1st Symposium on Intermediate-energy Heavy Ion Collisions (iHIC2018)

Bayesian inference on Skyrme EDF from HICs

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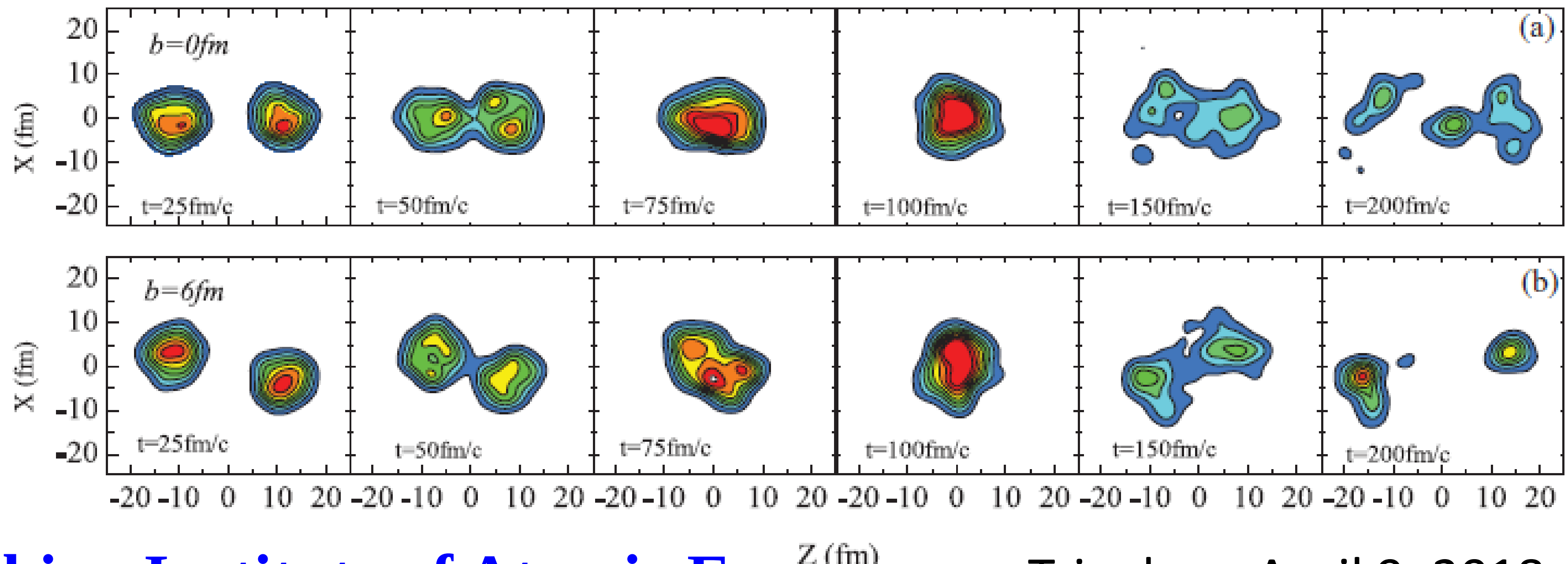


China Institute of Atomic Energy

Tsinghua, April 9, 2018

Outline

1, Nuclear equation of state, Skyrme interaction and its constraints

2, preliminary constraints on Skyrme parameter sets from isospin sensitive HIC observables

- Sensitivity study of model parameter *x* and observables *O* in transport model

- CI-DR(n/p) at 120AMeV, R_{diff} at 50AMeV, and Ri(y) at 35, 50 AMeV

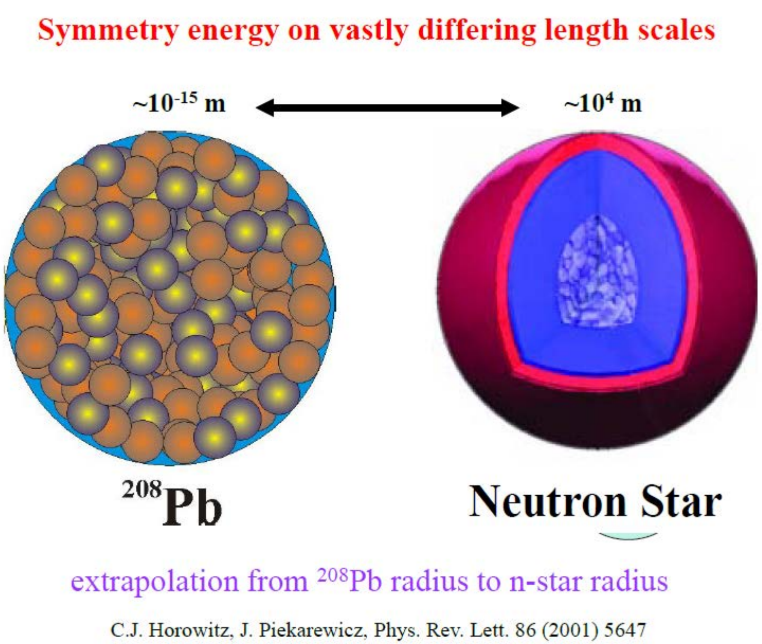
3, Summary and outlook

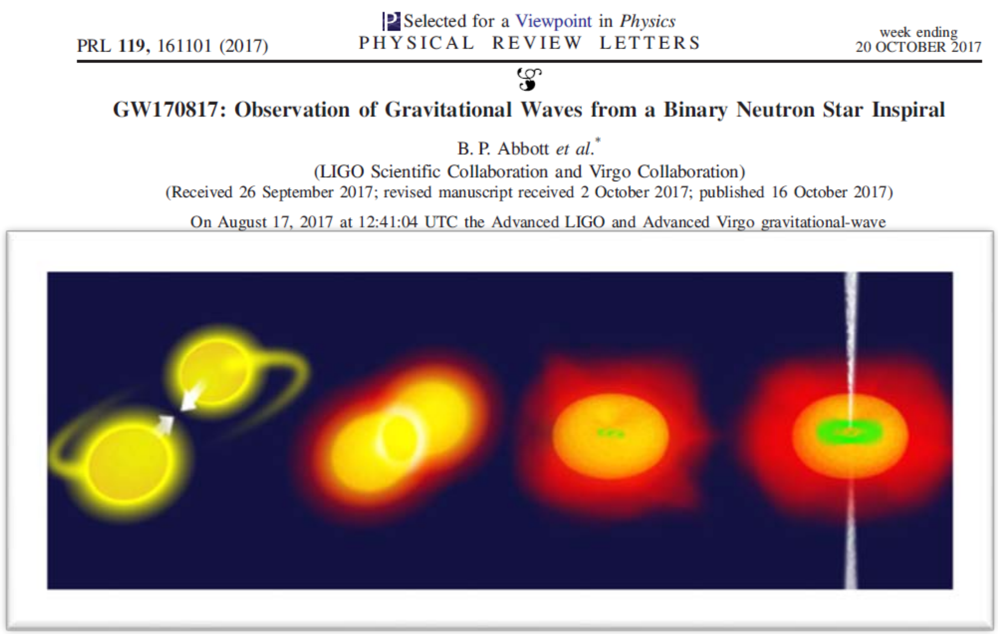
Isospin asymmetric Equation of State

$$E(\rho,\delta)=E(\rho,\delta=0)+S(\rho)\delta^2+O(\delta^4)$$

It is a fundamental properties of nuclear matter, and is very important for understanding

