Out-of-equilibrium system and relativistic hydro in heavy-ion collisions

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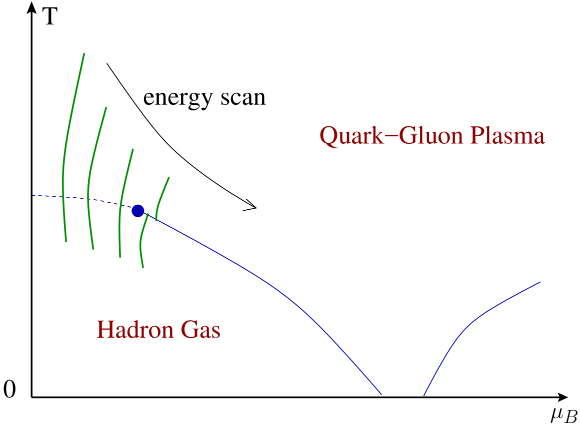
arXiv:1712.03856, 1703.10694, with Jean-Paul Blaizot

A ‘standard’ picture for heavy-ion collisions



The system is assumed in thermal equilibrium ⇒ hydrodynamics

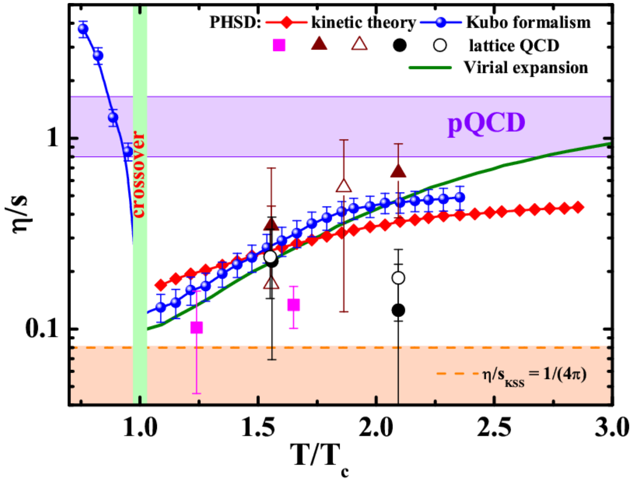
Hydrodynamics is an effective theory for system in/close to equilibrium

$$\text{e.g.,}\quad \partial_\mu T^{\mu\nu} = 0, \quad T^{\mu\nu} = \begin{pmatrix} \mathcal{E} & 0 & 0 & 0 \\ 0 & \mathcal{P} & 0 & 0 \\ 0 & 0 & \mathcal{P} & 0 \\ 0 & 0 & 0 & \mathcal{P} \end{pmatrix} + \Pi^{\mu\nu}$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \nabla^2 \underbrace{+\nabla^3 + \nabla^4 + \nabla^5 + \nabla^6 + \dots}_{\text{not included in simulations}}$$

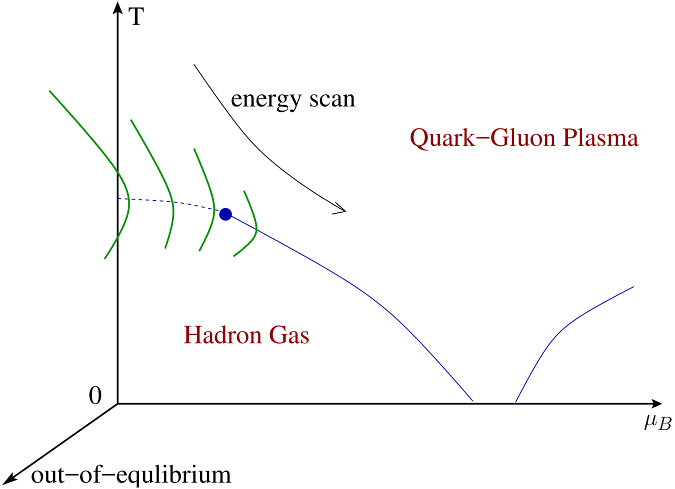
- Gradient expansion: Knudsen number $\sim \nabla$
- 2nd order in gradients: Israel-Stewart, BRSSS, DNMR
  Baier, Romatschke, Son, Starinets, Stephonov, Niemi, Denicol, ...
- Hydro predictions are compatible with heavy-ion experiments.
  P. Romatschke, M.Luzum, K. Dusling, D. Teaney, U. Heinz, H. Song
- η/s, etc. are estimated based on flow measurements:
  M.Luzum, J. Ollitrault, B. Schenke, C. Gale, S. Jeon, ...

