Pion transport in heavy ion collisions

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- Introduction
  - Pion yield and nuclear matter equation of state
  - Charged pion ratio and nuclear symmetry energy
- Pion production in transport models
- Threshold effects on charged pion ratio
- Pion in-medium effects on charged pion ratio
- Summary and conclusions

Based on work with **Jun Xu** [PRC 81,024910 (2010); 87, 067601 (2013)], **Taesoo Song** [PRC 91, 014901 (2015)] and **Zhen Zhang** [PRC 95, 064604 (2017); 97,014610 (2018)]

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Microscopic Theory of Pion Production and Sidewards Flow in Heavy-Ion Collisions

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(Received 9 July 1984)

Nuclear collisions from 0.3 to 2 GeV/nucleon are studied in a microscopic theory based on Vlasov's self-consistent mean field and Uehling-Uhlenbeck's two-body collision term which respects the Pauli principle. The theory explains simultaneously the observed collective flow and the pion multiplicity and gives their dependence on the nuclear equation of state.

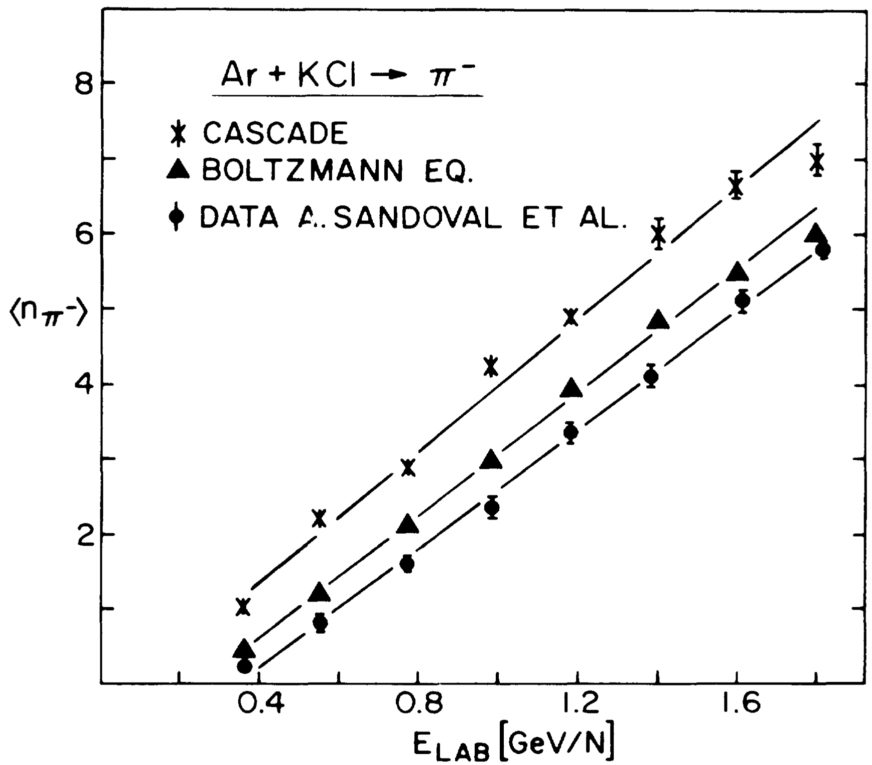
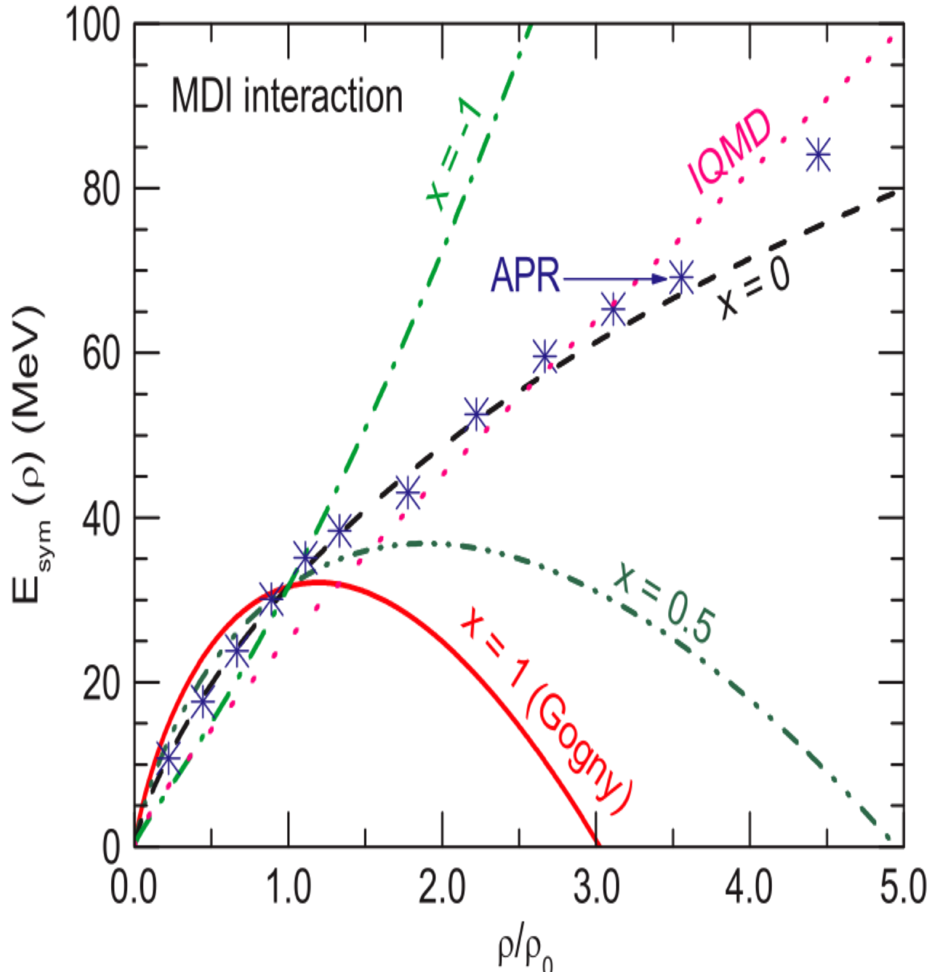
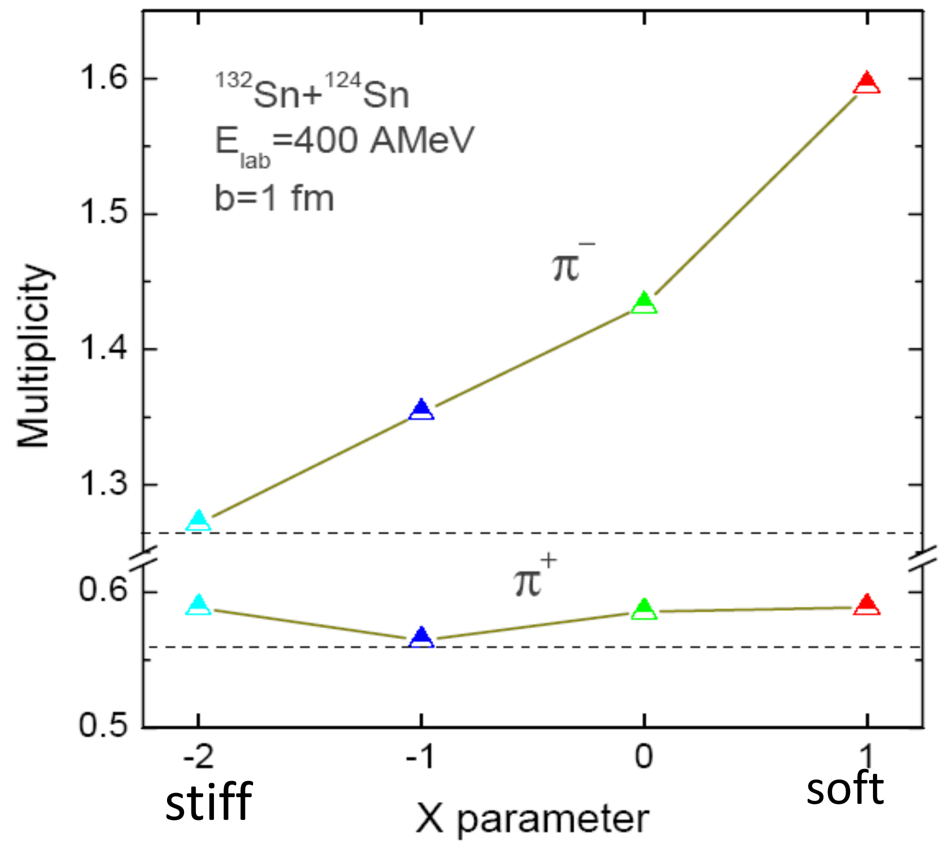


FIG. 2. Pion multiplicity for central collisions ($b < 2.4$ fm) of Ar+KCl. The data (Ref. 6, circles) are compared to the present theory in the "cascade mode" (crosses) and to the same theory with compression energy and phase-space Pauli blocking included (triangles).

Near-threshold pion production with high energy radioactive beams (IBUU)

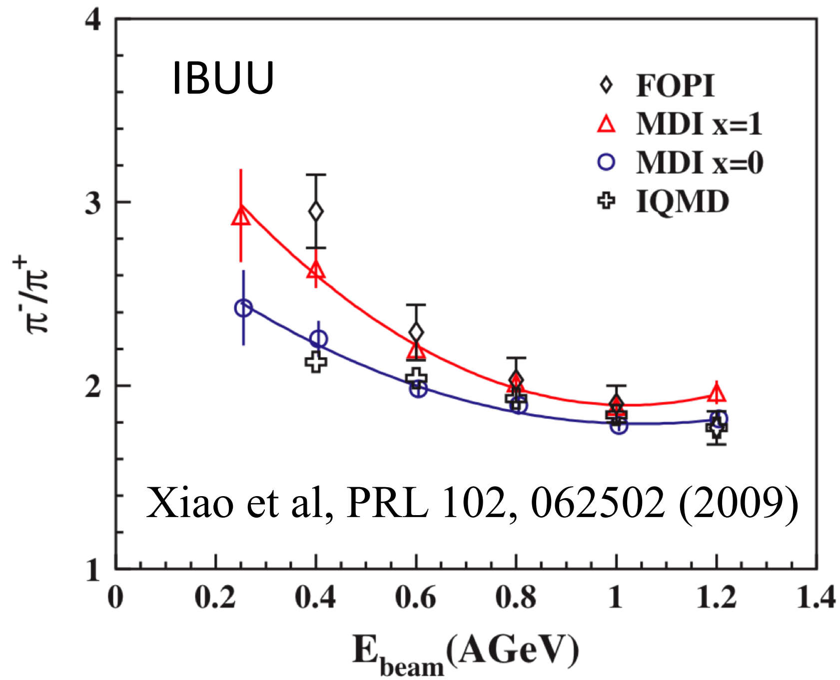
B. A. Li, PRL 88, 192701 (2002)

