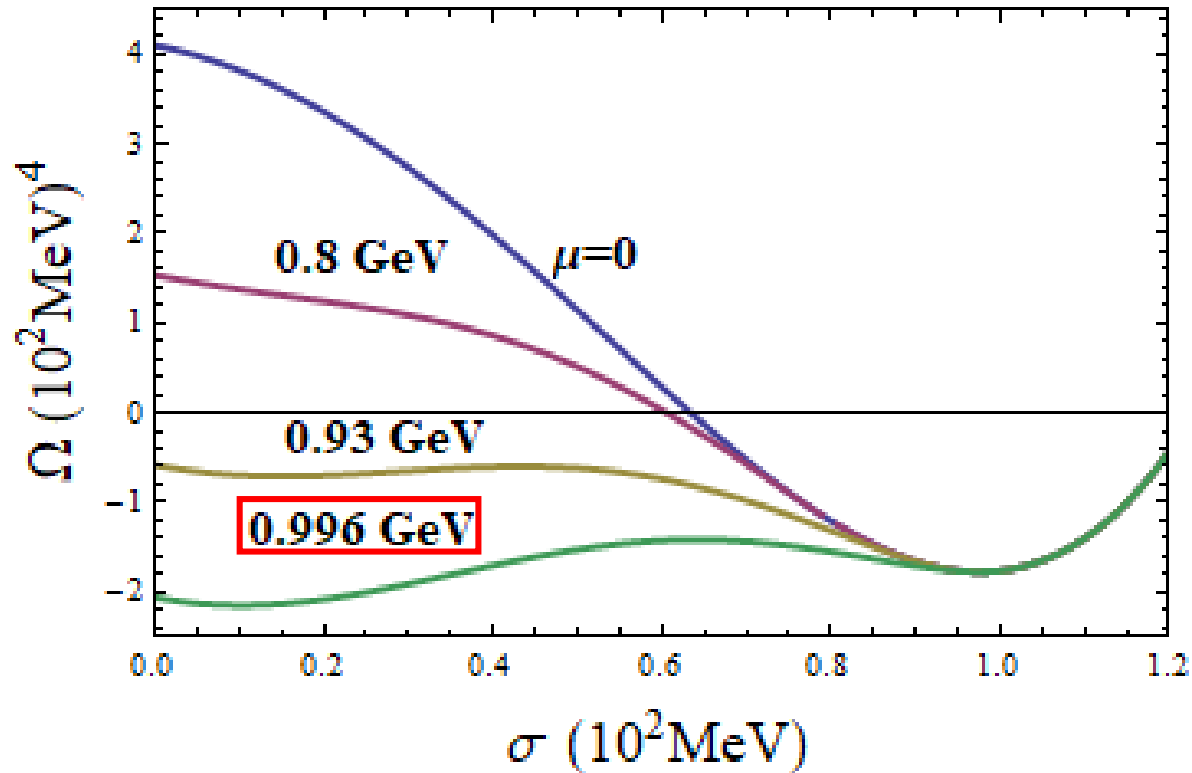
$$\mathcal{L}_s = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma) - \frac{\lambda}{4} (\sigma^2 - v^2)^2 + c \sigma + \Omega_{QM}^t(\sigma)$$

$$\Omega_{QM}^t(\sigma) = \frac{1}{12\pi^2} \left[2p_F^3 \mu_B - 3 \left(p_F \mu_B m_q^2 - N_c m_q^4 \operatorname{arccosh} \left(\frac{\mu_B}{N_c m_q} \right) \right) \right] \theta(\mu_B - N_c m_q)$$

$$p_F(\sigma(x)) = \left((\mu_B / N_c)^2 - m_q^2(x) \right)^{1/2} \quad m_q(x) = g_q \sigma(x).$$