$$\Rightarrow \frac{\eta}{s} = 1 \sim 3 \times \left(\frac{1}{4\pi}\right)$$



QGP is strongly coupled medium!

A 'modified' picture for heavy-ion collisions

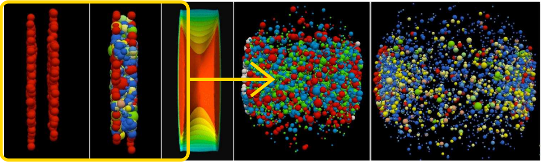


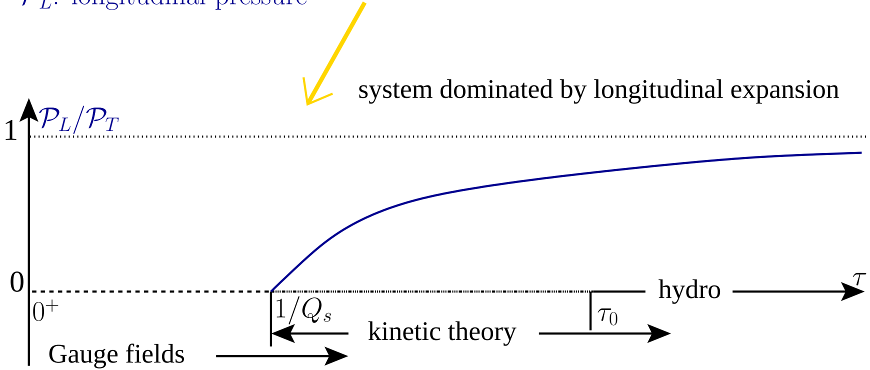
- Is it possible the system is out-of-equilibrium?
- For the system evolution, is it still fluid dynamics to be considered?

Out-of-equilibrium QGP system in heavy-ion collisions

$\mathcal{P}_T$: transverse pressure

$\mathcal{P}_L$: longitudinal pressure





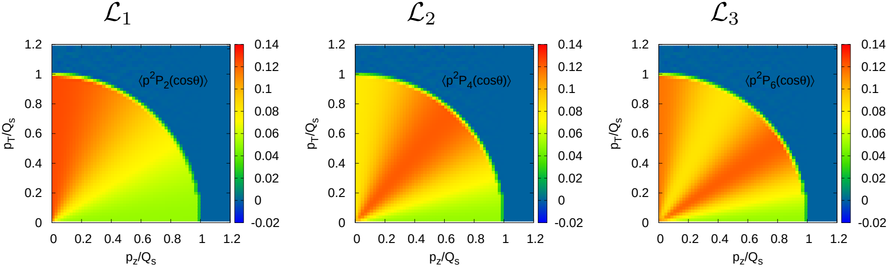
Thermalization is even more changllenged in small colliding systems !

Beyond pressure anisotropy: $\mathcal{L}$-moment

p^2-moment weighted with Legendre Polynomial P_{2n}:

$$\mathcal{L}_n = \int \frac{d^3p}{(2\pi)^3 p^0} p^2 P_{2n}(p_z/p_\perp) f(\tau, \vec{p}_\perp, p_z),$$

- Lowest order moment conincides with energy density: $\mathcal{L}_0 = \mathcal{E}$.
- First order moment describes isotropization: $\mathcal{L}_1 = \mathcal{P}_L - \mathcal{P}_T$.
- Higher order moments capture finer structure of f (or δf).