- *properties of nuclear structure*
- *properties of neutron star*
- *properties of heavy ion reaction mechanism*





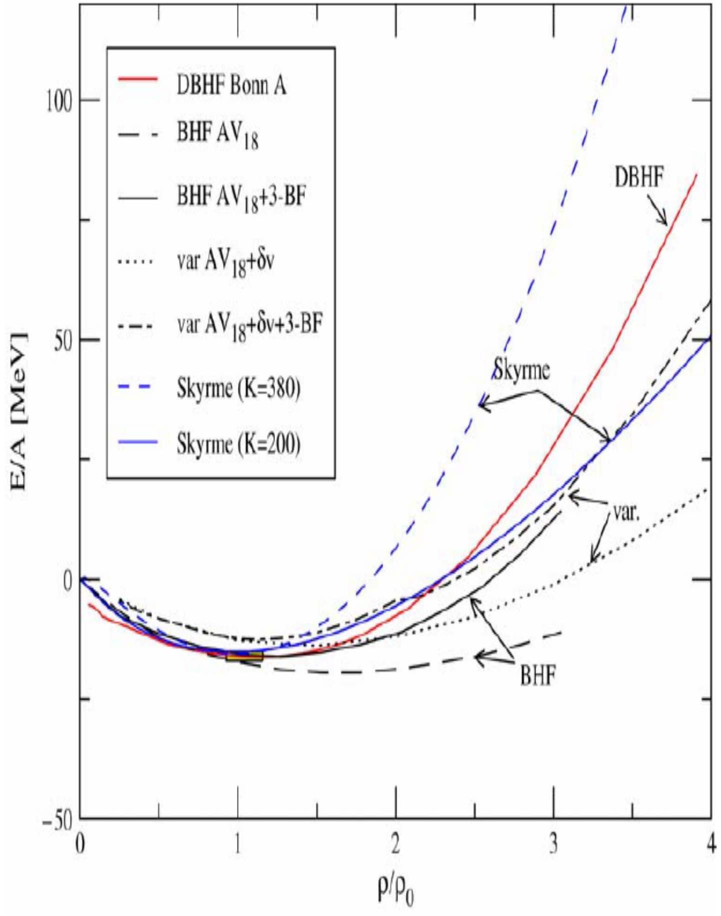
Theoretical predictions on the properties of nuclear matter

- Effective field theory approaches (Based on Chiral perturbation theory, ……)

- Ab initio approaches (Based on the high precision free space nucleon-nucleon interaction) DBHF, SCGF, QMC

  - Phenomenological density functional (Based on Gogny or **Skyrme** force or RMF, ……)

C. Fuchs / Progress in Particle and Nuclear Physics 56 (2006) 1–103



• Skyrme interaction (1956)

E. Chabanat et al. / Nuclear Physics A 627 (1997) 710–746

$$
\begin{aligned}
V(\boldsymbol{r}_1,\boldsymbol{r}_2) = \; & t_0\,(1+x_0P_\sigma)\,\delta(\boldsymbol{r}) && \text{central term} \\
& +\frac{1}{2}t_1\,(1+x_1P_\sigma)\left[\boldsymbol{P}'^2\delta(\boldsymbol{r})+\delta(\boldsymbol{r})\boldsymbol{P}^2\right] && \\
& +t_2\,(1+x_2P_\sigma)\,\boldsymbol{P}'\cdot\delta(\boldsymbol{r})\boldsymbol{P} && \text{non-local terms} \\
& +\frac{1}{6}t_3\,(1+x_3P_\sigma)\left[\rho(\boldsymbol{R})\right]^\sigma\delta(\boldsymbol{r}) && \text{density-dependent term} \\
& +iW_0\boldsymbol{\sigma}\cdot\left[\boldsymbol{P}'\times\delta(\boldsymbol{r})\boldsymbol{P}\right] && \text{spin–orbit term}\,.
\end{aligned}
$$

Effective Skyrme

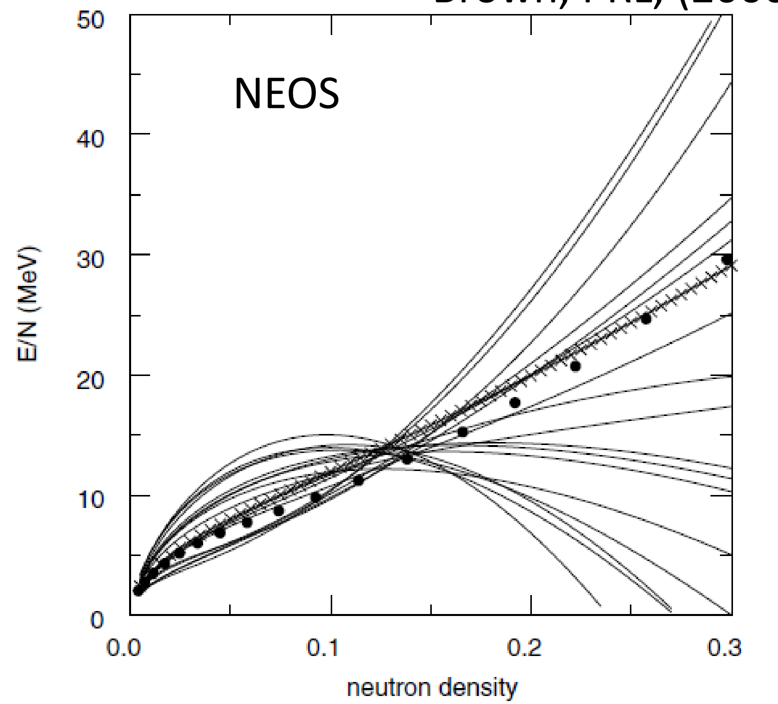
• simple, effectively taken into account the complicated correlations

• parameter are determined by fitting, *rho*$_0$, *K*$_0$, *m*$_s$*, *kappa*, *mass*, ……