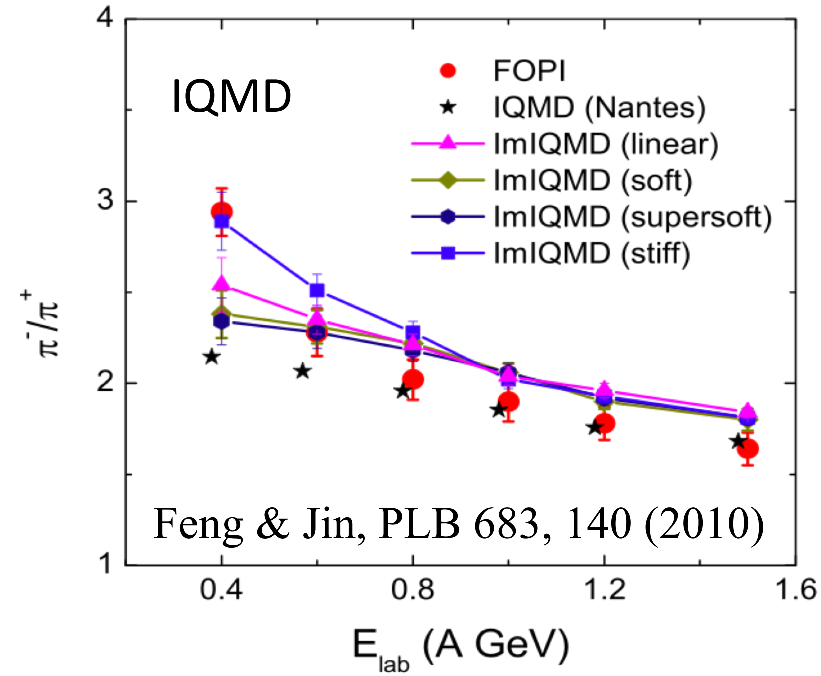


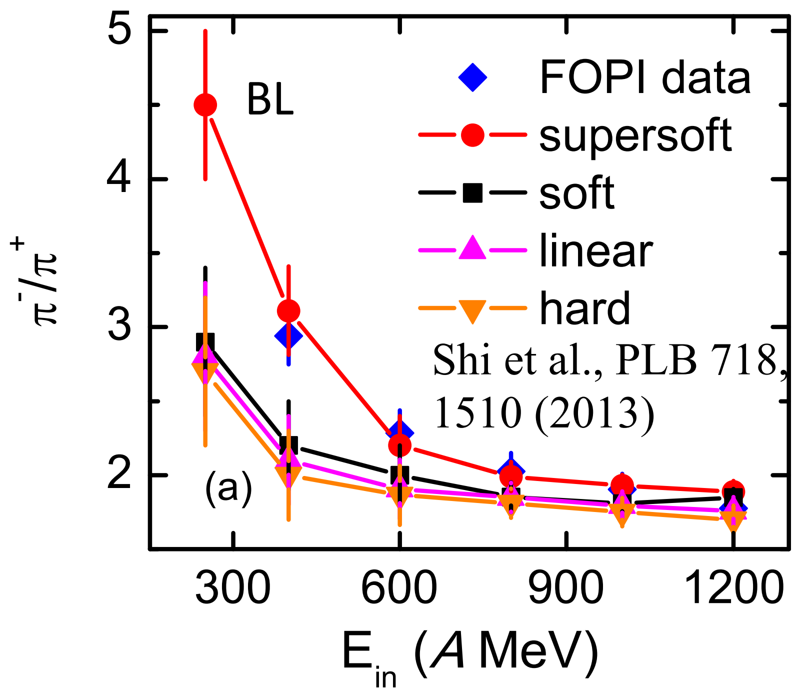


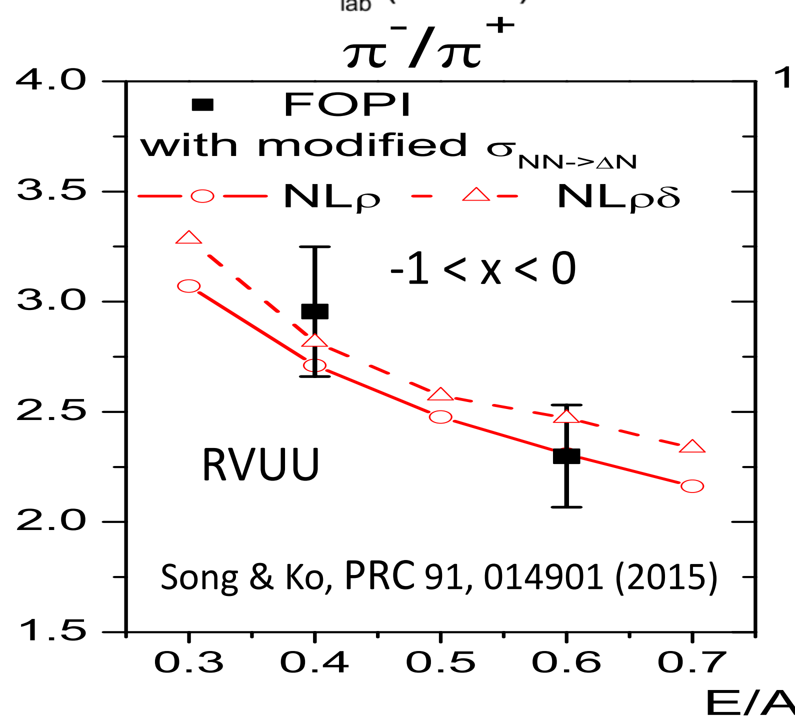
π^- yield is sensitive to the symmetry energy $E_{sym}(\rho)$ since they are mostly produced in the neutron-rich region, with softer one (x=1) giving more π^- than stiffer one (x=-1).

Conflicting results on symmetry energy from charged pion ratio





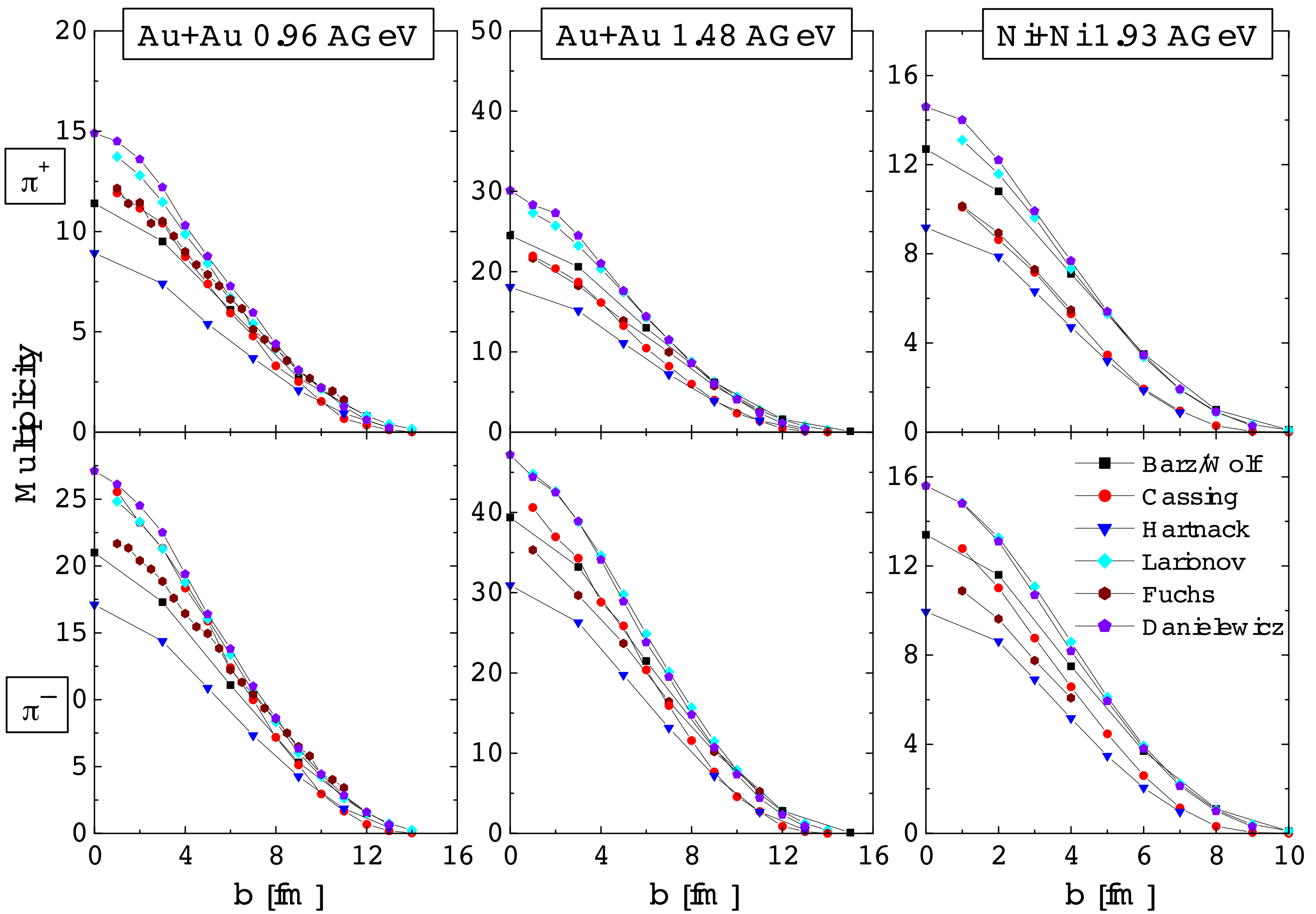




Transport model predictions for pion production in HIC

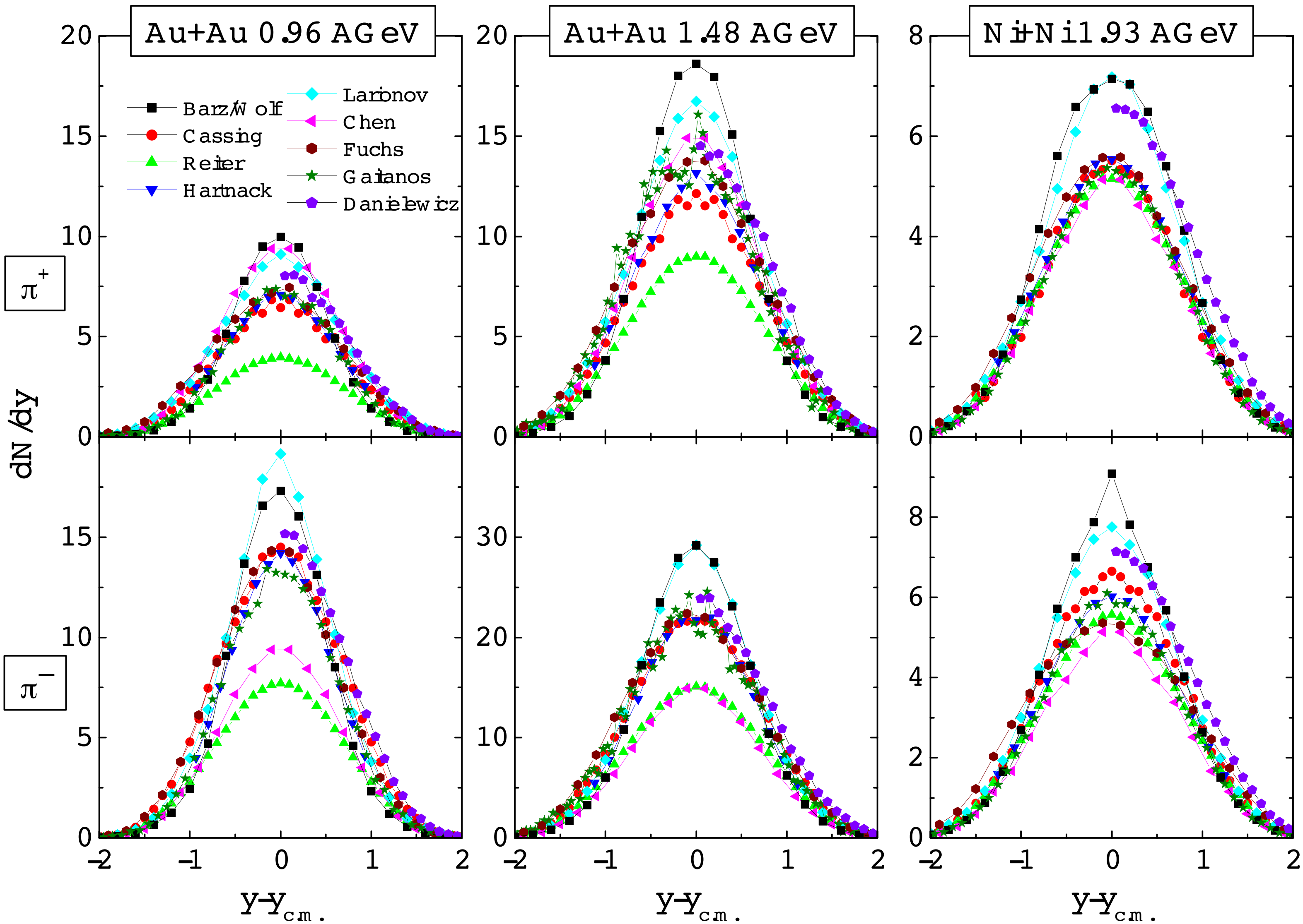
- Centrality dependence

Kolomeitsev et al., J. Phys. G 31, S741 (2005)



- Results from different transport models can differ by ~ 2.

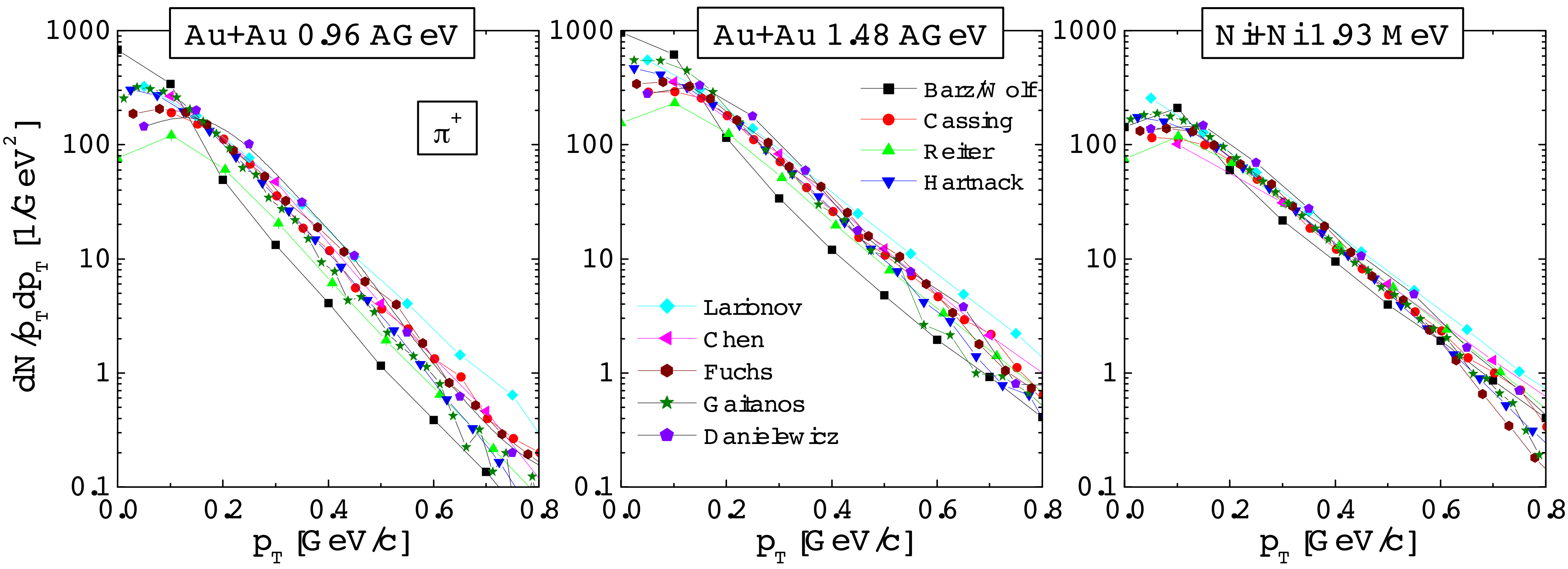
- Rapidity distributions: Impact parameter b = 1fm, Delta width Γ_Δ = 120 MeV



- Results from different transport models can differ by ~ 2, particularly at midraipidy.