Equation of motion for $\mathcal{L}_n$

Consider relaxation time approximation, τ_R indep. of $\vec{p}$:

$$\left[\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right] f(\mathbf{p},\tau) = -\frac{f(\mathbf{p},\tau) - f_{\rm eq}(p/T)}{\tau_R}, \qquad \tau_R = \tau_R(T)$$

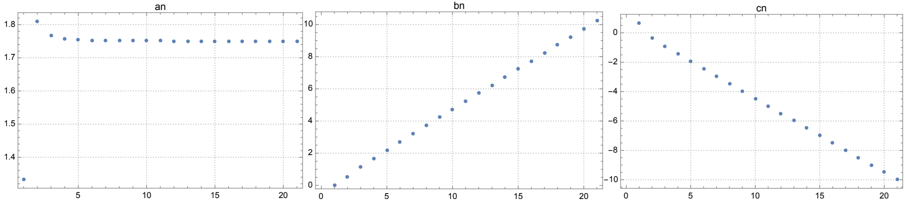
which is equivalent to

$$\begin{aligned}
\frac{\partial \mathcal{L}_n}{\partial \tau} =& -\frac{1}{\tau}\left[a_n\mathcal{L}_n + b_n\mathcal{L}_{n-1} + c_n\mathcal{L}_{n+1}\right] - \frac{\mathcal{L}_n}{\tau_R} \quad (n \geq 1)\\
\frac{\partial \mathcal{L}_0}{\partial \tau} =& -\frac{1}{\tau}\left[a_0\mathcal{L}_0 + c_0\mathcal{L}_1\right],
\end{aligned}$$

where

$$\begin{aligned}
a_n =& \frac{2(14n^2+7n-2)}{(4n-1)(4n+3)}, \quad b_n = \frac{(2n-1)2n(2n+2)}{(4n-1)(4n+1)},\\
c_n =& \frac{(1-2n)(2n+1)(2n+2)}{(4n+1)(4n+3)},
\end{aligned}$$

$1/(\tau/\tau_R)$ defines Knudsen number.



$a_0 = 4/3$, $a_1 = 38/21$, $a_n + b_n + c_n = 2$ ...

Truncation of the coupled equations

$$
\begin{aligned}
\frac{\partial \mathcal{L}_n}{\partial \tau} &= -\frac{1}{\tau}\left[a_n\mathcal{L}_n + b_n\mathcal{L}_{n-1} + c_n\mathcal{L}_{n+1}\right] - \frac{\mathcal{L}_n}{\tau_R} \quad (n \geq 1)\\
\frac{\partial \mathcal{L}_0}{\partial \tau} &= -\frac{1}{\tau}\left[a_0\mathcal{L}_0 + c_0\mathcal{L}_1\right],
\end{aligned}
$$

Truncate at n-th order (ignore all $\mathcal{L}$-moments higher than n-th order) :

- at $n = 0$

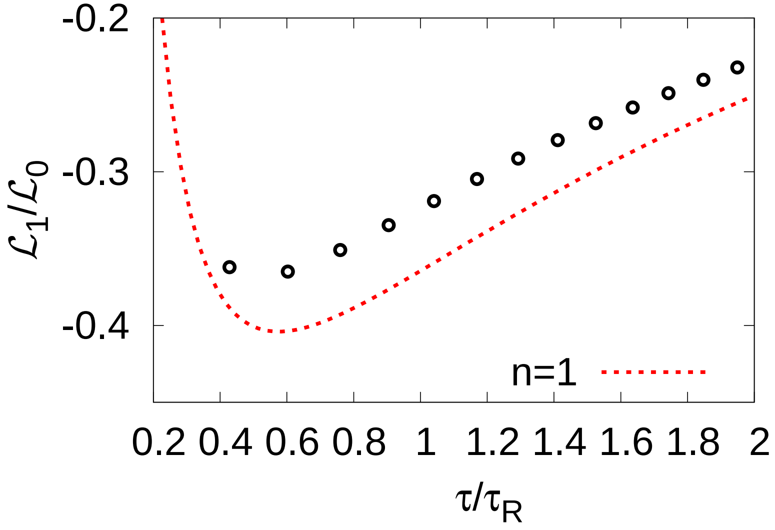
$$
\frac{\partial \mathcal{E}}{\partial \tau} + \frac{4}{3}\frac{\mathcal{E}}{\tau} = 0 \quad \rightarrow \quad \mathcal{E} \sim \tau^{-4/3} \qquad \text{ideal hydro}
$$