Chiral symmetry for different μ_B

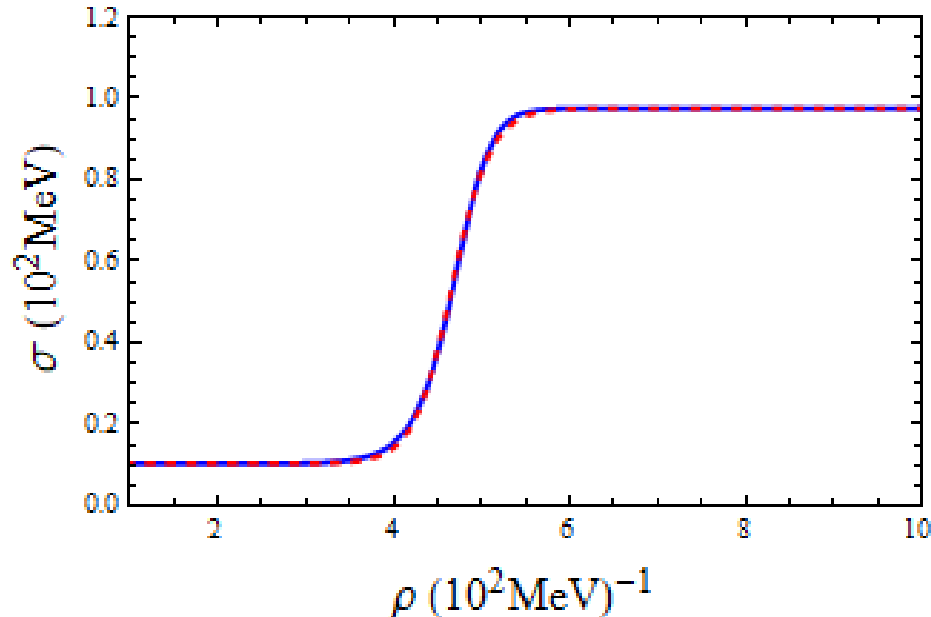


Chiral symmetry is gradually restored by increasing chemical potential – first order



Bubble dynamics for chiral restoration

One bubble generation



- 1) Fine tuned: $10^{-4}\%$ for true vacuum;
- 2) Well-fitted to solitonic solution

$$\sigma(\rho) = \frac{\sigma_T}{2} \left(1 - \tanh\left(\frac{\rho - R_B}{d_w}\right) \right) + \frac{\sigma_F}{2} \left(1 + \tanh\left(\frac{\rho - R_B}{d_w}\right) \right)$$

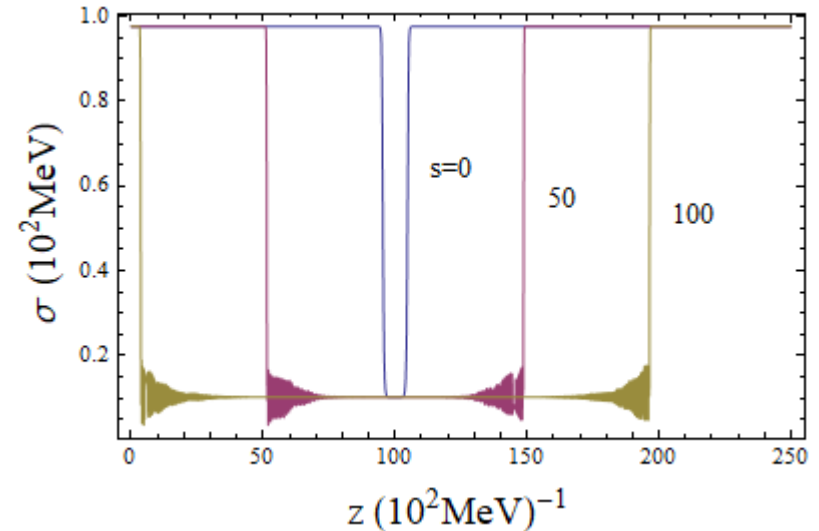
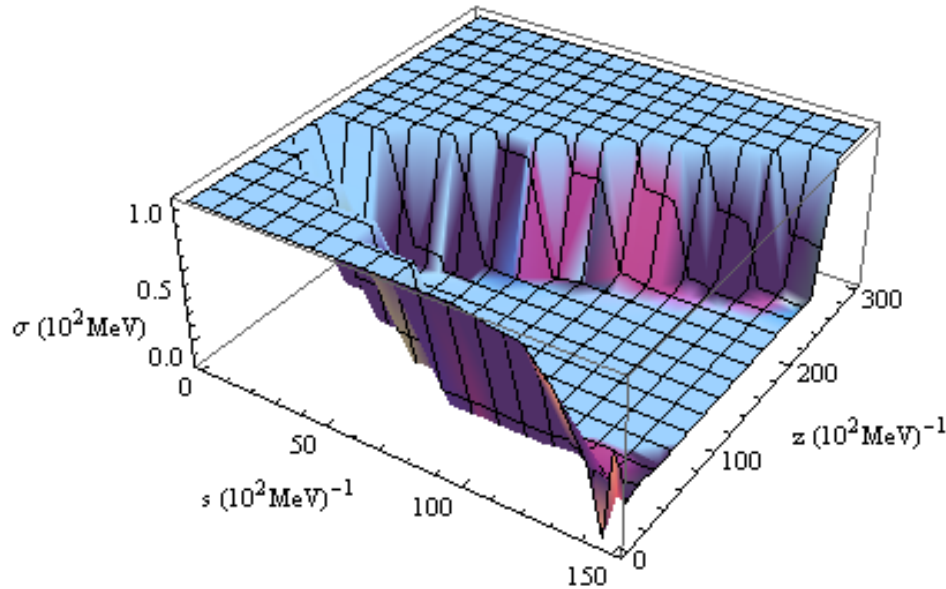
$$R_B = 4.66 * 10^{-2} \text{MeV}^{-1} \text{ and } d_w = 4.38 * 10^{-3} \text{MeV}^{-1}$$