• **Many applications in nuclear structure studies**

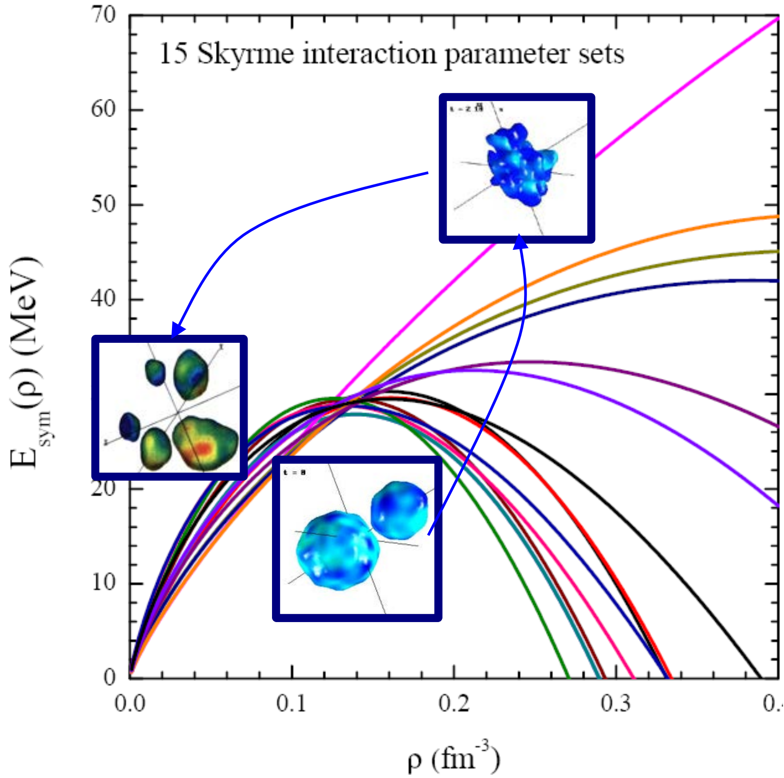
• ***parameterization is not unique and there exists >240 sets (drawback)***

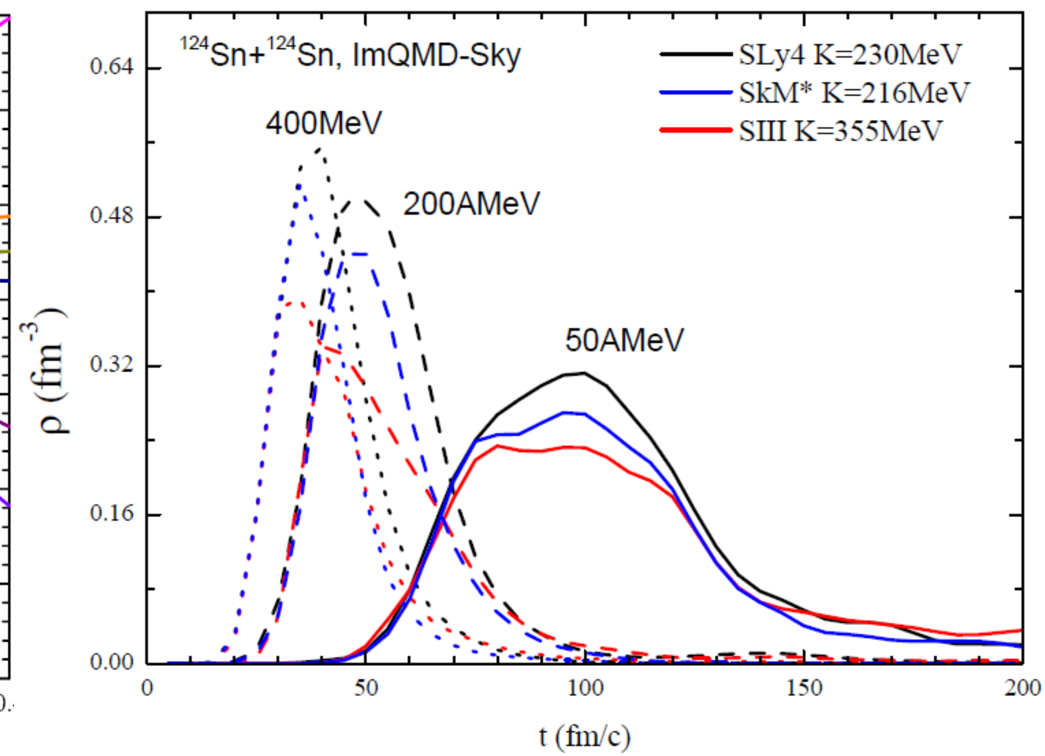
Brown, PRL, (2000)



Can we learn the information of equation of state away from normal density?

Yes, we can!   Heavy ion collisions + Transport model calculations





Input of transport models:
nucleonic potential or interaction parameter (→EoS)

A, BUU type: *f(r,p,t)* one body phase space density

$$\frac{\partial f}{\partial t} + v \cdot \nabla_{\mathbf{r}} f - \nabla_{\mathbf{r}} U \nabla_{\mathbf{p}} f$$

Mean field

$$= -\frac{1}{(2\pi)^6} \int d^3p_2 d^3p_{2'} d\Omega \frac{d\sigma}{d\Omega} v_{12}$$
$$\times \{[f f_2 (1-f_{1'})(1-f_{2'})] - [f_{1'} f_{2'} (1-f)(1-f_2)]$$
$$\times (2\pi)^3 \delta^3 (\mathbf{p} + \mathbf{p}_2 - \mathbf{p}_{1'} - \mathbf{p}_{2'})\}$$

Two-body collision: occurs between test part.

$$f(\mathbf{r},\mathbf{p}) \cong \frac{1}{\tilde{N}} \sum_{i=1}^{N\tilde{N}} \delta(\mathbf{r}-\mathbf{r}_i)\delta(\mathbf{p}-\mathbf{p}_i)$$

Solved with test particle methods

B, QMD type: solve N-body equation of motion

$$\phi_i(\mathbf{r}_i,t) = \frac{1}{(2\pi\sigma_r^2)^{3/4}} \exp[-\frac{(\mathbf{r}-\mathbf{r}_i)^2}{4\sigma_r^2} + \frac{i\mathbf{p}_i \cdot \mathbf{r}}{\hbar}]$$

nucleon

$$\dot{\mathbf{r}_i} = \frac{\mathbf{p}_i}{m} + \nabla_{\mathbf{p}_i} \sum_j \langle V_{ij} \rangle = \nabla_{\mathbf{p}_i} \sum_j \langle H \rangle$$

$$\dot{\mathbf{p}_i} = -\nabla_{\mathbf{r}_i} \sum_j \langle V_{ij} \rangle = -\nabla_{\mathbf{r}_i} \sum_j \langle H \rangle$$