- at $n = 1$

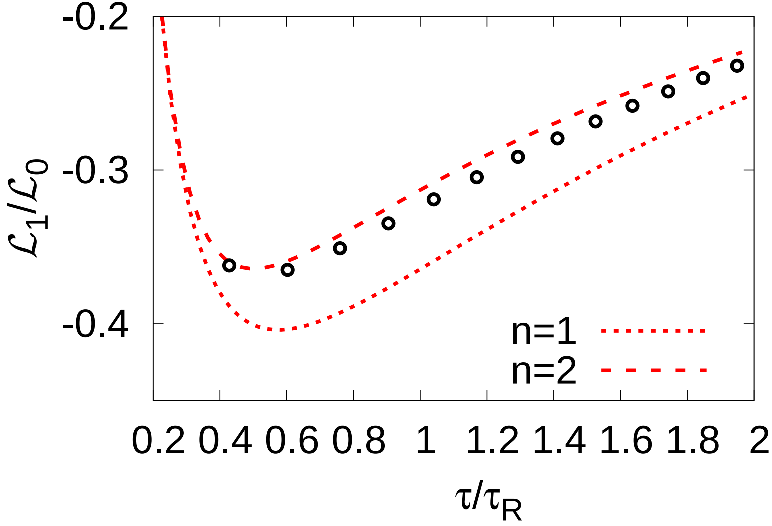
$$
\begin{aligned}
\frac{\partial \mathcal{L}_0}{\partial \tau} &= -\frac{1}{\tau}\left[a_0\mathcal{L}_0 + c_0\mathcal{L}_1\right]\\
\frac{\partial \mathcal{L}_1}{\partial \tau} &= -\frac{1}{\tau}\left[a_1\mathcal{L}_1 + b_1\mathcal{L}_0\right] - \frac{\mathcal{L}_1}{\tau_R}
\end{aligned}
$$

- ...