- Transverse momentum spectra

Impact parameter b = 1fm, Rapidity $|y_{c.m.}| \leq 0.5$,
Delta width Γ_Δ = 120 MeV



- Results from different transport models can differ by ~ 2, particularly at low energy collisions.

In-medium threshold effects on pion production

$$U_{asy}^{\Delta^{++}} = U_{asy}^{p}, \quad U_{asy}^{\Delta^{+}} = \frac{2}{3}U_{asy}^{p} + \frac{1}{3}U_{asy}^{n}, \quad U_{asy}^{\Delta^{0}} = \frac{1}{3}U_{asy}^{p} + \frac{2}{3}U_{asy}^{n}, \quad U_{asy}^{\Delta^{-}} = U_{asy}^{n}$$

- $pn \rightarrow p\Delta^0$

  Initial-state potential: U_p+U_n

  Final-state potential: $U_p+U_{\Delta 0}=U_p+U_p/3+2/3U_n$

  → difference in initial and final potentials: $(U_n-U_p)/3>0$ in neutron-rich matter

  → reduced production threshold

- First studied by Ferini, Colonna, Gaitanos and Di Toro (NPA 762, 147 (2005)) in a relativistic transport model

Relativistic Vlasov-Uehling-Uhlenbeck model

Ko, NPA 495, 321 (1989)

$$\frac{\partial}{\partial t} f + \vec{v} \cdot \nabla_r f - \nabla_r H \cdot \nabla_p f = \mathcal{C}[f]$$

Mean-field potential $$H = \sqrt{m^{*2} + p^{*2}} + g_\omega \omega^0 \pm g_\rho (\rho_3)_0$$

Collisional integral *C*[*f*] includes nucleon-nucleon elastic scattering NN → NN based on empirical cross sections as well as inelastic scattering NN → NΔ and its inverse reaction NΔ → NN using cross sections from the one-boson exchange model of Huber and Aichelin [NPA 573, 587 (1994)]

Delta resonances satisfy a similar RVUU equation with mean-field potentials related to those of nucleons via their isospin structures in terms of those of nucleons and pions

$$\begin{aligned}
m^*_{\Delta^{++}} &= m_\Delta - g_\sigma \sigma - g_\delta \delta_3, & p^{\mu *}_{\Delta^{++}} &= p^\mu_\Delta - g_\omega \omega^\mu - g_\rho \rho^\mu_3 \\
m^*_{\Delta^{+}} &= m_\Delta - g_\sigma \sigma - \frac{1}{3} g_\delta \delta_3, & p^{\mu *}_{\Delta^{+}} &= p^\mu_\Delta - g_\omega \omega^\mu - \frac{1}{3} g_\rho \rho^\mu_3 \\
m^*_{\Delta^{0}} &= m_\Delta - g_\sigma \sigma + \frac{1}{3} g_\delta \delta_3, & p^{\mu *}_{\Delta^{0}} &= p^\mu_\Delta - g_\omega \omega^\mu + \frac{1}{3} g_\rho \rho^\mu_3 \\
m^*_{\Delta^{-}} &= m_\Delta - g_\sigma \sigma + g_\delta \delta_3, & p^{\mu *}_{\Delta^{-}} &= p^\mu_\Delta - g_\omega \omega^\mu + g_\rho \rho^\mu_3, \\
m^2_\delta \delta_3 &= g_\sigma (\phi_p - \phi_n) & m^2_\rho \rho^\mu_3 &= g_\rho (j^\mu_p - j^\mu_n)
\end{aligned}$$

Medium modification of Delta production threshold

Threshold energy for NN → NΔ (1+2 → 3+4) is determined by requiring the kinetic momenta of final nucleon and Delta are zero in the frame where their total kinetic momentum vanishes ($\mathbf{p}_3^* + \mathbf{p}_4^* = 0$)

$$\sqrt{s_{\rm th}} = \sqrt{(m_3^* + \Sigma_3^0 + m_4^* + \Sigma_4^0)^2 - |\mathbf{\Sigma}_3 + \mathbf{\Sigma}_4|^2}$$

where Σ^μ is vector self energy of nucleon or Delta. Since the initial energy of the two nucleons is

$$\sqrt{s_{\rm in}} = \sqrt{(E_1^* + \Sigma_1^0 + E_2^* + \Sigma_2^0)^2 - |\mathbf{\Sigma}_1 + \mathbf{\Sigma}_2|^2}$$

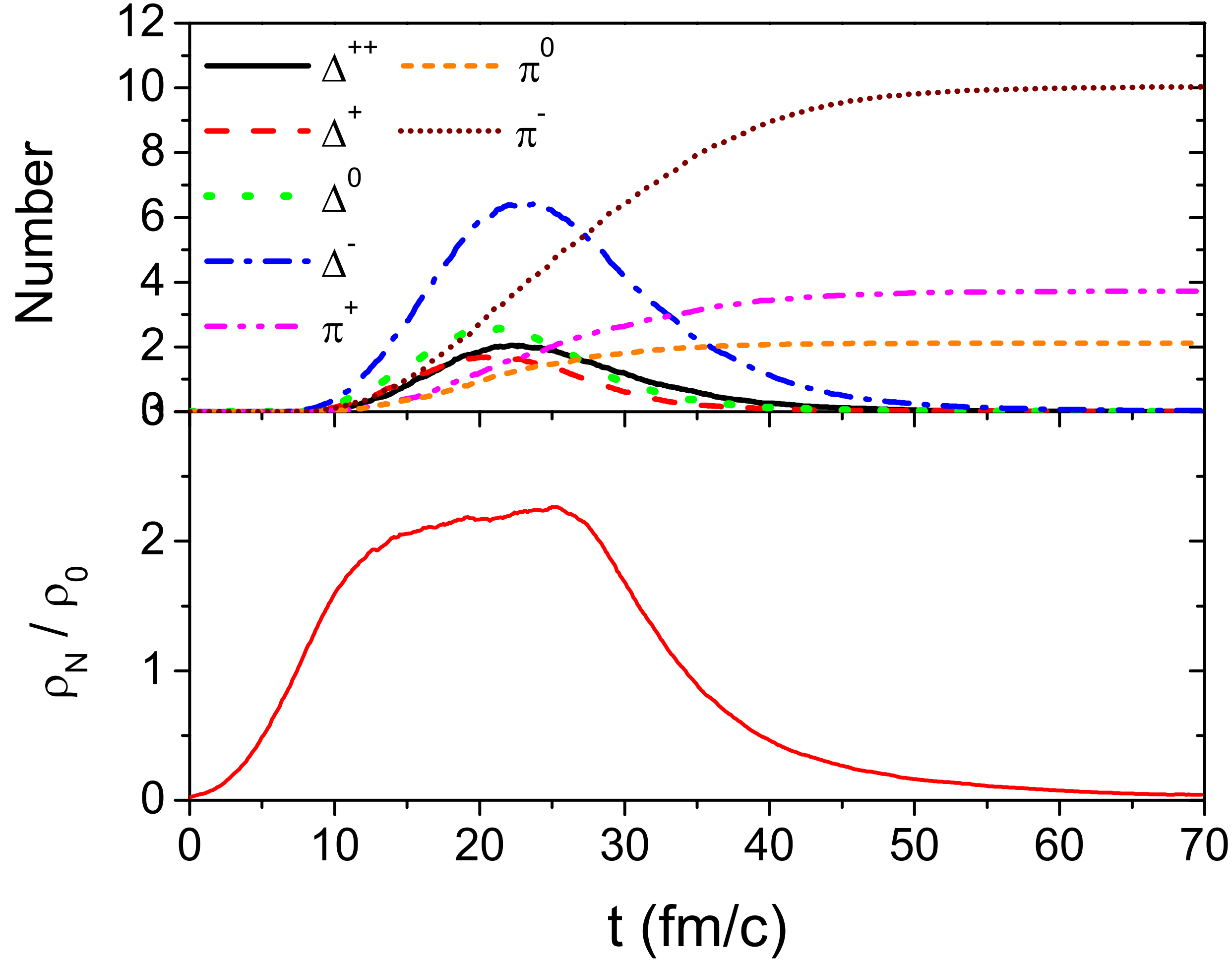
difference between the initial and threshold energies in static nuclear matter ($\mathbf{\Sigma}_i = 0$, $\mathbf{p}_i^* \approx 0$) is

$$\sqrt{s_{\rm in}} - \sqrt{s_{\rm th}} \simeq E_1^* + E_2^* + \Sigma_1^0 + \Sigma_2^0 - m_3^* - m_4^* - \Sigma_3^0 - \Sigma_4^0$$

In nonrelativistic limit

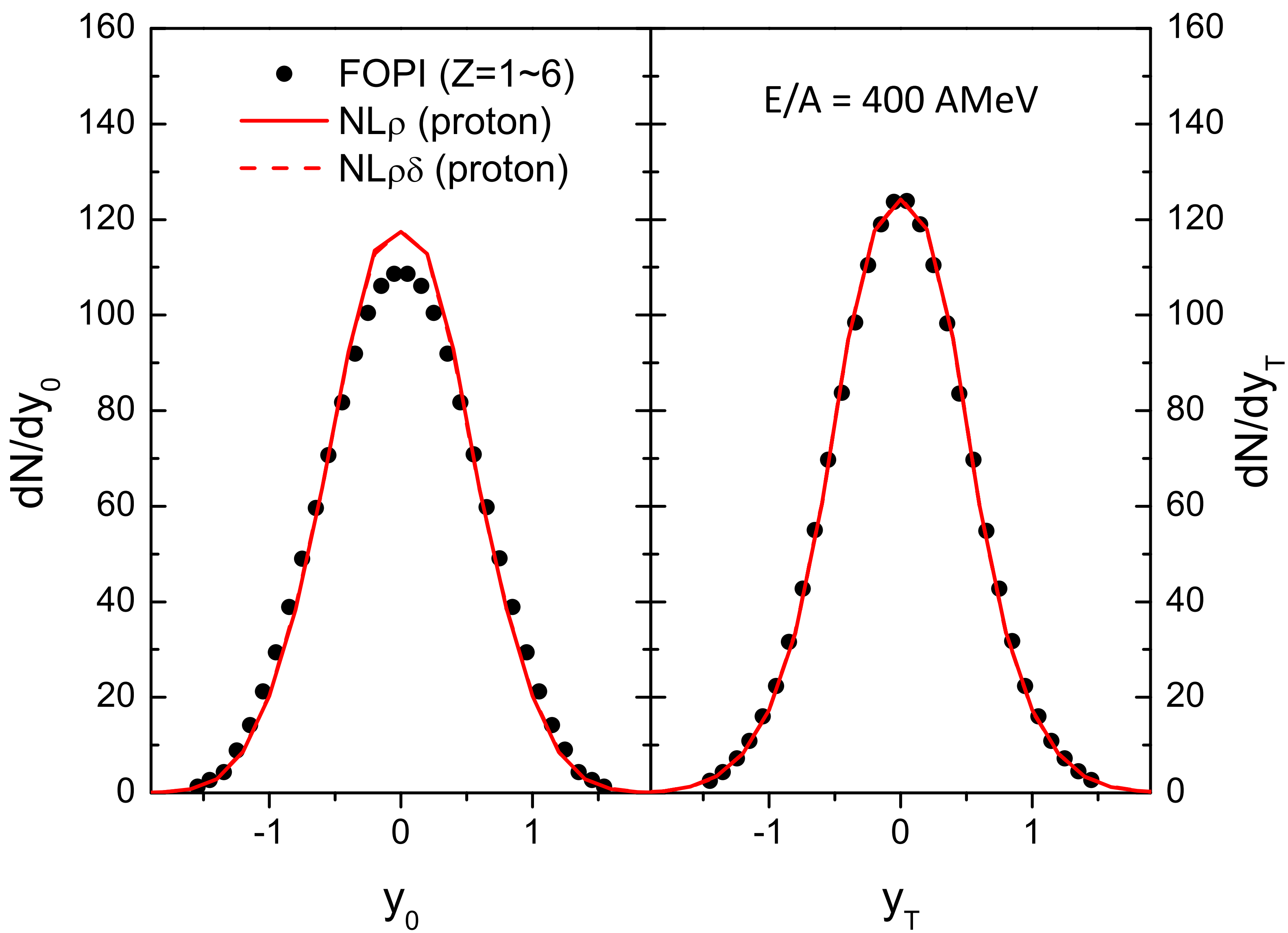
$$\begin{aligned} \sqrt{s_{\rm in}} - \sqrt{s_{\rm th}} &\simeq m_1 + m_2 - m_3 - m_4 + \Sigma_1^s + \Sigma_2^s - \Sigma_3^s - \Sigma_4^s \\ &+ \frac{|\mathbf{p_1^*}|^2}{2m_1^*} + \frac{|\mathbf{p_2^*}|^2}{2m_2^*} + \Sigma_1^0 + \Sigma_2^0 - \Sigma_3^0 - \Sigma_4^0 \end{aligned}$$