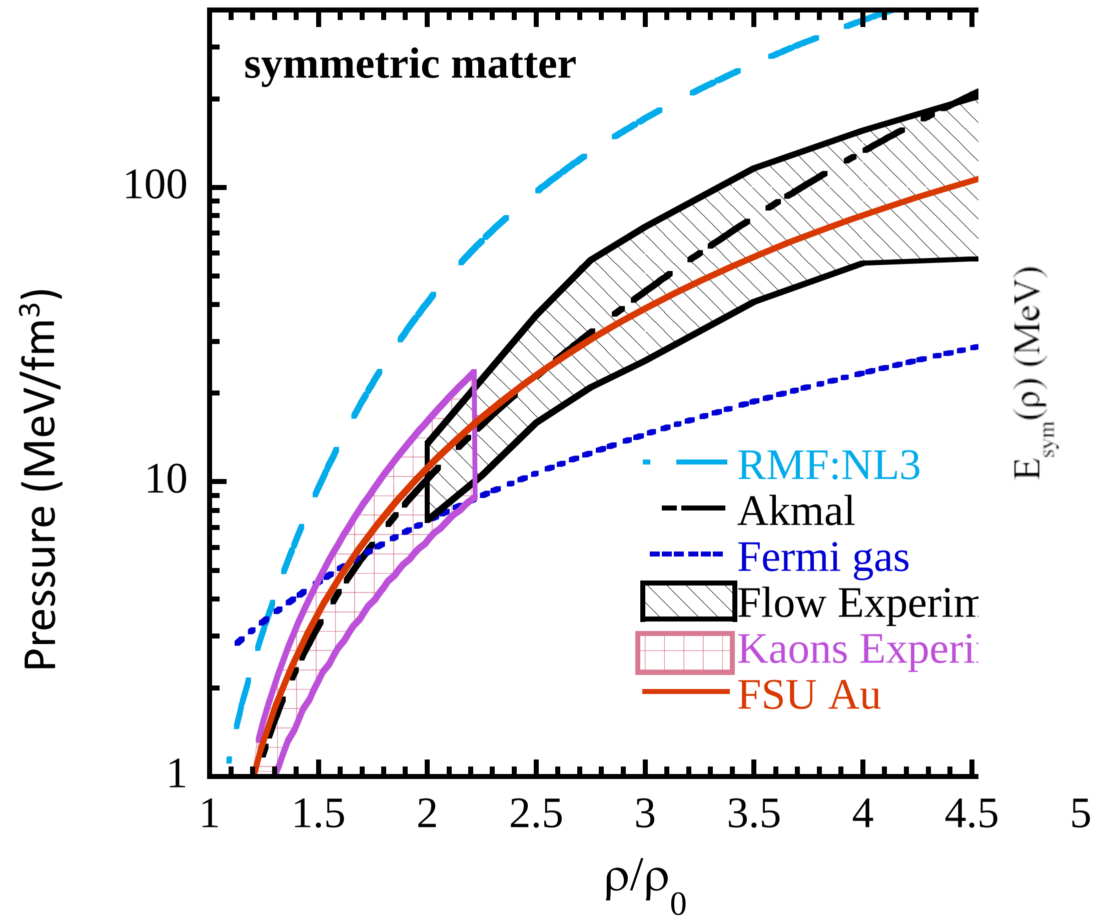
Two body collision: occurs between nucleons

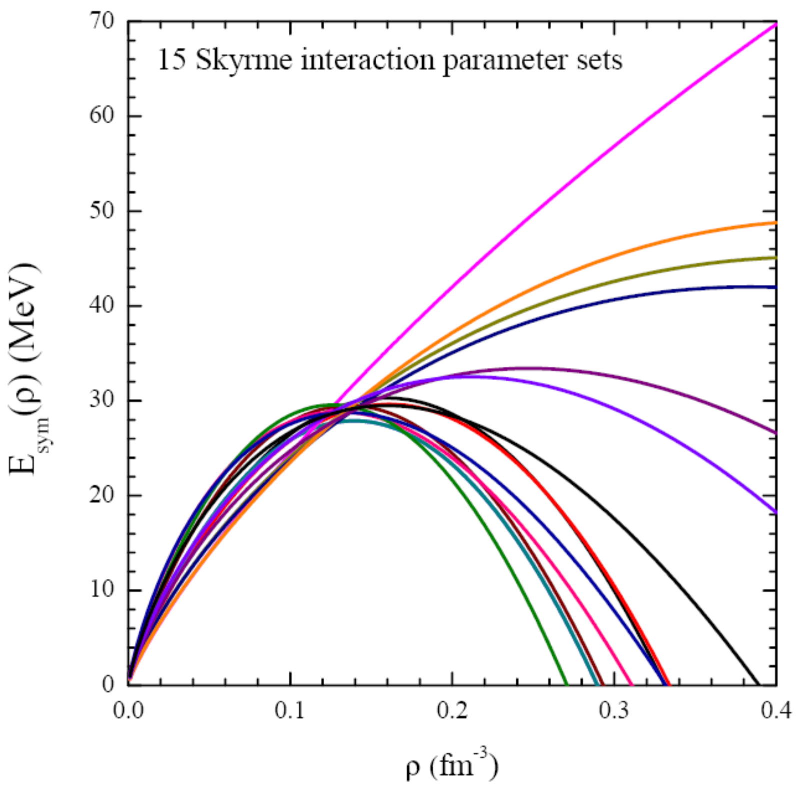
Rearrange whole nucleon-> large fluctuation

• symmetric part is relatively well constrained

$$E/A(\rho,\delta) = E/A(\rho,0) + \delta^2 \cdot S(\rho)\ ; \quad \delta = (\rho_n - \rho_p)/(\rho_n + \rho_p) = (N-Z)/A$$

Danielewicz, Lacey, Lynch, Science 298,1592 (2002)