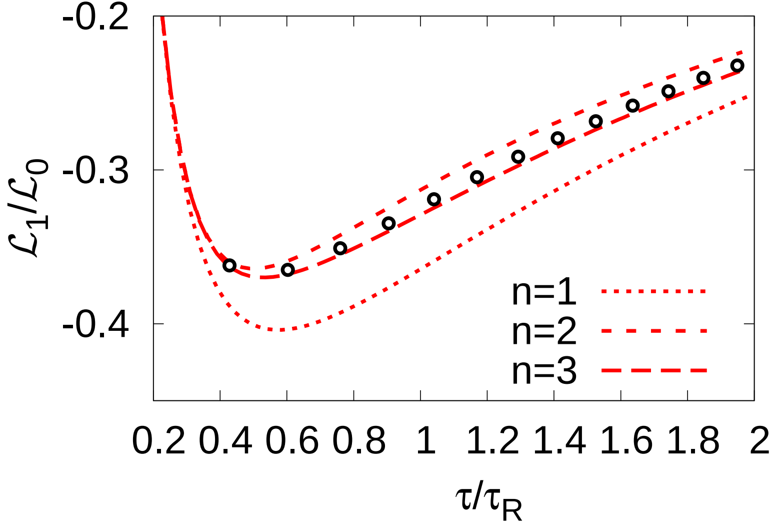
Convergence of truncation



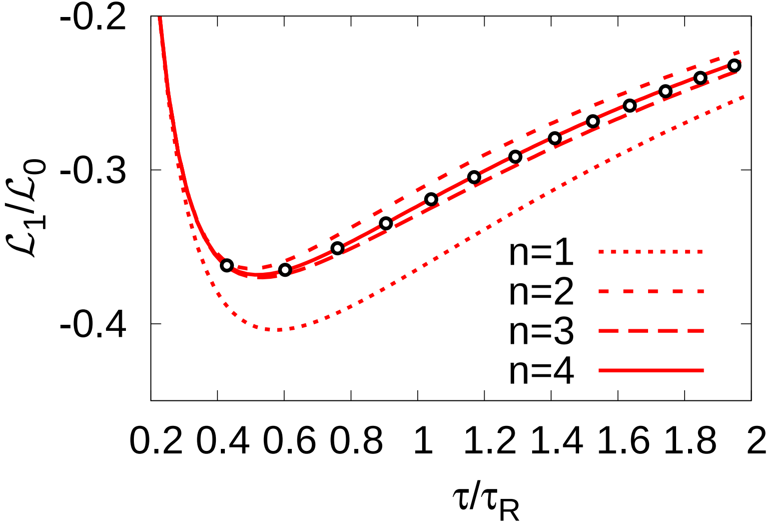
Convergence of truncation



Convergence of truncation



Convergence of truncation



The free-streaming fixed points: $\tau/\tau_R \to 0$

$$\frac{\partial \mathcal{L}_n}{\partial \tau} = -\frac{1}{\tau}\left[a_n \mathcal{L}_n + b_n \mathcal{L}_{n-1} + c_n \mathcal{L}_{n+1}\right]$$

For infinite n:

- $\mathcal{L}_0 = \mathcal{L}_1 = \mathcal{L}_2 = \mathcal{L}_3 = \dots$

$$\Rightarrow \mathcal{L}_n = \mathcal{L}_n(\tau_0)\left(\frac{\tau_0}{\tau}\right)^2 \qquad \to \tau\partial_\tau \ln \mathcal{L}_n = 2$$

- $\mathcal{L}_n(\tau) = P_{2n}(0)\mathcal{L}_0(\tau)$,

$$\Rightarrow \mathcal{L}_n(\tau) = \mathcal{L}_n(\tau_0)\left(\frac{\tau_0}{\tau}\right) \qquad \to \tau\partial_\tau \ln \mathcal{L}_n = 1$$

For finite n,

$$\begin{pmatrix} a_0 & c_0 & 0 & 0 & \dots \\ b_1 & a_1 & c_1 & 0 & \dots \\ 0 & b_2 & a_2 & c_2 & \dots \\ \dots & \dots & \dots & \dots & \dots \end{pmatrix} \Rightarrow \lambda \approx 2(\text{unstable}) \text{ and } \approx 1(\text{stable})$$

The hydro fixed points: $\tau/\tau_R \to \infty$

Restore hydro EoM via truncation order by order:

- Truncation at $n=1$ gives 2nd order hydro: $(c_0\mathcal{L}_1 = \Pi = \Pi^\xi_\xi)$

$$\partial_\tau \mathcal{L}_0 = -\frac{1}{\tau}(a_0\mathcal{L}_0 + c_1\mathcal{L}_1) \quad \to \quad \partial_\tau \mathcal{E} + \frac{\mathcal{E}+\mathcal{P}_L}{\tau} = 0$$

$$\partial_\tau \mathcal{L}_1 = -\frac{1}{\tau}(a_1\mathcal{L}_1 + b_1\mathcal{L}_0) - \frac{1}{\tau_R}\mathcal{L}_1 \quad \to \quad \Pi = -\eta\sigma - a_1\frac{\tau_R}{\tau}\Pi - \tau_R\partial_\tau\Pi\,.$$

* Note $a_1 = 38/21 = \beta_{\pi\pi}$ is 2nd transport coefficient in DNMR.

* Also compatible with BRSSS with respect to conformal symmetry.