Pion production in Au+Au collisions at E = 400 AMeV and b= 1fm



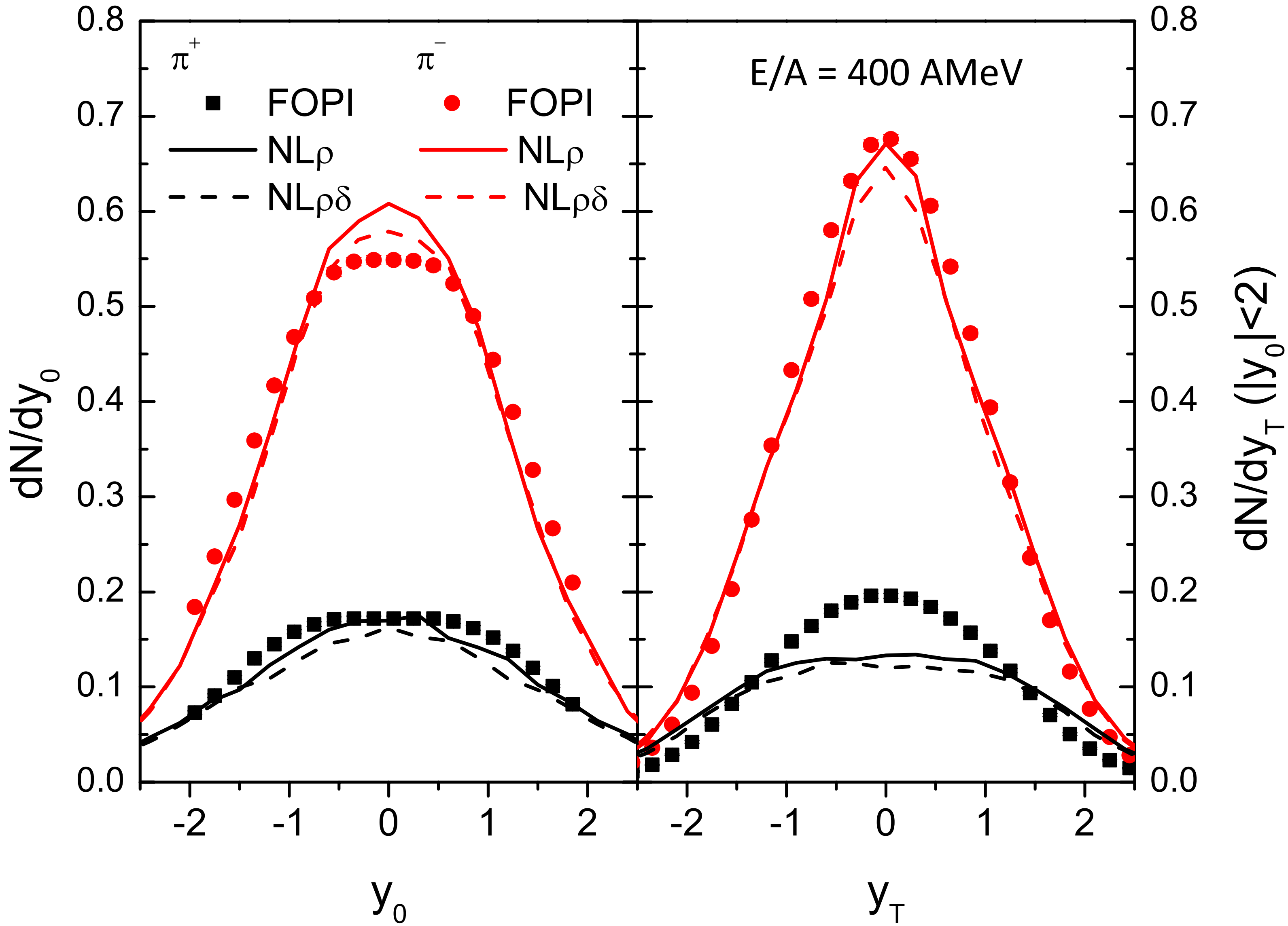
- Deltas are produced during high density stage and decay to pions as the matter expands.

Proton longitudinal and transverse rapidity distributions



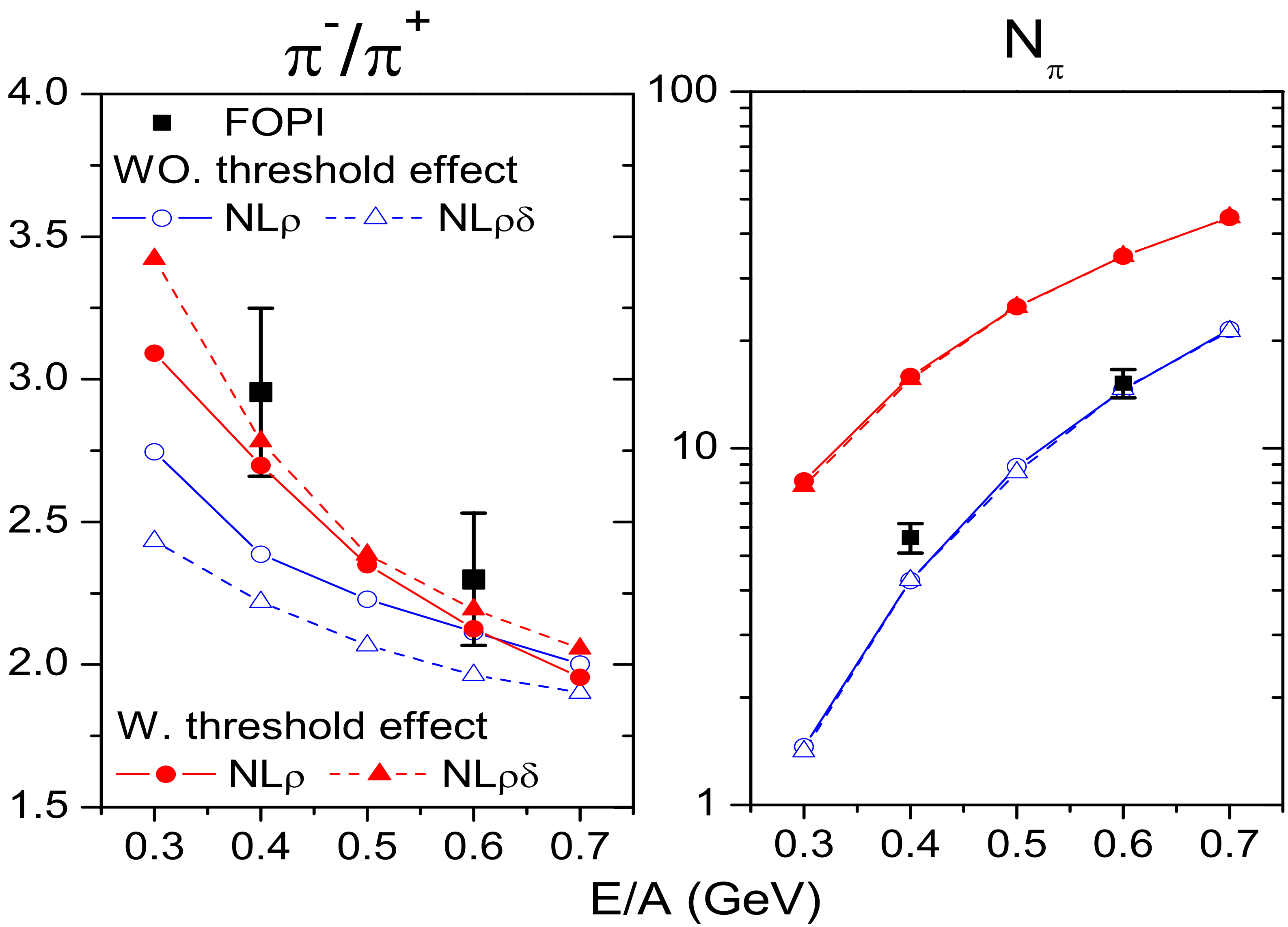
$$y_0 = \ln\left(\frac{1+\beta_z}{1-\beta_z}\right) / \left(\frac{1+\beta_0}{1-\beta_0}\right), \qquad y_T = \ln\left(\frac{1+\beta_x}{1-\beta_x}\right) / \left(\frac{1+\beta_0}{1-\beta_0}\right)$$

Pion longitudinal and transverse rapidity distributions



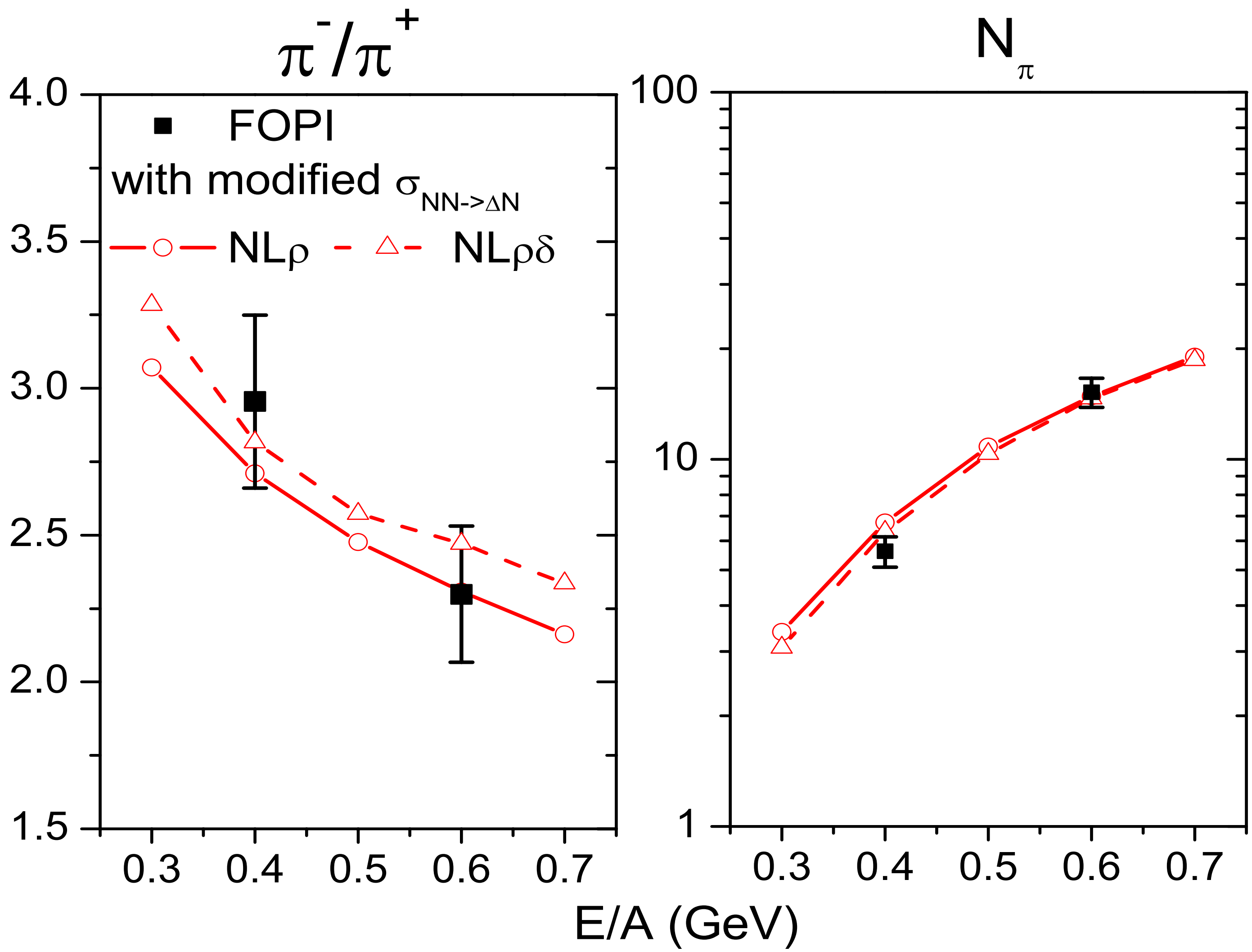
- Discrepancy in π^+ at small y_T due to neglect of pion in-medium effects? [Xiong, Ko & Koch, PRC 47, 788 (1993)]

In-medium threshold effects on π^-/π^+ ratio



- In-medium threshold effects increase the total pion yield, the π^-/π^+ ratio, and reverse the effect of symmetry energy.