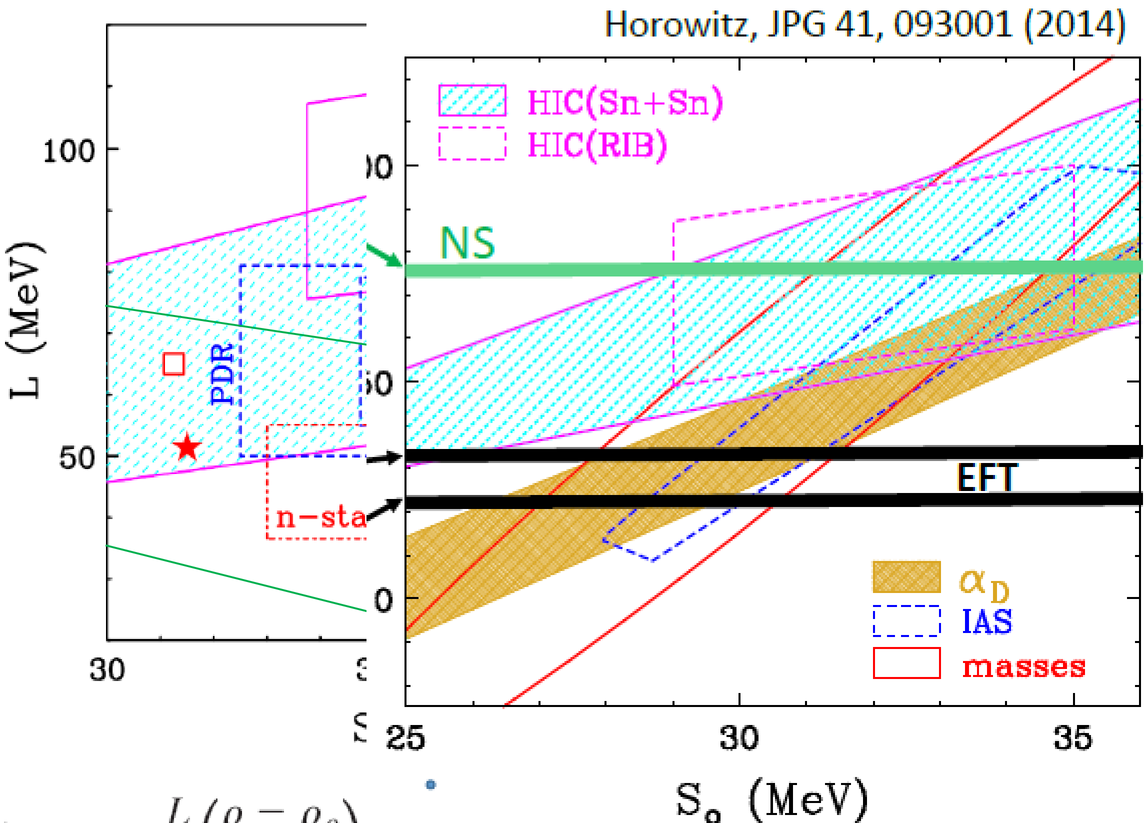


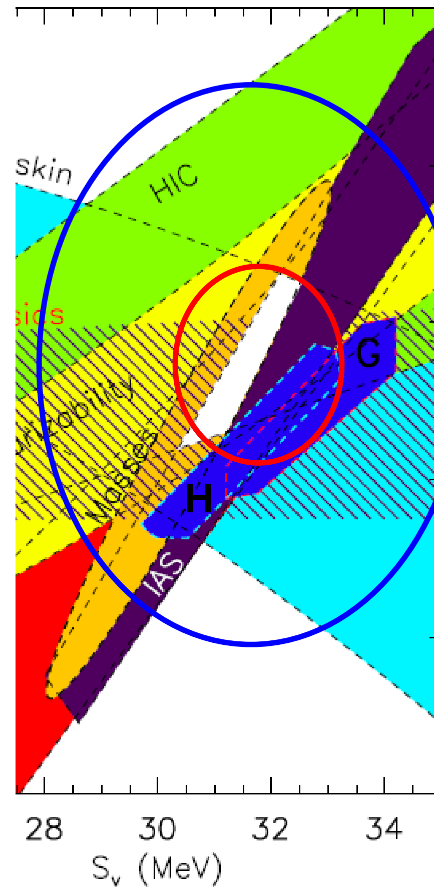


Progress on the constraints of symmetry energy

M.B.Tsang, Yingxun Zhang, et.al., PRL2009

Lattimer, EPJA 50 (2014) 40





$$S(\rho) = S_0 + \frac{L}{3}\left(\frac{\rho - \rho_o}{\rho_o}\right) + \frac{\quad}{18}\left(\frac{\quad}{\rho_o}\right) + \ldots,$$

Consensus on symmetry energy have been obtained at subsaturation density.

Uncertainties on the constraints still need to be understand and constraints on symmetry energy need to be improved! (L+- ΔL)

Parameter correlations in S(ρ): {L, ms*, fi} in Skyrme-HF

- Density dependent of symmetry energy from SHF

$$S(\rho) = \frac{1}{3}\frac{\hbar^2}{2m}(\frac{3\pi^2}{2})^{2/3}\rho^{2/3} - \frac{1}{8}t_0(2x_0+1)\rho - \frac{1}{48}t_3(2x_3+1)\rho^{\sigma+1} - \frac{1}{24}(\frac{3\pi^2}{2})^{2/3}(3\Theta_v - 2\Theta_s)\rho^{5/3} \quad \text{(C8)}$$

$$S(\rho) = \frac{1}{3}\epsilon_F \rho^{2/3} + A_{sym}\rho + B_{sym}\rho^{\sigma+1} + C_{sym}(m_s^*, m_v^*)\rho^{5/3} \quad \text{(C11)}$$

- **Density dependent of symmetry depends not only on density, effective mass splitting but also isoscalar effective mass**

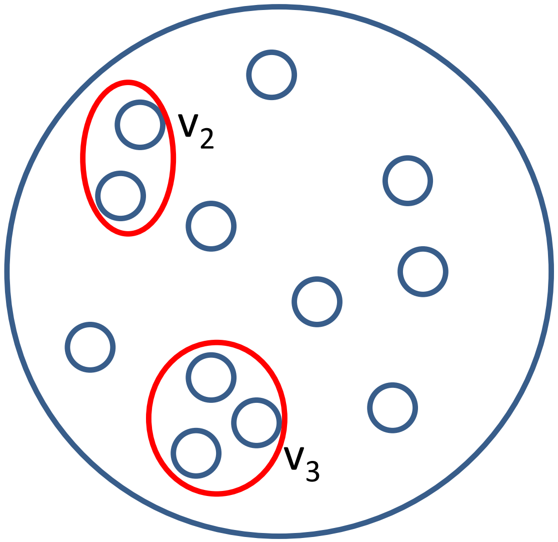
$$f_I = \frac{1}{2\delta}(\frac{m}{m_n^*} - \frac{m}{m_p^*}) = \frac{m}{k}\frac{\partial((U_n - U_p)/2\delta)}{\partial k} = \frac{\partial U_{sym}}{\partial E_k}.$$

$$\ldots (2x_1+1)]\frac{\rho}{2}$$

One should constrain the symmetry energy in multi-dimension parameter space

Model: transport code, ImQMD

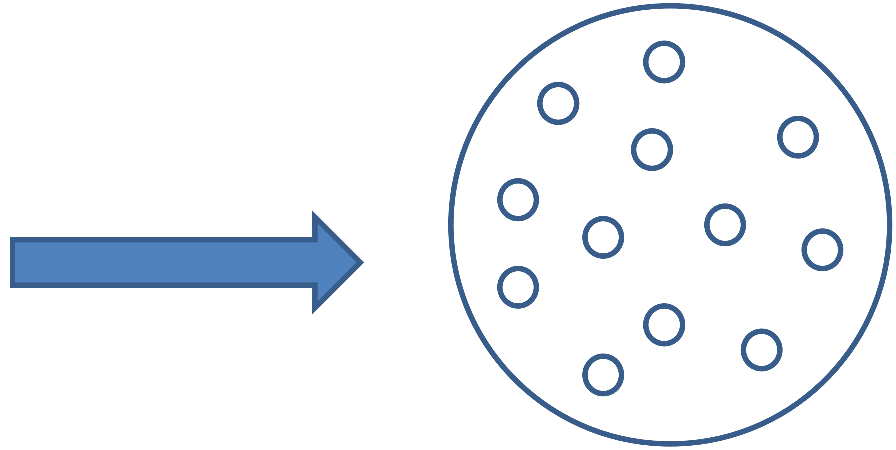
Ab-initio



$$\Psi(r_1, r_2, r_3, ..., r_N)$$

$$<\Psi|H|\Psi> = E$$

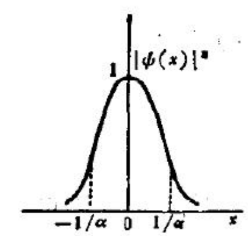
QMD approaches



Hartree approximation
Mean field level

$$\Psi \approx \Phi = \Pi_i \phi_i$$

$$H \rightarrow H_{eff}$$



$$\frac{\partial < p_i >}{\partial t} = - < \nabla_i V(R_1, ...R_N) > \approx -\frac{\partial U(\bar{R}_1 ... \bar{R}_N)}{\partial \bar{R}_i} \qquad \frac{d < R_i >}{dt} = \frac{< p_i >}{m}.$$