Bubble dynamics for chiral restoration

Two bubble evolution

Stephen William Hawking, J. M. Stewart, and I. G. Moss, [Phys. Rev. D 26, 2681 \(1982\)](#).



- 1) Bubble walls expand relativistically and nearly symmetrically;
- 2) Oscillations have developed before bubble collision.



Preliminary results for the GW energy emission

$$\frac{dE}{d\omega d\Omega} = G\omega^2 |T^{zz}(\hat{\mathbf{k}}, \omega)\sin^2\theta + T^{xx}(\hat{\mathbf{k}}, \omega)\cos^2\theta - T^{yy}(\hat{\mathbf{k}}, \omega) - 2T^{xz}(\hat{\mathbf{k}}, \omega)\sin\theta\cos\theta|^2$$

Inside light-cone

$$T_1^{xx}(\hat{\mathbf{k}}, \omega) = \frac{1}{2} \int_0^\infty s^2 ds \int_{-\infty}^\infty dz e^{i\omega_z z} \left[\frac{\partial\varphi}{\partial s} \right]^2 \int_1^\infty du (u^2 - 1) e^{i\omega s u} \{J_0[\omega_x s (u^2 - 1)^{1/2}] - J_2[\omega_x s (u^2 - 1)^{1/2}]\} C(su),$$

$$T_1^{yy}(\hat{\mathbf{k}}, \omega) = \frac{1}{2} \int_0^\infty s^2 ds \int_{-\infty}^\infty dz e^{i\omega_z z} \left[\frac{\partial\varphi}{\partial s} \right]^2 \int_1^\infty du (u^2 - 1) e^{i\omega s u} \{J_0[\omega_x s (u^2 - 1)^{1/2}] + J_2[\omega_x s (u^2 - 1)^{1/2}]\} C(su),$$