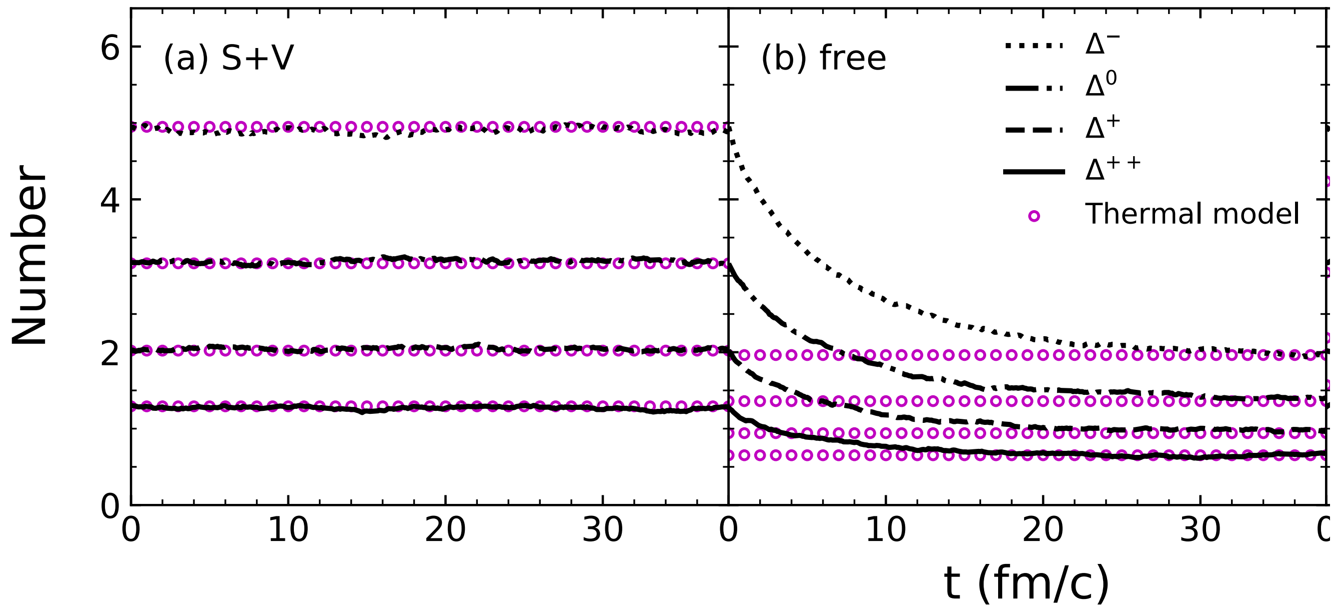
Effects of in-medium Delta production cross sections

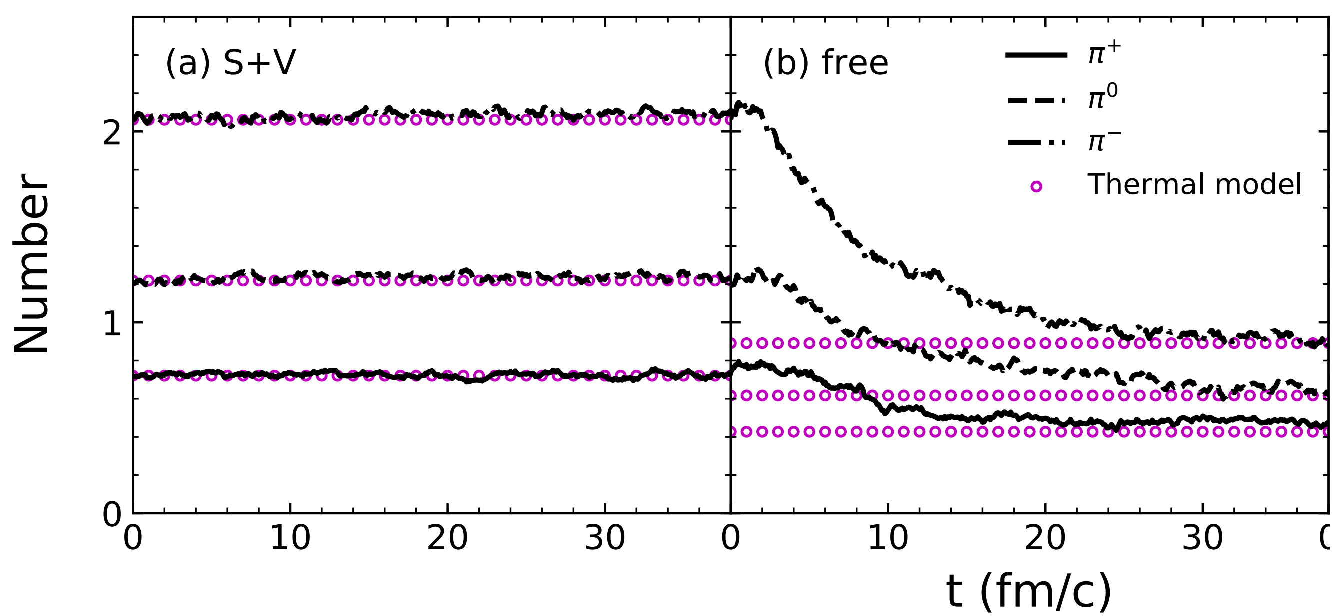


- Reproducing total pion yield requires density-dependent Delta production cross section $\sigma_{NN\to N\Delta}(\rho) = \sigma_{NN\to N\Delta}(0)\exp(-1.65\rho/\rho_0)$, similar to those by Larionov and Mosel, NPA 728, 135 (2003) and Prassa et al., NPA 789, 311 (2007).

Effects of energy conservation on chemical equilibrium in hot dense symmetric nuclear matter

Zhang & Ko, PRC 97, 014910 (2018)





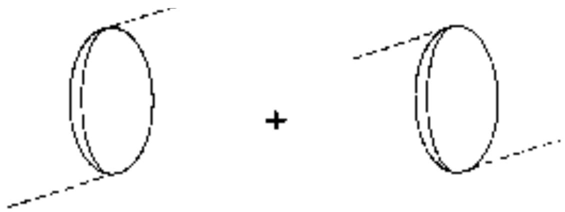
Nucleons, Deltas and pions in a box at T= 60 MeV, $\rho = 0.24\ \mathrm{fm}^{-3}$, $\rho_I = 0.096\ \mathrm{fm}^{-3}$

- Including potentials in the energy conservation during collisions keeps correct equilibrium distributions.
- Treating collisions as in free space, as done in all transport models, leads to equilibrium distributions without potential effects.

Pion in nuclear matter

Brown & Weise, PR 22, 279 (1975)

- Pion p-wave selfenergy

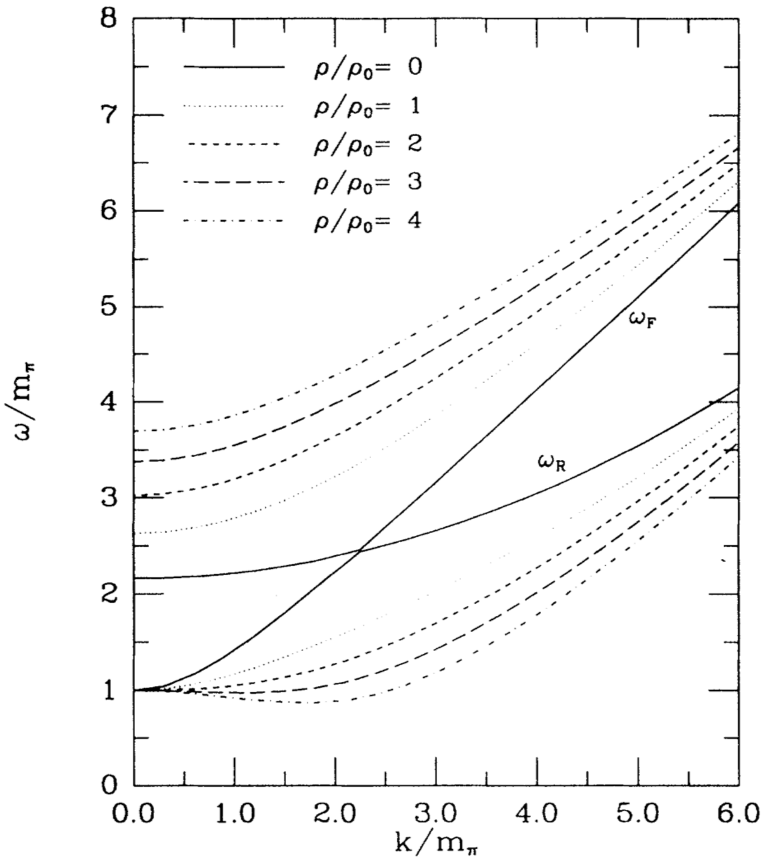


$$\Pi_0(\omega,k) \approx \frac{4}{3}\left(\frac{f_\Delta}{m_\pi}\right)^2 k^2 F^2(k)\rho \frac{\omega_0}{\omega^2-\omega_0^2}$$

$$\omega_0 \approx \frac{k^2}{2m_\Delta} + m_\Delta - m_N$$

Including short-range repulsion through the Migdal parameter g' ~ 0.3

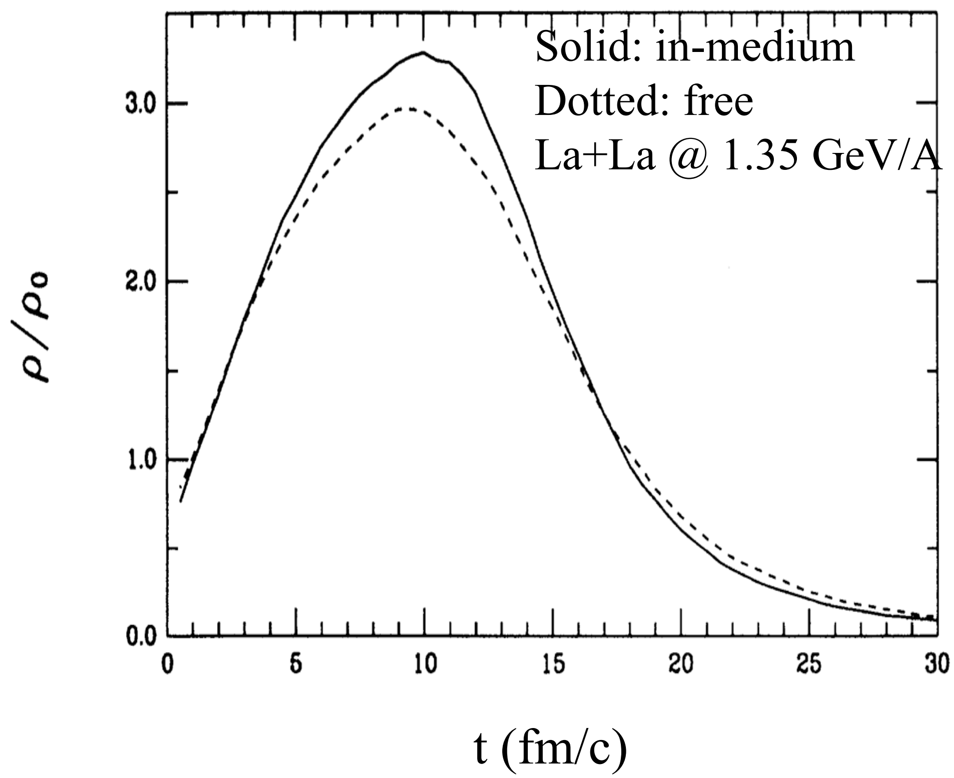
$$\Pi^{m_t}(\omega,k) = \frac{\Pi_0^{m_t}}{1-g'\Pi_0^{m_t}/k^2}$$

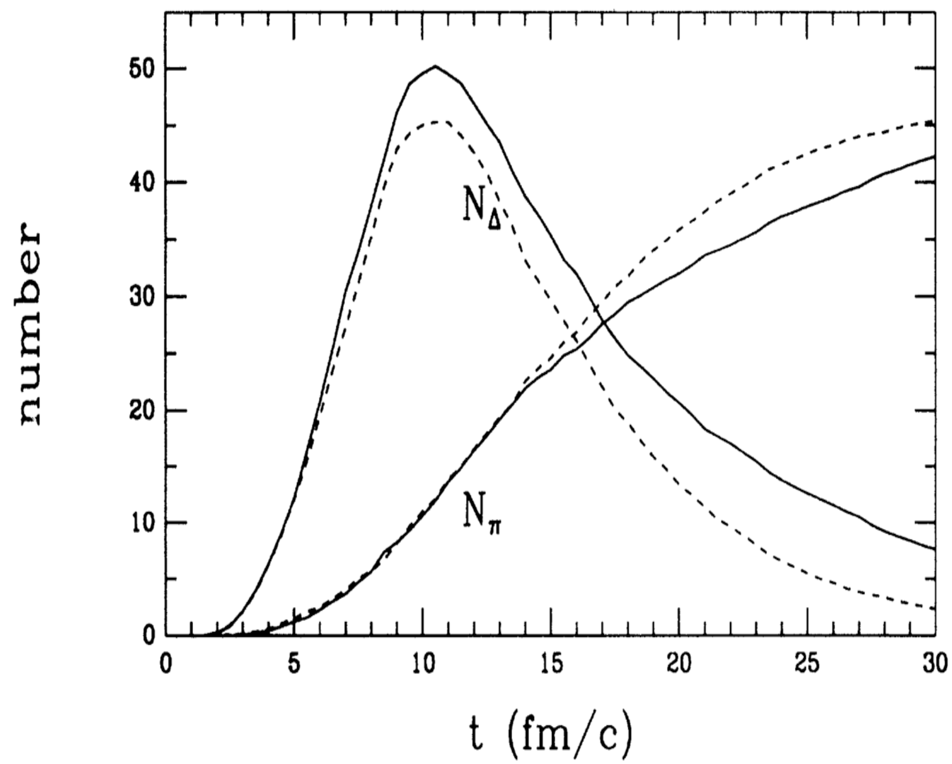


- Leads to a softening of the pion dispersion relation

Pion medium effects on HIC dynamics

Xiong, Ko & Koch, PRC 47, 788 (1993)