We can use EDF in QMD approach

$R_{i0} =< R_i >$ and $p_{i0} =< p_i >$ are the centroid of wave packets in coordinate and momenta

Nucleon-nucleon collision integral part is simulated with cascade

In the new version of ImQMD code, nucleons are represented by Gaussian wavepackets

- ***ImQMD with standard Skyrme interaction***

the potential energy U that includes the full Skyrme potential energy without the spin-orbit term:

$$U = U_\rho + U_{md} + U_{coul} \qquad (2)$$

and U_{coul} is the Coulomb energy. The nuclear contributions are represented in a local form with

$$U_{\rho,md} = \int u_{\rho,md} d^3r$$

and

$$u_\rho = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\eta+1}\frac{\rho^{\eta+1}}{\rho_0^\eta} + \frac{g_{sur}}{2\rho_0}(\nabla\rho)^2 + \frac{g_{sur,iso}}{\rho_0}[\nabla(\rho_n - \rho_p)]^2 + A_{sym}\rho^2\delta^2 + B_{sym}\rho^{\eta+1}\delta^2$$

Skyrme type Momentum Dependent Interaction

$$\delta(r_1 - r_2)(p_1 - p_2)^2$$

Y.X. Zhang, M.B.Tsang, Z.X. Li, HLiu, PLB732,186 (2014)

$$\begin{aligned} u_{md} &= u_{md}(\rho\tau) + u_{md}(\rho_n\tau_n) + u_{md}(\rho_p\tau_p) \qquad (5) \\ &= C_0 \int d\vec{p}\, d\vec{p}'\, f(\vec{r},\vec{p}) f(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2 \\ &\quad + D_0[\int d\vec{p}\, d\vec{p}'\, f_n(\vec{r},\vec{p}) f_n(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2 + \int d\vec{p}\, d\vec{p}'\, f_p(\vec{r},\vec{p}) f_p(\vec{r},\vec{p}')(\vec{p}-\vec{p}')^2] \end{aligned}$$

Interaction parameter →Nuclear matter parameters in SHF

$$g_{\rho\tau} = \frac{3}{5}\left(\frac{m_0}{m_s^*} - 1\right)\epsilon_F^0,$$

$$\gamma = \frac{K_0 + \frac{6}{5}\epsilon_F^0 - 10 g_{\rho\tau}}{\frac{9}{5}\epsilon_F^0 - 6 g_{\rho\tau} - 9E_0}$$

$$\beta = \frac{\left(\frac{1}{5}\epsilon_F^0 - \frac{2}{3}g_{\rho\tau} - E_0\right)(\gamma + 1)}{\gamma - 1}$$

$$\alpha = E_0 - \epsilon_F^0 - \frac{8}{3}g_{\rho\tau} - \beta$$

$$C_{sym} = -\frac{1}{24}\left(\frac{3\pi^2}{2}\right)^{2/3}(3\Theta_v - 2\Theta_s)\rho^{5/3}$$

$$B_{sym} = \frac{3S_0 - L - \frac{1}{3}\epsilon_F^0 + 2C_{sym}(m_s^*, m_v^*)}{-3\sigma}$$

$$A_{sym} = S_0 - \frac{1}{3}\epsilon_F^0 - B_{sym} - C_{sym}(m_s^*, m_v^*)$$

$$\Theta_s = \left(\frac{m}{m_s^*} - 1\right)\frac{8\hbar^2}{m}\frac{1}{\rho_0}, \quad \Theta_v = \left(\frac{m}{m_v^*} - 1\right)\frac{4\hbar^2}{m}\frac{1}{\rho_0}.$$

7 nuclear matter parameters!

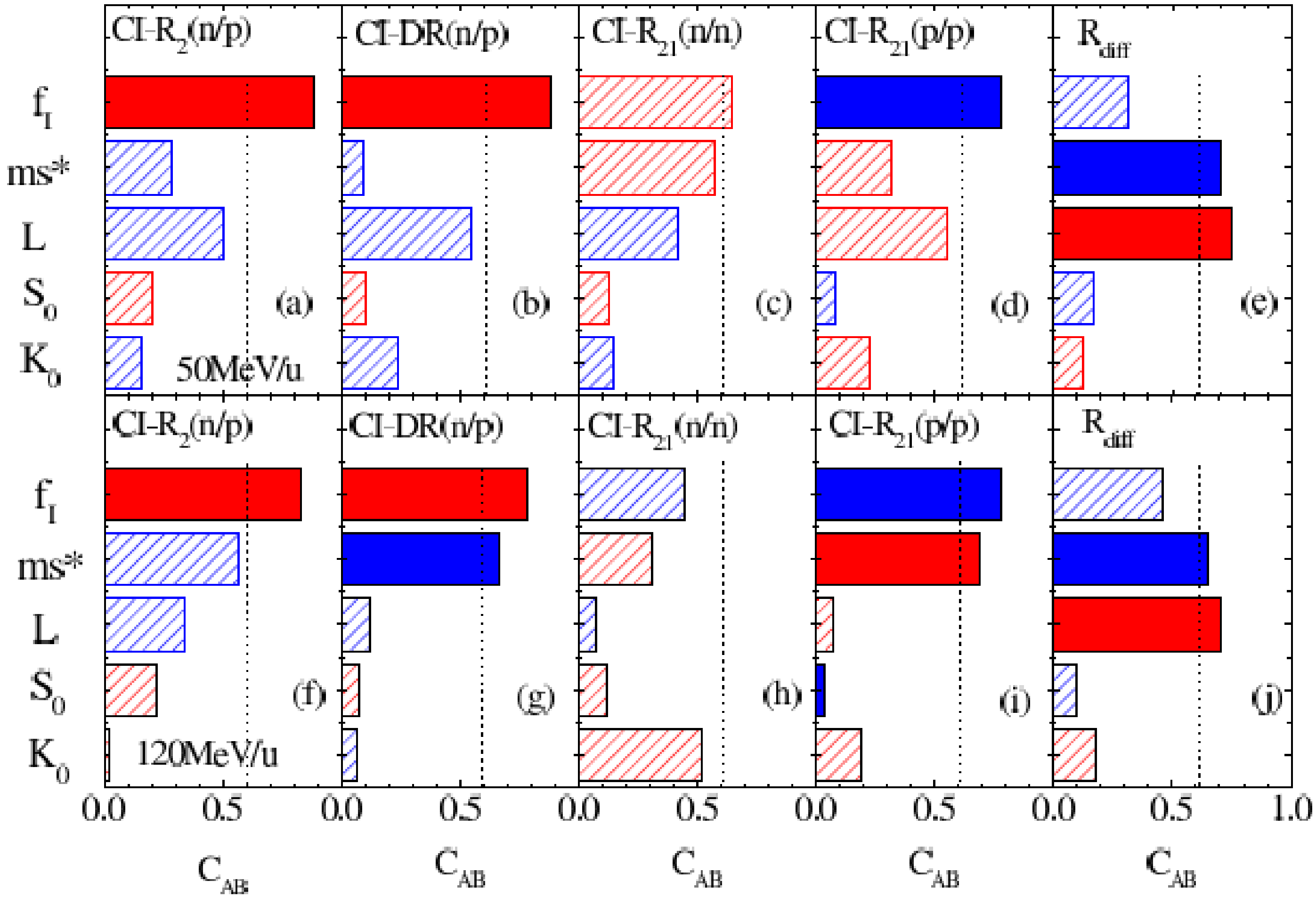
$$C_0 = \frac{1}{16\hbar^2}[t_1(2 + x_1) + t_2(2 + x_2)] = \frac{1}{16\hbar^2}\Theta_v$$