- Truncation at $n=2$ gives 3rd order hydro: $(c_0\mathcal{L}_2 = \Sigma = \Sigma^\xi_\xi)$

$$\Pi = -\eta\sigma - \tau_\pi\left[\partial_\tau\Pi + \frac{4}{3}\Pi\nabla\cdot u\right] + \underline{\frac{3\tau_\pi}{4\eta}(a_1 - 4/3)}\Pi\Pi + \underline{\frac{3c_1\tau_\pi}{4\eta}}\Sigma\Pi$$

The hydro fixed points: $\tau/\tau_R \to \infty$

Ansatz expansion form (hydro gradient expansion)

$$\mathcal{L}_n = \sum_{i=n} \frac{\alpha_n^{(i)}}{\tau^i}$$

$\alpha_n^{(i)}$ is transport coefficient (of order $n+i$).

The leading and subleading coefficients can be found analytically.

- E.g., $\mathcal{P}_L - \mathcal{P}_T = \mathcal{L}_1 = \frac{2\eta}{\tau} + \frac{4}{3\tau^2}(\lambda_1 - \eta\tau_\pi) + O(1/\tau^3)$
- E.g., $\mathcal{L}_2 = \frac{4}{3\tau^2}(\lambda_1 + \eta\tau_\pi) + O(1/\tau^3)$
- Conformal gas and relaxation time approximantion $\tau_R \propto p^2$:

$$\alpha_n^{(n)} = (-1)^n \frac{(2n)!}{(4n+1)!!}\Gamma(2n+4)\left(\frac{\eta}{s}\right)^n \frac{T^{4-n}}{2\pi^2}$$

Factorial growth of $\alpha_n^{(n)} \sim n!$: gradient expansion is asymptotic!

H. Grad, M. Heller, M. Spalinski, G. Denicol, J. Noronha

The hydro fixed points: $\tau/\tau_R \to \infty$

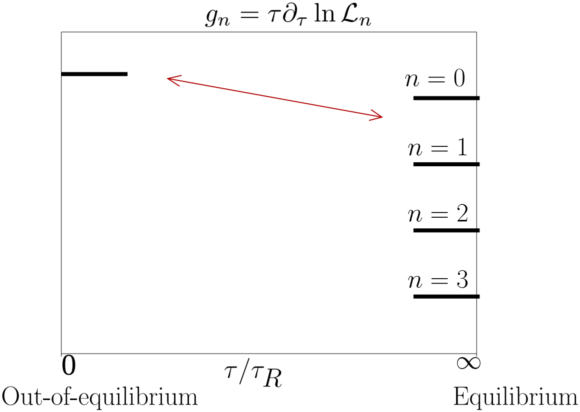
Asymptotic decay rate determined by the leading term: $\mathcal{L}_n \sim \alpha_n^{(n)}/\tau$

$$\Rightarrow \tau\partial_\tau \ln \mathcal{L}_n = -\frac{4+2n}{3} \quad (\tau_R \propto 1/T)$$

$$\Rightarrow \tau\partial_\tau \ln \mathcal{L}_n = -\frac{4+3n}{3} \quad (\tau_R \text{ constant})$$

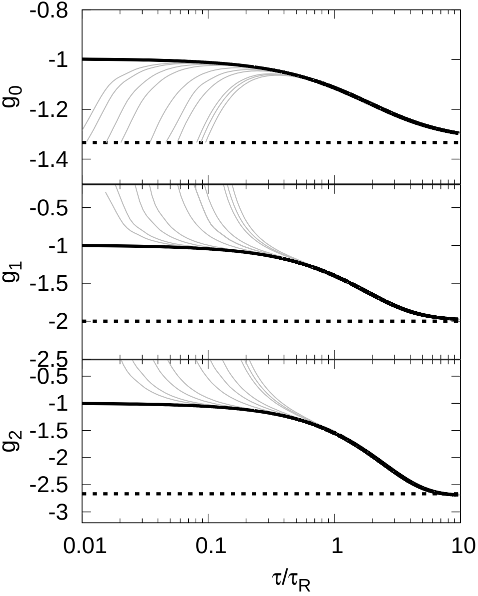
These are stable fixed points in the hydro regime.