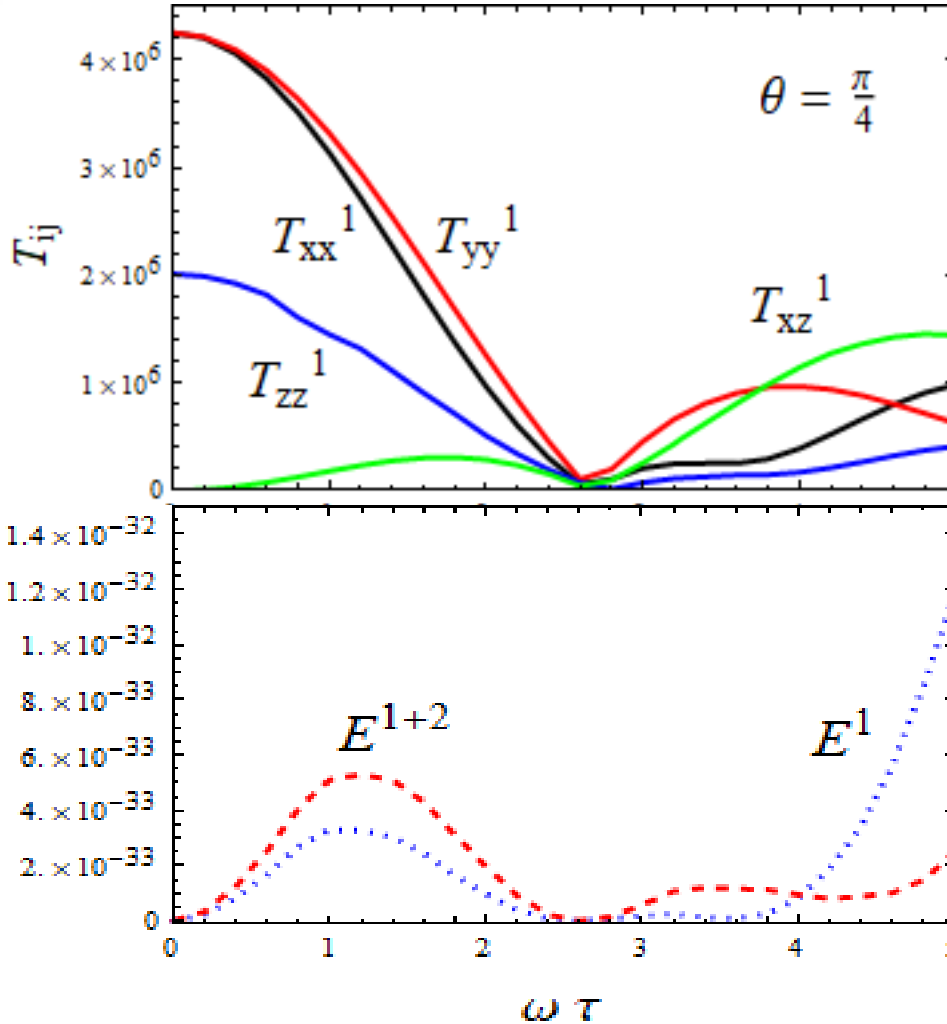
$$T_1^{zz}(\hat{\mathbf{k}}, \omega) = \int_0^\infty s^2 ds \int_{-\infty}^\infty dz e^{i\omega_z z} \left[\frac{\partial\varphi}{\partial z} \right]^2 \int_1^\infty du e^{i\omega s u} J_0[\omega_x s (u^2 - 1)^{1/2}] C(su),$$

$$T_1^{xz}(\hat{\mathbf{k}}, \omega) = -i \int_0^\infty s^2 ds \int_{-\infty}^\infty dz e^{i\omega_z z} \left[\frac{\partial\varphi}{\partial s} \right] \left[\frac{\partial\varphi}{\partial z} \right] \int_1^\infty du (u^2 - 1)^{1/2} e^{i\omega s u} J_1[\omega_x s (u^2 - 1)^{1/2}] C(su).$$

$$\hat{\mathbf{k}}_x = \sin\theta, \quad \hat{\mathbf{k}}_y = 0, \quad \hat{\mathbf{k}}_z = \cos\theta \quad C(t) = \begin{cases} 1, & 0 \leq t \leq \tau_c, \\ \exp[-(t - \tau_c)^2 / \tau_0^2], & \tau_c \leq t \leq \tau \end{cases}$$