$$D_0 = \frac{1}{16\hbar^2}[t_2(2x_2 + 1) - t_1(2x_1 + 1)] = \frac{1}{16\hbar^2}(\Theta_s - 2\Theta_v)$$

Sensitivity analysis of model parameter x={} to O={}

Sensitivity analysis C_{AB} of x={} and O={}

Y.X.Zhang,M.B.Tsang, Z.X.Li, PLB749,262(2015)



Blue:
negative correlation

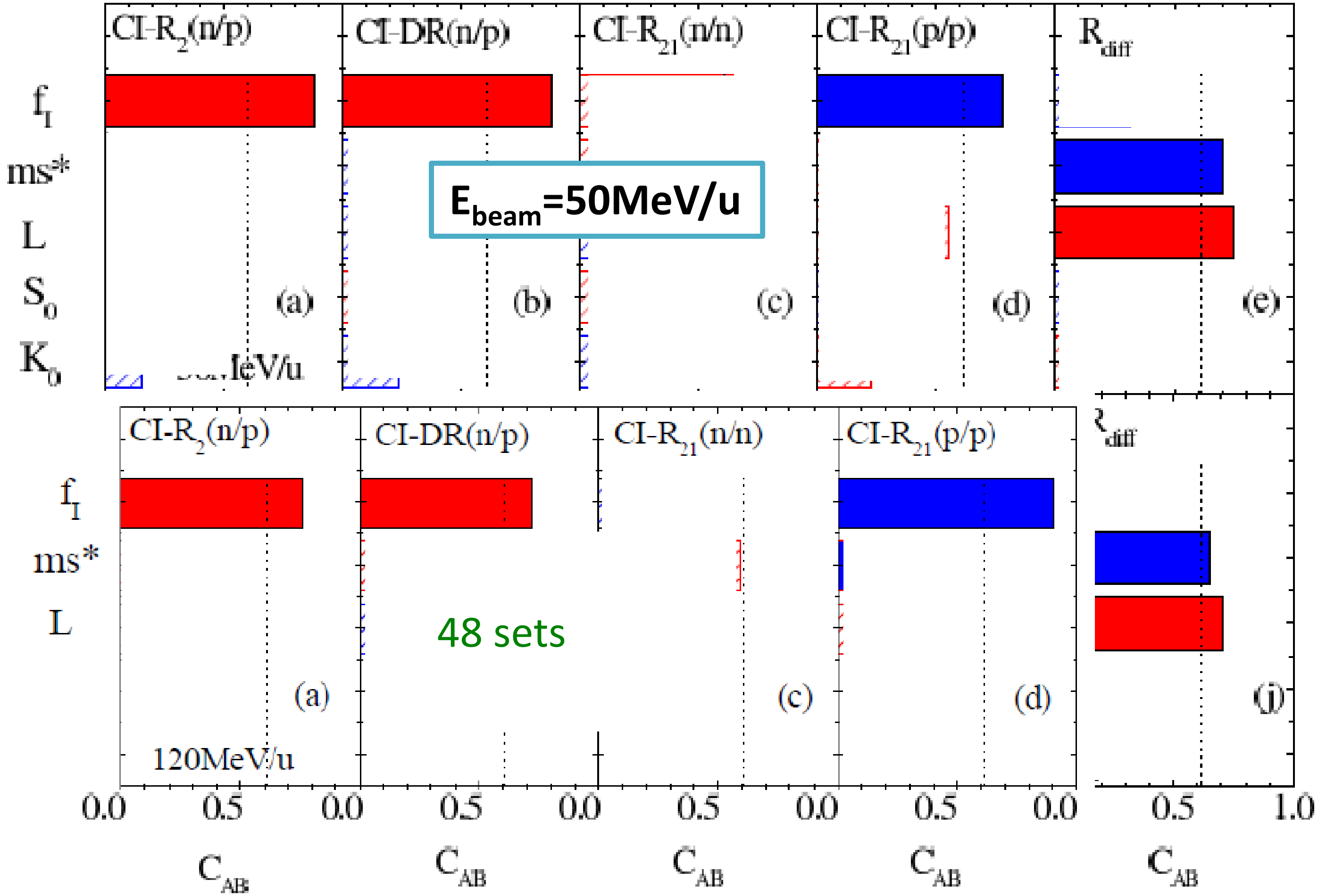
Red:
Positive correlation

The ratios are constructed with Ek>40MeV

- **Ms* also play important roles for isospin diffusion, and neutron to proton yield ratio observables at 120MeV/u.**

Sensitivity analysis C_{AB} of x={} and O={}

Y.X.Zhang,M.B.Tsang, Z.X.Li, PLB749,262 (2015)



Blue:
negative correlation

Red:
Positive correlation

The ratios are constructed with Ek>40MeV

- **Ms* also play important roles for isospin diffusion, and neutron to proton yield ratio observables at 120MeV/u, one can reasonable determine {K_0, S_0, L,ms*, fi} by combination analysis.**

Bayesian inference on the $\{K_0, S_0, L, ms^*, f_I\}$ in multi-dimension parameter space for Skyrme parameter set

$$P(X|D) = \frac{P(D|X)P(X)}{P(D)}$$

- Prior information of X

P(X)

- Likelihood function

P(D|X)

$$L(x) \sim \exp\left(-\sum_a \frac{\left(y_{M,a}(x) - y_a^{exp}\right)^2}{2\sigma_a^2}\right)$$

- Likelihood function

$$L(x) \sim \exp\left(-\sum_a \frac{\left(y_{M,a}(x) - y_a^{exp}\right)^2}{2\sigma_a^2}\right)$$