A simple summary on the fixed points

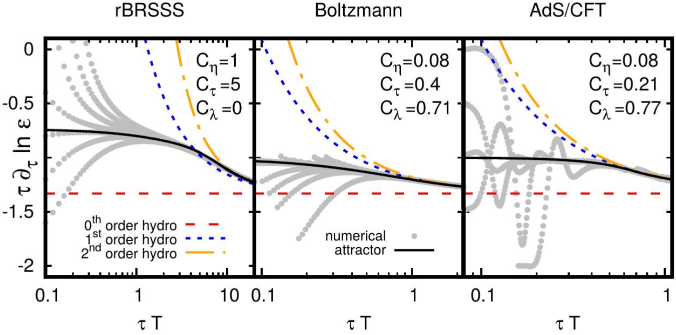


- Degenerated fixed point of all g_n in the free-streaming limit.
- Hydro fixed points of g_n split according to n.
- System evolves between there two types of fixed points – attractor.

Attractor solution



Some general discussions on attractor solution



P. Romatschke

- Attractor solution exists, with or without conformal symmetry
- Non-hydro modes decay exponentially, w.r.t. attractor solutions.
  P. Romatschke, A. Kurkela, U. Wiedemann

$\Rightarrow$ fluid dynamics can be extended to out-of-equilibrium system!

- Attractor corresponds to Borel resummed hydro gradient expansion.
  M.Heller, M. Spalinski, R. Janik, P. Witaszczky, G. Basar, G. Dunne, ...

Renormalization of η/s

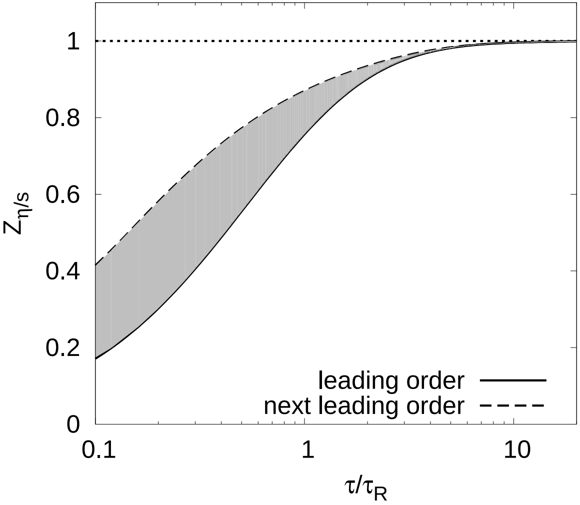
Effects from higher order moments/viscous hydro (leading order):

$$\partial_\tau \mathcal{L}_0 = -\frac{1}{\tau}(a_0\mathcal{L}_0 + c_0\mathcal{L}_1)\,,$$
$$\partial_\tau \mathcal{L}_1 = -\frac{1}{\tau}(a_1\mathcal{L}_1 + b_0\mathcal{L}_0) - \underbrace{\left[1 + \frac{c_1\tau_R}{\tau}\frac{\mathcal{L}_2}{\mathcal{L}_1}\right]}_{Z^{-1}_{\eta/s}}\frac{\mathcal{L}_1}{\tau_R} \qquad (\mathcal{L}_2 \text{ in 2nd hydro})\,,$$

$$g_2(\tau/\tau_R) = -a_2 - b_2\frac{\mathcal{L}_2}{\mathcal{L}_1} - \frac{\tau}{\tau_R}\,.$$