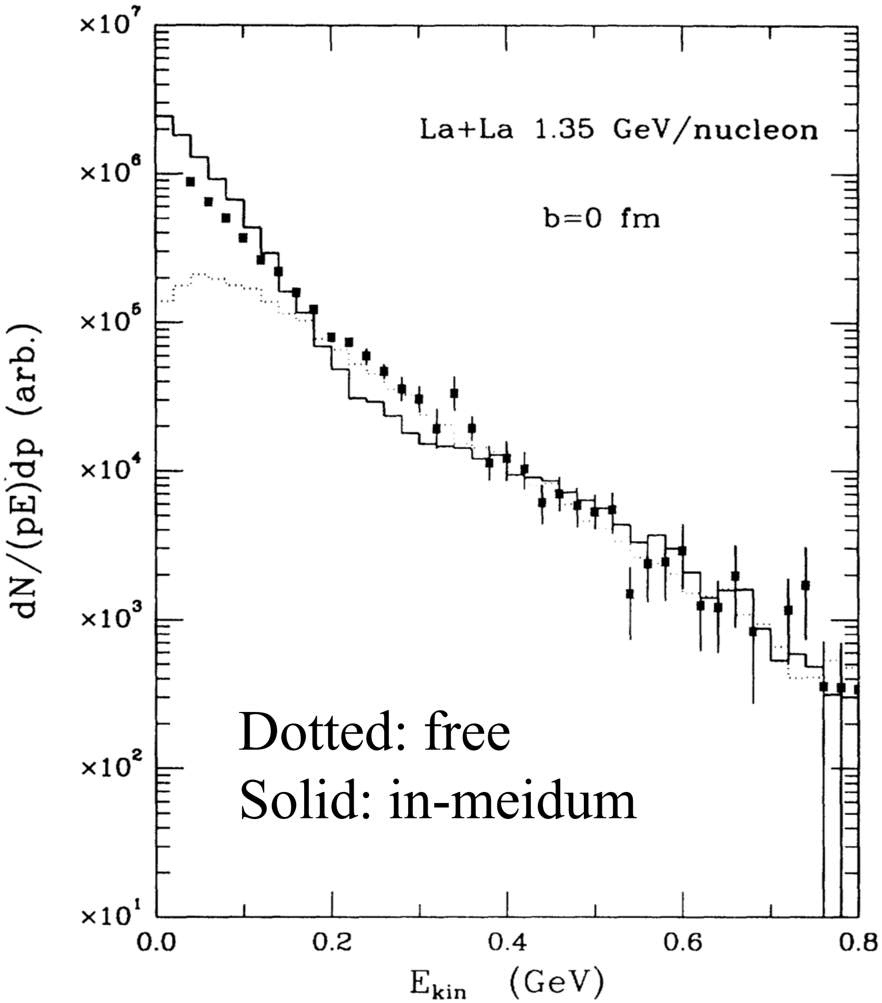




- Including pion in-medium dispersion relations has little effect on the time evolutionof density, pion and delta numbers.

Pion medium effects on pion p_T spectrum

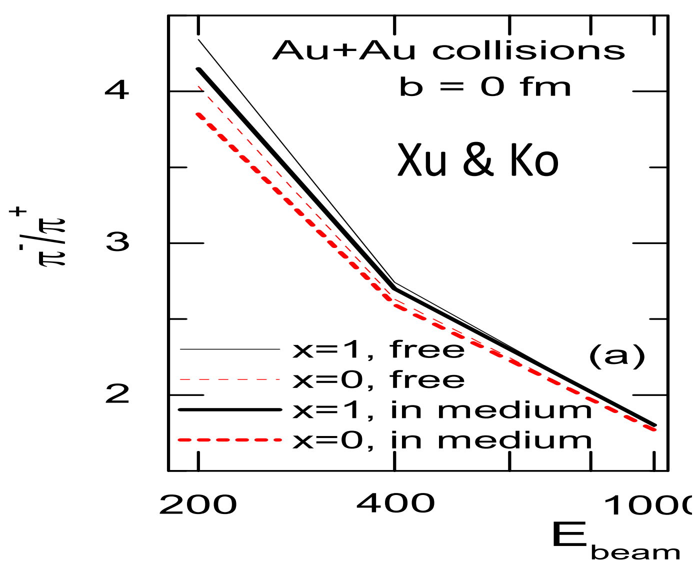
Xiong, Ko & Koch, PRC 47, 788 (1993)

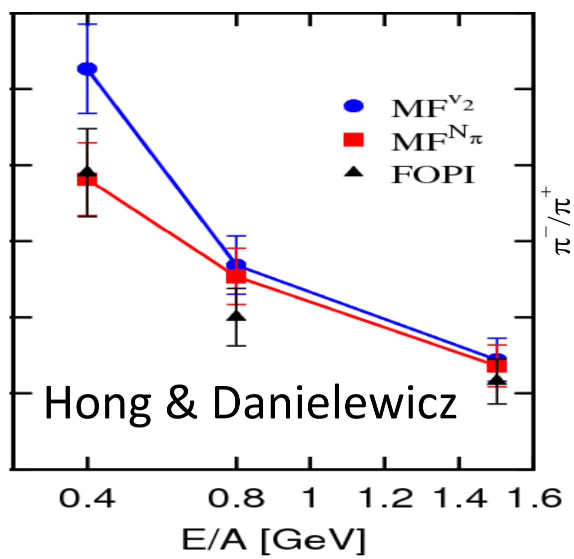


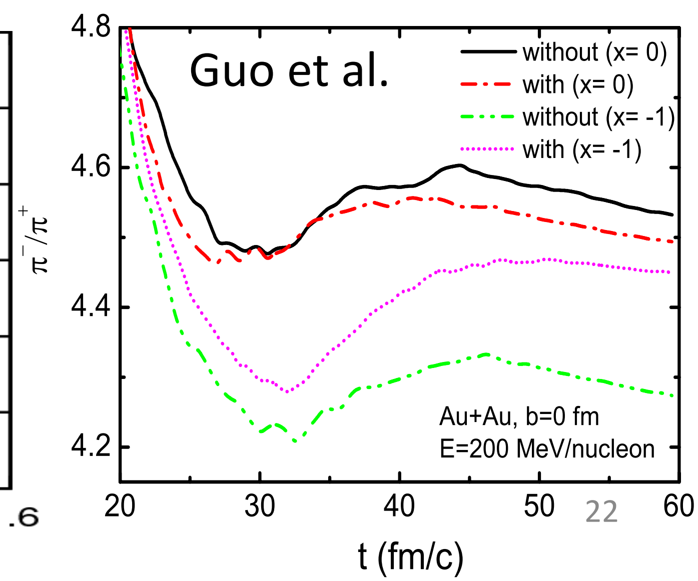
- Including pion in-medium effects does not affect the total pion yield but enhances the production of pions of low kinetic energies

Pion potential effects on charged pion ratio

- Xu & Ko, PRC 81, 024910 (2010); Xu, Chen, Ko, Li & Ma, PRC 87, 067601 (2013): Thermal model → Including both pion s- and p-wave interactions, which have opposite effects, decreases the π^-/π^+ ratio.
- Hong and Danielewicz, PRC 90, 024605 (2014): pBUU → π^-/π^+ ratio is insensitive to stiffness of symmetry energy after including pion s-wave potential.
- Guo, Yong, Liu & Zuo, PRC 91, 054616 (2015): IBUU → pion s- and p-wave potentials and symmetry potential have opposite effects. (p-wave potential essentially vanishes in this study because of average over the pion and Delta-hole branches.)
- Feng, EJPA 53, 30 (2017): LQMD → similar to Guo et al.

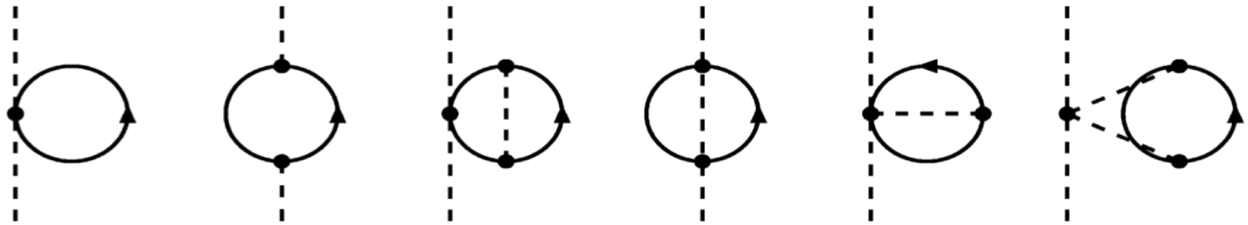






Pion in nuclear matter (I)

- Pion s-wave selfenergies: Kaiser & Weise, PLB 512, 283 (2001)



$$\Pi^-(\rho_n,\rho_p) = \rho_n[T^-_{\pi N} - T^+_{\pi N}] - \rho_p[T^-_{\pi N} + T^+_{\pi N}] + \Pi^-_{\rm rel}(\rho_n,\rho_p) + \Pi^-_{\rm cor}(\rho_n,\rho_p)$$

$$\Pi^+(\rho_p,\rho_n) = \Pi^-(\rho_n,\rho_p)$$

$$\Pi^0(\rho_n,\rho_p) = -(\rho_p + \rho_n)T^+_{\pi N} + \Pi^0_{\rm cor}(\rho_n,\rho_p)$$

Isospin even and odd πN-scattering matrices extracted from energy shift and width of 1s level in pionic hydrogen atom

$$T^+_{\pi N} \approx 1.847\,\mathrm{fm} \quad \text{and} \quad T^-_{\pi N} \approx -0.045\,\mathrm{fm}$$

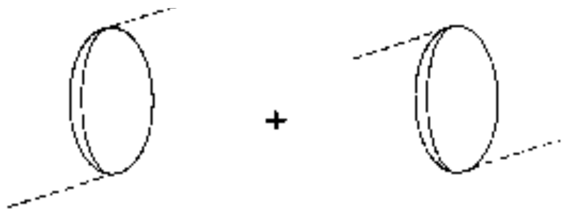
At normal nuclear density ρ=0.165 fm^{-3} and isospin asymmetry δ=0.2 such as in Pb,

$$U_\pi = \Pi/(2m_\pi) \qquad U_{\pi^-} = 14\ \mathrm{MeV},\ U_{\pi^+} = -1\ \mathrm{MeV},\ U_{\pi^0} = 6\ \mathrm{MeV}$$

Pion in nuclear matter (II)

Brown & Weise, PR 22, 279 (1975)