7 Nuclear matter parameters:

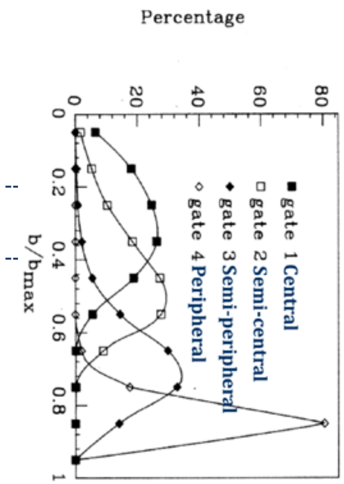
$$\{\rho_0,\ E_0,\ K_0,\ S_0,\ L,\ m_s^*,\ m_v^*\}$$

Use rho$_0$=0.16, E_0 =-16MeV

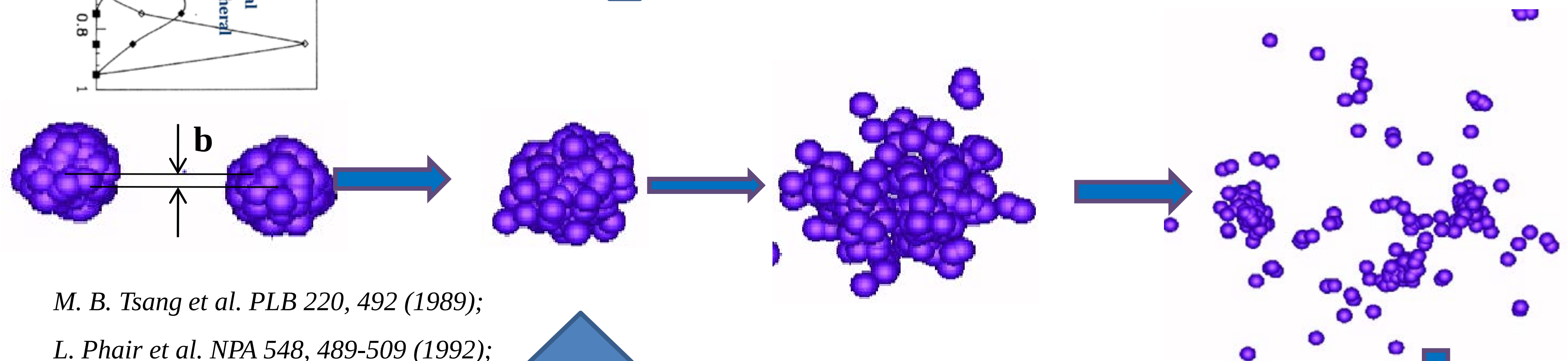
- K_0= 200-280 MeV
- S_0= 25- 35MeV
- L=30-120 MeV,
- m_s*/m=0.6-1.0
- f_i=-0.5-0.4

120 set points (training points)

Considering the Impact parameter smearing



Estimate impact parameter



M. B. Tsang et al. PLB 220, 492 (1989);

L. Phair et al. NPA 548, 489-509 (1992);

J.F. Lecolley et al. PLB 325, 317-321 (1994);

J. Lukasik et al. PRC 55, 1906 (1997);

T. X. Liu et al. PRC 86, 024605 (2012).

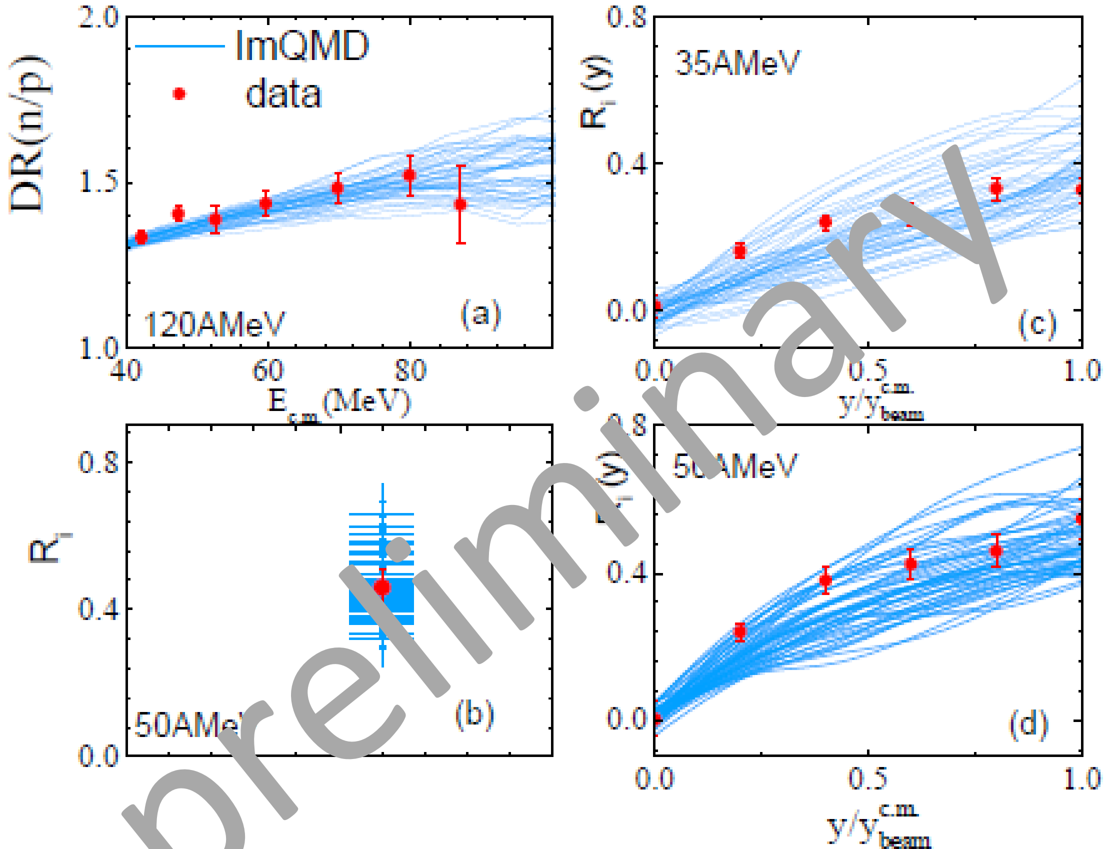
…

1. M， 2. E_{t12} ， 3. θ_{flow}…

Go back to the process

Observables

$$\{\rho_0,\ E_0,\ K_0,\ S_0,\ \boxed{L,\ m_s^*,\ m_v^*}\}$$



obtained with ImQMD, for 4 Obs from 88 sets

Data:

D.D.Coupland, et.al., PRC94,011601(2016)

Z.Y.Sun, et.al., PRC82, 051603(2010)

T.X.Liu, et.al., PRC76,034603(2007)

Tsang, et.al., PRL92,062701(2006)

4, Summary and outlook

1), A new version ImQMD can use the ‘real’ Skyrme EDF was developed

2), parameter correlation should be carefully considered in the reliable constraints on symmetry energy in HICs

3), Bayesian inference on Skyrme EDF from HIC together with neuron skin and NS properties is needed.

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NSCC-TJ for calculations

Thanks for your attention!