Vary with frequency

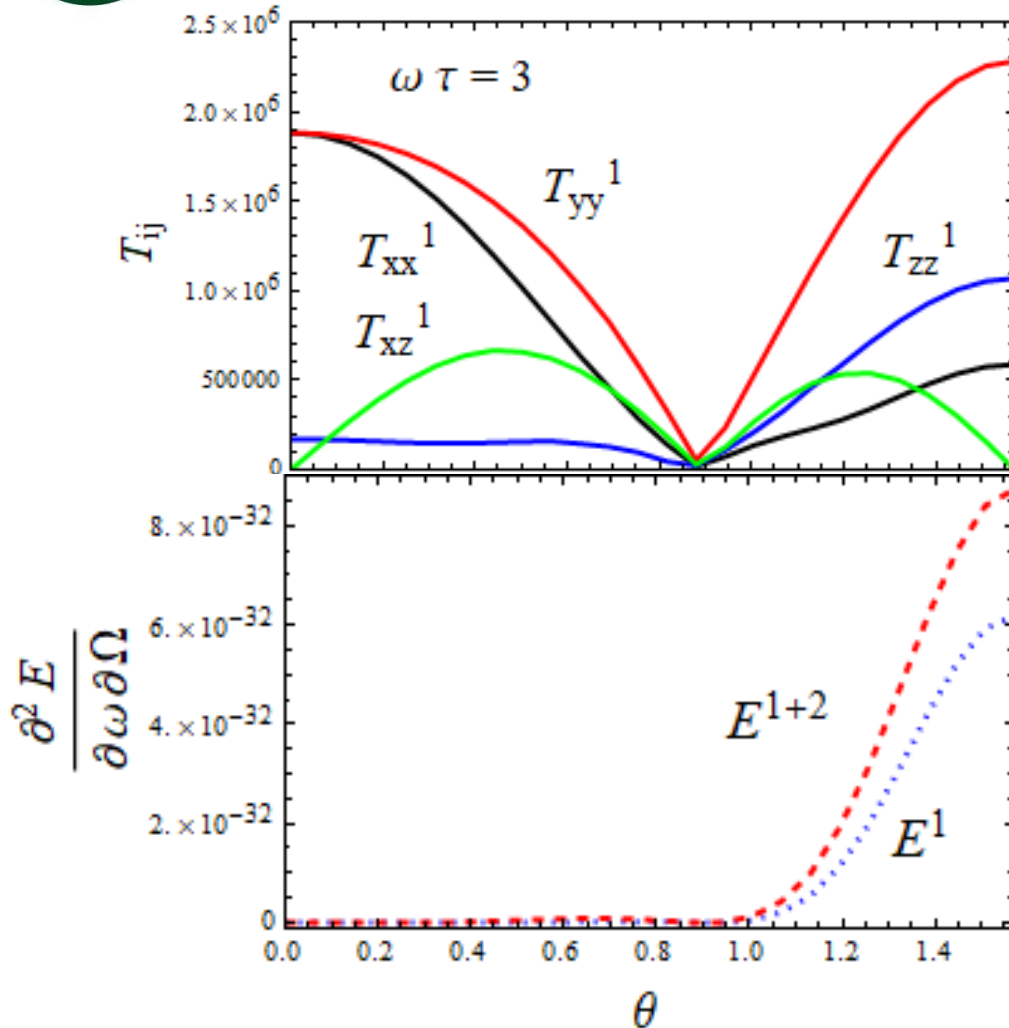


- 1) Transverse terms dominate at low frequency
- 2) Cross term contribute at large frequency
- 3) All show minima at the same point

- 1) Inside light-cone dominates GW emission at low frequency
- 2) Large cancellation between inside and outside light-cone contribution
- 3) Oscillation for the total emission



Vary with angle



- 1) Transverse terms dominate at low angle
- 2) Cross term jumps twice
- 3) All show minima at the same point

- 1) Inside light-cone dominates GW emission
- 2) Only large angle is important for the emission

Consistent with

[A. Kosowsky, M.S. Turner, and R. Watkins, Phys. Rev. D45, 4514 \(1992\).](#)



Summary and prospective

- **GW emission** from **first-order QCD** transition is preliminarily studied by following **bubble dynamics**
- This applies to the **collapse of neutron star** with large chemical potential in the inner core, maybe to **EM emission** if first-order transition is realized in HIC
- Angle integrated GW emission、 multi-bubble dynamics、 bubble generation rate
- More realistic study: non-Lorentz invariant dynamics、 contributions from **pion interactions**、 including **nuclear matter phase**
- Further extensions: **color superconductivity**、 **liquid-gas transition**、 **neutron star-black hole binary**

Thanks!