- Pion p-wave selfenergy

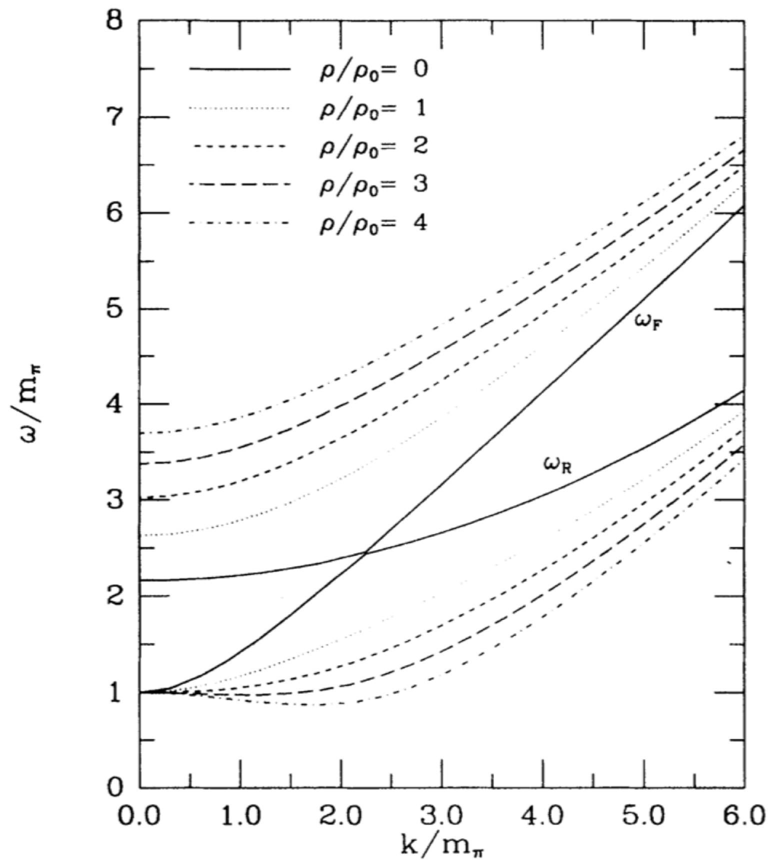


$$\Pi_0(\omega,k) \approx \frac{4}{3}\left(\frac{f_\Delta}{m_\pi}\right)^2 k^2 F^2(k)\rho \frac{\omega_0}{\omega^2-\omega_0^2}$$

$$\omega_0 \approx \frac{k^2}{2m_\Delta} + m_\Delta - m_N$$

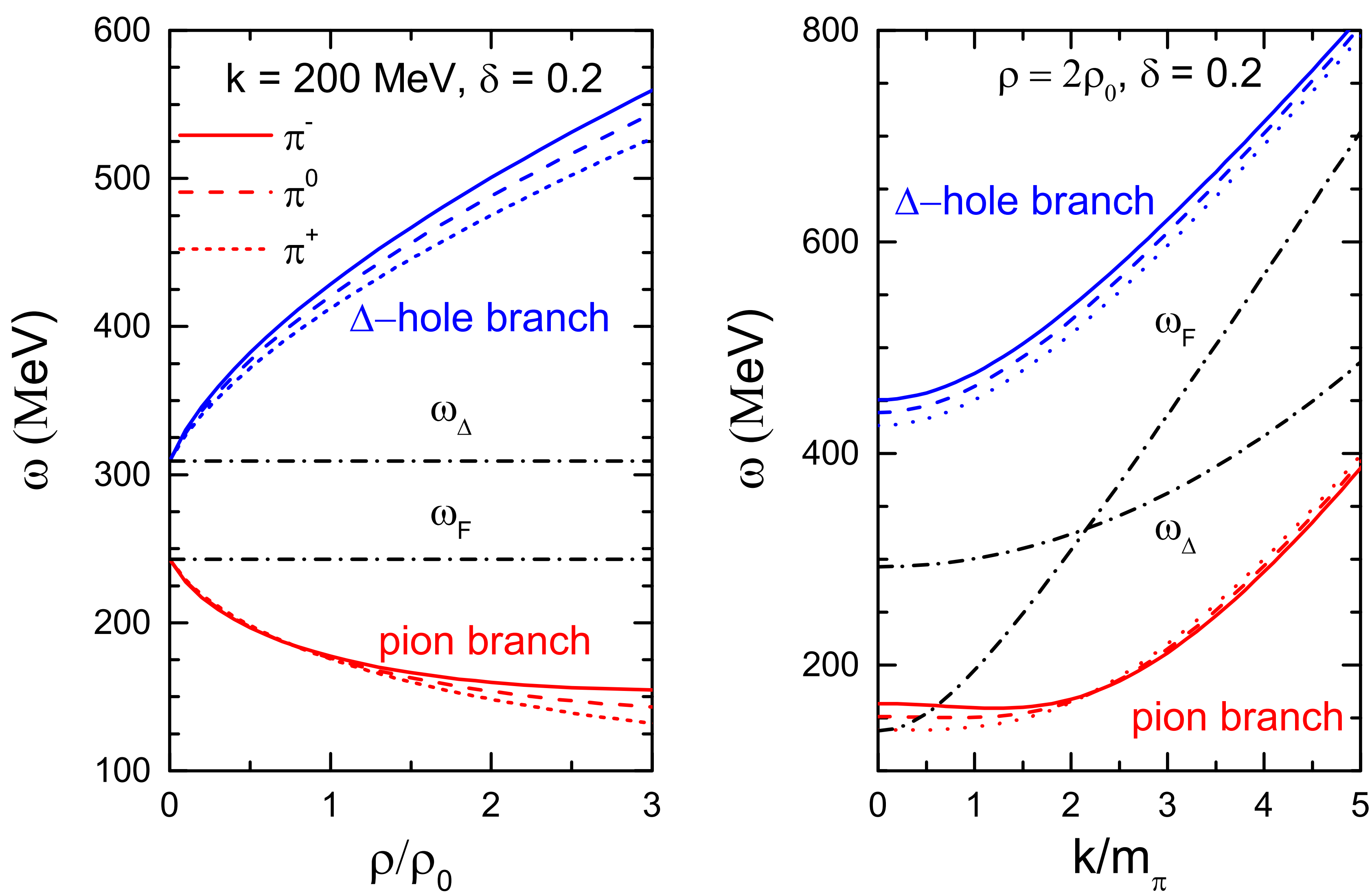
Including short-range repulsion through the Migdal parameter G' ~ 0.3

$$\Pi^{m_t}(\omega,k) = \frac{\Pi_0^{m_t}}{1-g'\Pi_0^{m_t}/k^2}$$



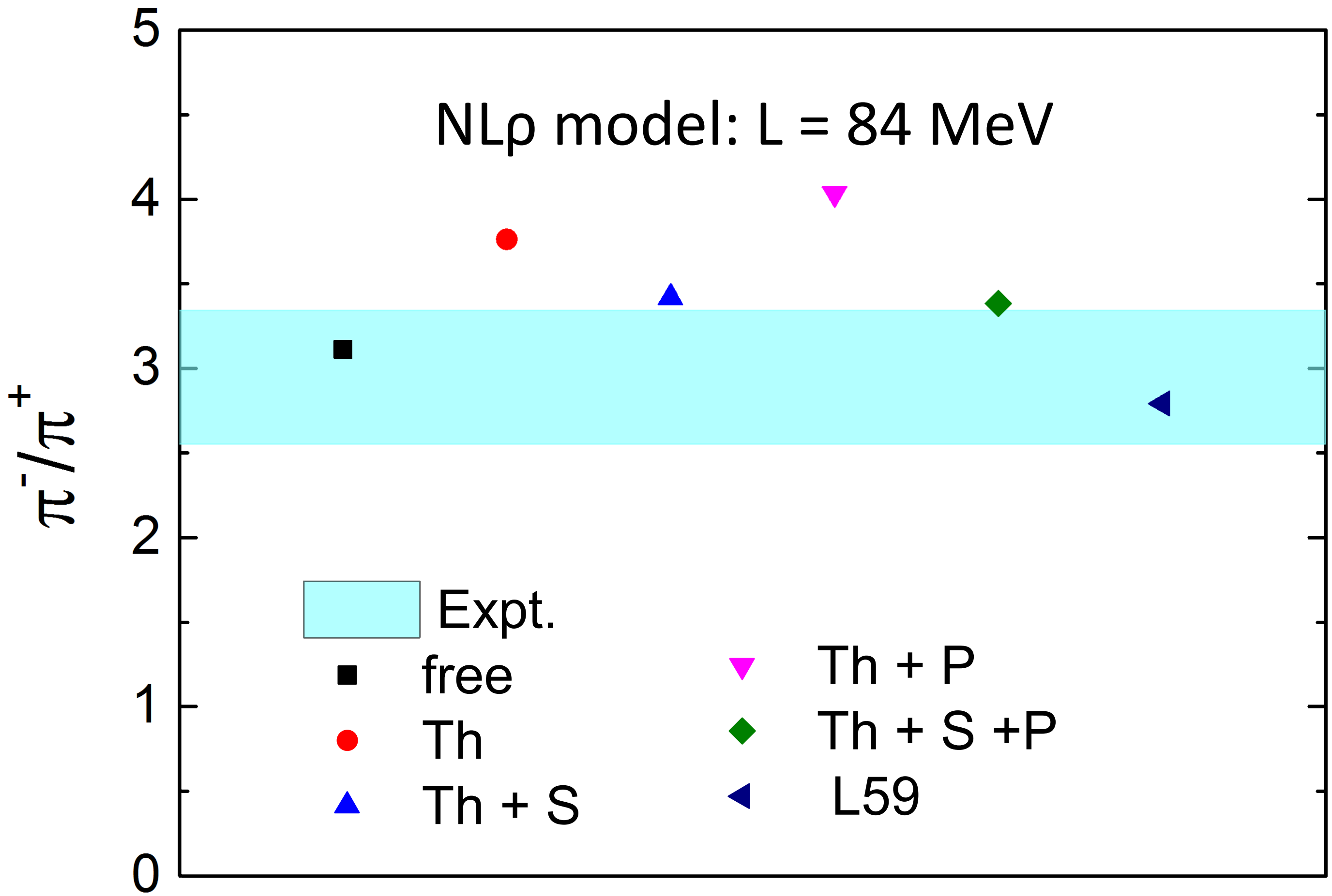
- Leads to a softening of the pion dispersion relation
- π^- has a more softened dispersion relation than π^+ in neutron-rich matter

Pion energy in asymmetric nuclear matter



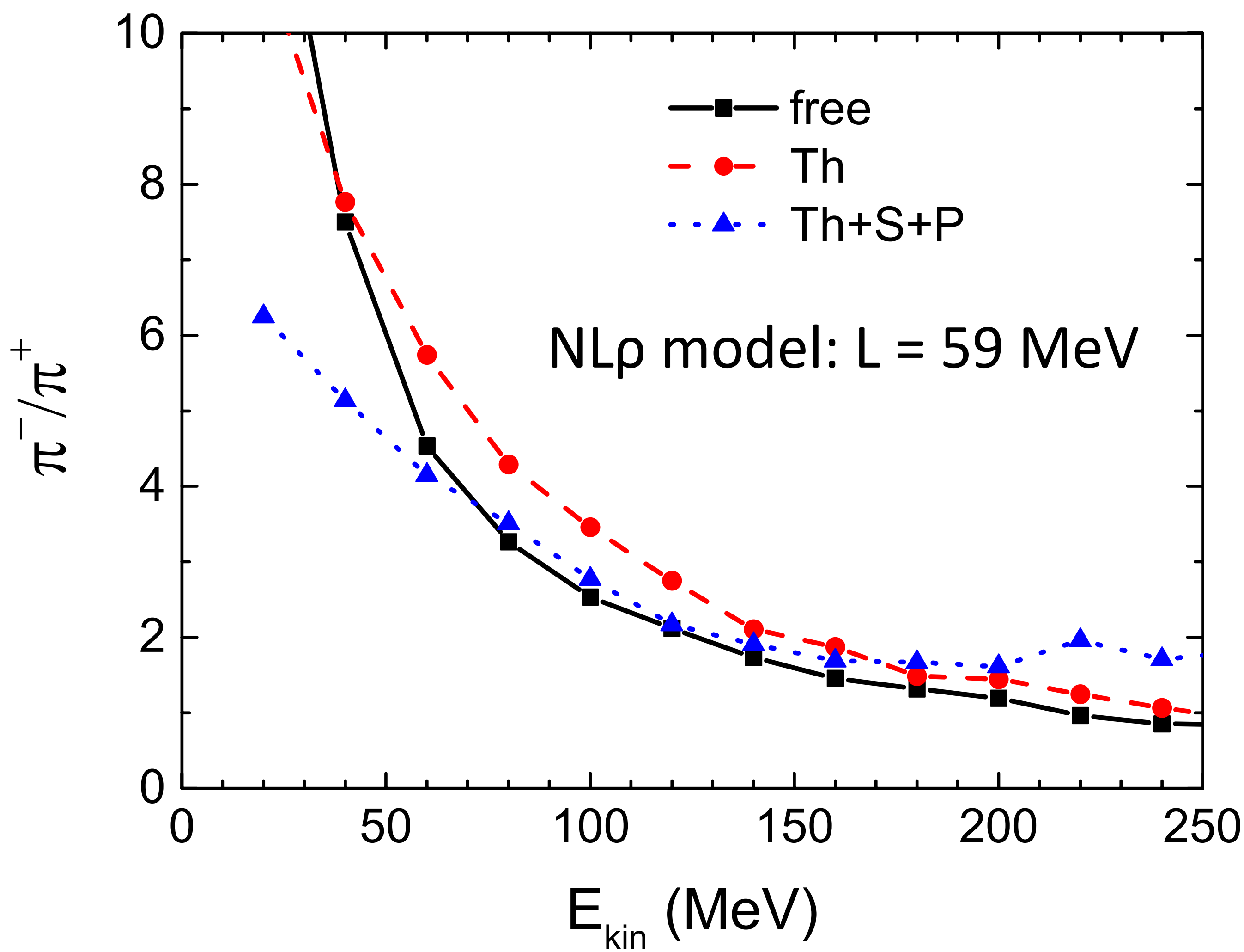
- Pion branch is lower in energy and thus more important.
- π^+ is lower than π^- and thus reduced π^-/π^+ ratio, opposite to that due to stiffness of symmetry energy.