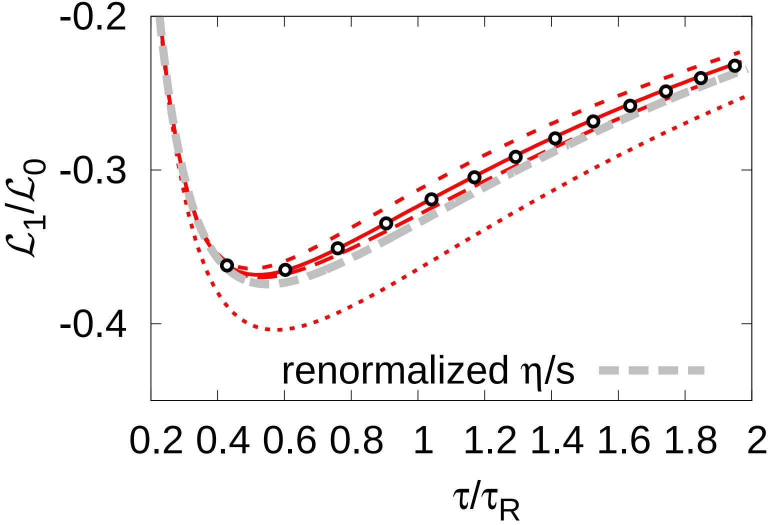
- Taking attractor solution for g_2: resummed of gradients.
- Effectively, for 2nd order hydro, η/s is renormalized.

Attractor solution



Out-of-equilibrium physics can be effectively absorbed into a reduced η/s.

E. Shuryak, M. Lublinsky


-0.2
-0.3
-0.4
$\mathcal{L}_1/\mathcal{L}_0$
renormalized η/s
0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
τ/τ$_R$

Summary

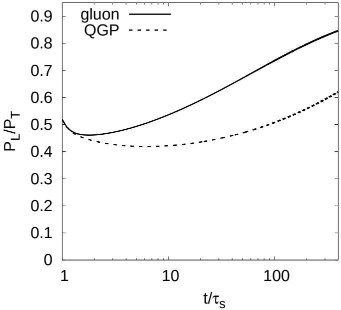
- $\mathcal{L}$-moments are proposed to quantify system thermalization.
- Coupled equations for $\mathcal{L}_n$ are derived, and valid truncations.

$\Rightarrow$ fluid dynamics for out-of-equilibrium system

- Attractor solution smoothly connect fixed points of $\mathcal{L}_n$ in two limits.
- The systems in heavy-ion collisions may be out-of-equilibrium:
  - Actual value of η/s could be larger than expectation.

Back-up slides

Kinetic theory description

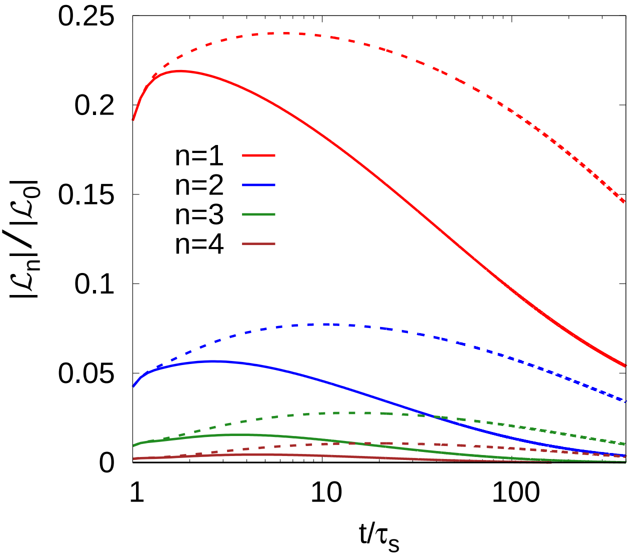


- In kinetic theory: $p^\mu \partial_\mu f(\tau, p_\perp, p_z) = -\mathcal{C}[f]$

$$\mathcal{P}_L = \int \frac{d^3p}{(2\pi)^3 p^0} p_z^2 f(\tau, p_\perp, p_z), \qquad \mathcal{P}_T = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3 p^0} (p_x^2 + p_y^2) f(\tau, p_\perp, p_z)$$

$$\mathcal{E} = \mathcal{P}_L + 2\mathcal{P}_T = \int \frac{d^3p}{(2\pi)^3 p^0} p^2 f(\tau, p_\perp, p_z)$$

Bjorken symmetry: boost invariance along ξ, translation invariance in $\vec{x}_\perp$.



Solved by kinetic theory with respect to QCD 2-to-2 scattering.