Charged pion ratio in Au+Au @ 400A MeV (I)



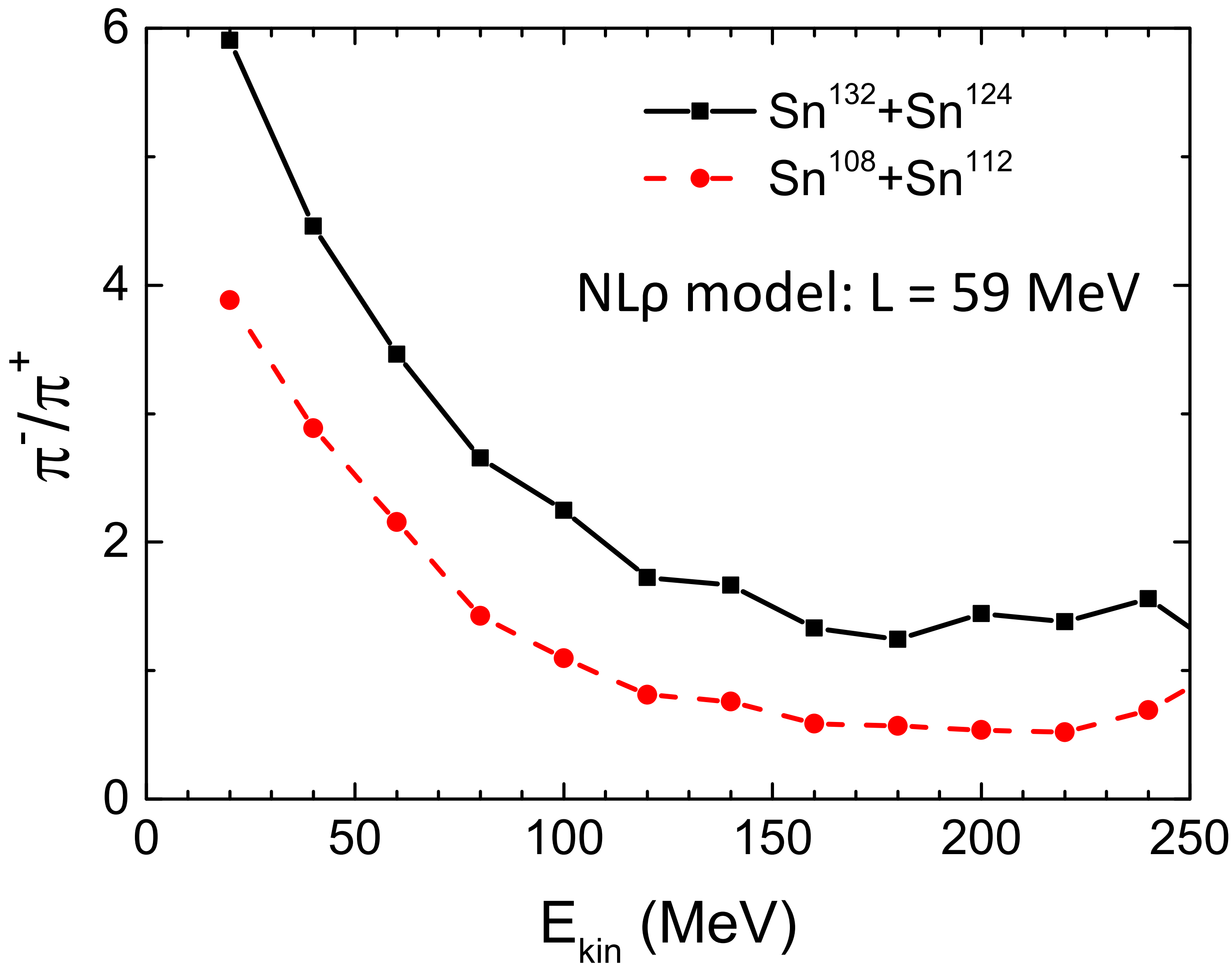
- Charged pion ratio is increased by threshold effect, reduced by s-wave potential, increased by p-wave potential, leading to a somewhat lager ratio compared to that without any medium effects.
- Reproducing FOPI data requires a small symmetry energy slope parameter L comparable with the constraints from nuclear structure and reactions as well as neutron star properties.

Charged pion ratio in Au+Au @ 400A MeV (II)



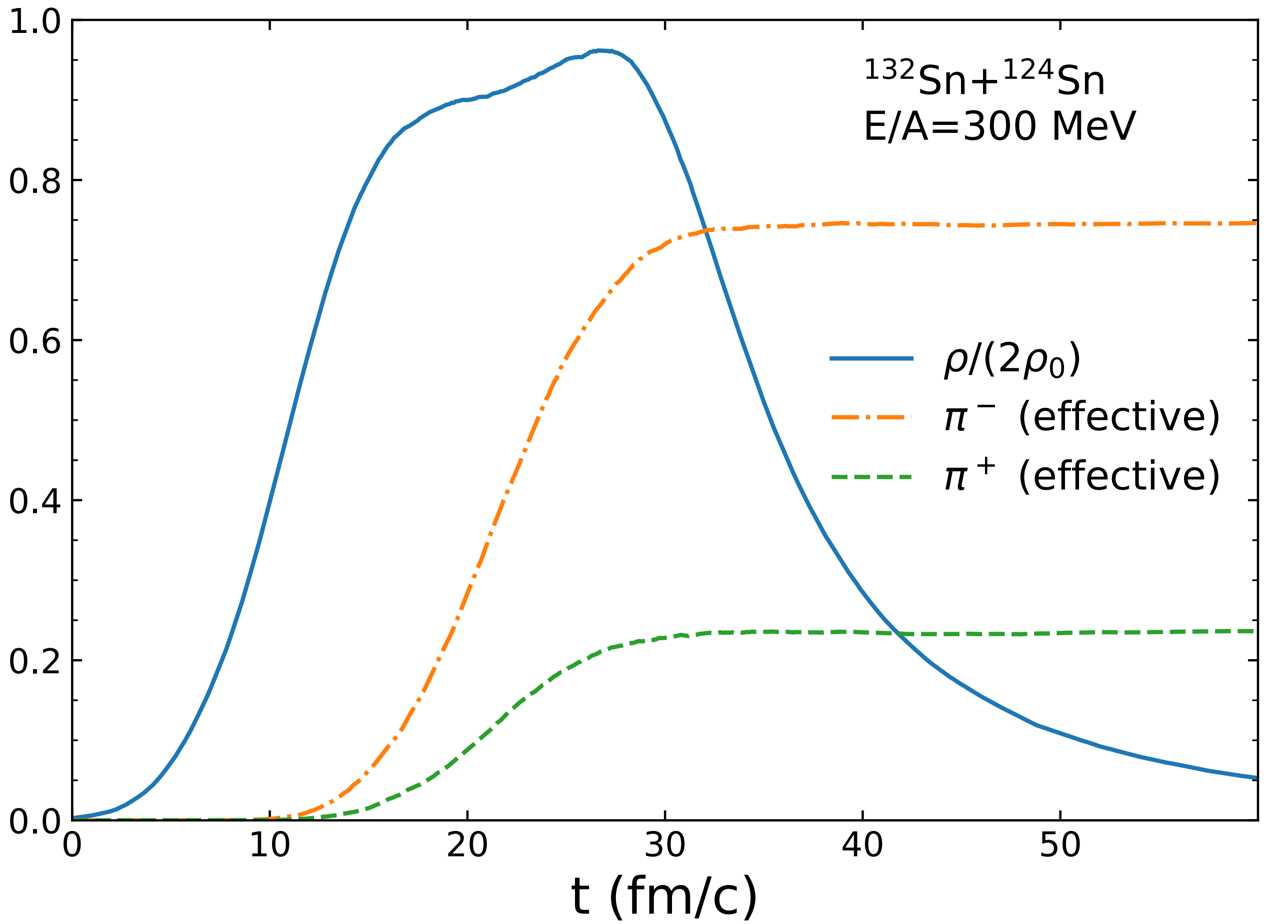
- Threshold effect enhances charged pion ratio at $E_{kin} < 50$ MeV.
- Pion potentials suppresses the ratio for $E_{kin} < 70$ MeV but enhances it for larger E_{kin}.
- Including both medium effects enhances the ratio by 2 at $E_{kin} = 250$ MeV.

Charged pion ratio in Sn+Sn @ 300A MeV



- Charged pion ratio is larger in collisions of more neutron-rich nuclei.
- The ratio decreases with energy of pion.

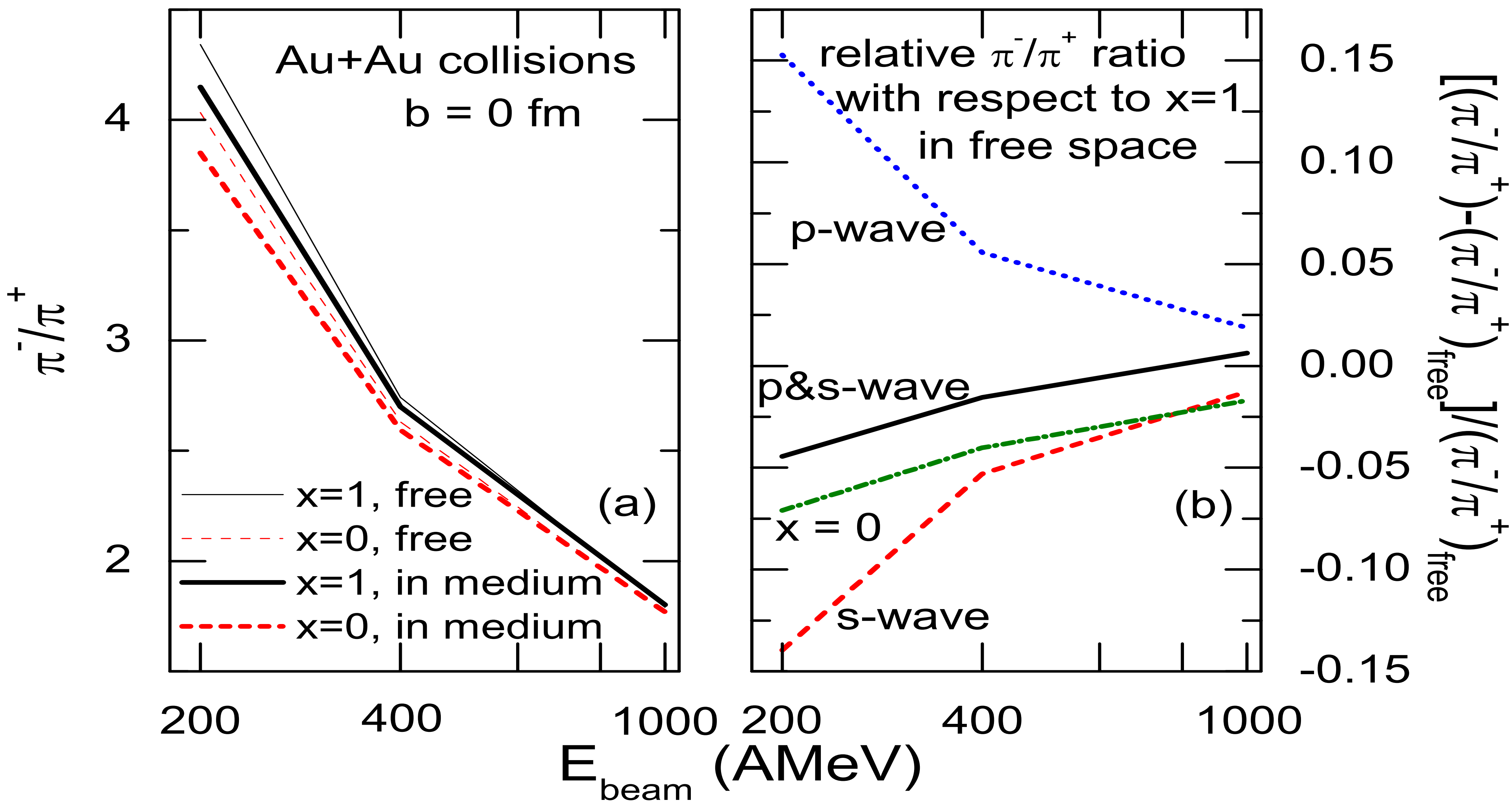
Time evolution of effective charged pion numbers



- Both effective π^- and π^+ numbers (including those in Delta resonances remain unchanged after maximum compression (chemical freeze out), due to constancy of entropy per particle (Xu & Ko, PLB 772, 290 (2017)).

Beam energy dependence of pion in-medium effect

Xu, Chen, Ko, Li & Ma, PRC 87, 067601 (2013)



- Assuming pion, nucleon, and Delta in thermal equilibrium at maximum compression.
- Pion in-medium effect decreases the π^-/π^+ ratio, and the effect is larger at lower collision energies.

Summary

- Nuclear symmetry energy affects the π^-/π^+ ratio in HIC (B. A. Li).

However,

- Results depend on the transport model used in a study.
- In-medium threshold effects increase the total pion yield and the π^-/π^+ ratio, and reverse the effect of symmetry energy (Ferini et al, Song and Ko).
- Charged pion ratio is reduced by pion s-wave potential and increased by pion p-wave potential. The net effect is a reduction of the ratio if keeping the total pion number unchanged. (Xu et al., Zhang and Ko).

On the other hand,

- Essentially all transport models do not include potential effect in scattering, leading thus to incorrect equilibrium pion abundance.
- Both symmetry energy effect and medium effect depend on pion kinetic energy.

→ Require better theoretical modeling of pion production in HIC to extract information on the stiffness of nuclear symmetry energy at high density from the ratio of charged pions.

→ **Comparison study of transport models for pion production is essential!**