三峡大学
CHINA THREE GORGES UNIVERSITY

Chiral Magnetic Effects and Chiral Mgnetic Current in Relativistic Heavy-Ion Collisions

1. Introductions
2. Magnetic field feature in heavy-ion collisions
3. Charge Separation in heavy-ion collisions
4. Chiral Electromagnetic Current in heavy-ion collisions

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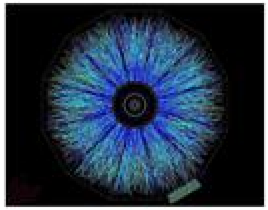
Sheng-Qin Feng, CTGU, iHIC 2018, Tsinghua University, Beijing, April 7-11, 2018

1. Introduction

Strong

Strong matter produced
in heavy ion collisions



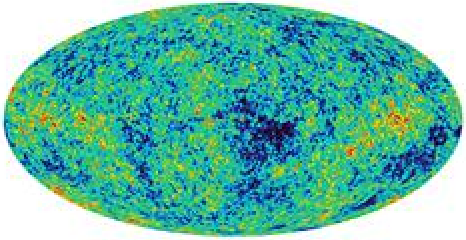
Topological charge changing transitions

induce difference between number of left- and right-handed fermions

Parity to be violated locally in microscopic domains in QCD at finite temperature

ElectroWeak Matter

Electroweak matter produced
in the early universe



Topological charge changing transitions

induce nonzero baryon + lepton number

Parity to be violated globally of weak interactions of the standard model

At high temperatures these transitions are unsuppressed (Sphalerons)
Manton ('83), Manton and Klinkhamer ('84), Mclerran and Shaposhnikov ('85)

How to observe topological charge changing transitions in hot quark matter?

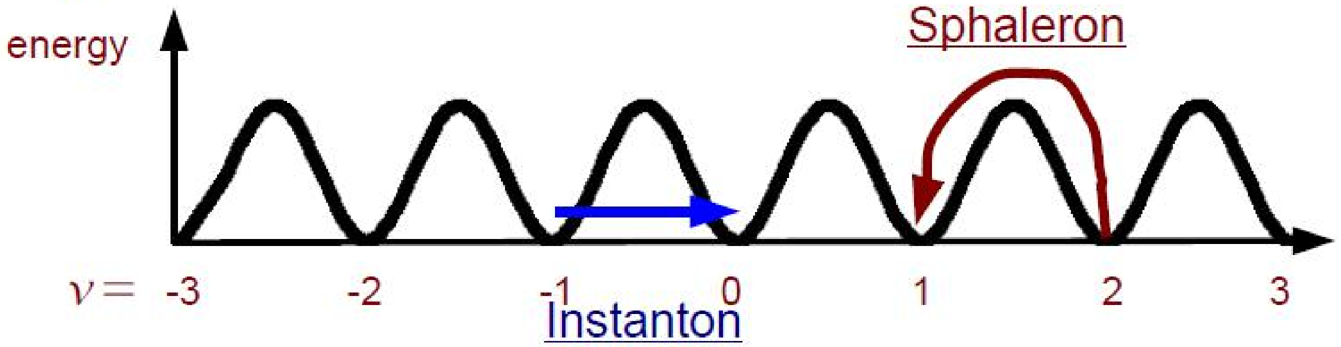
Kharzeev, Mclerran, Warringa, （‘08)

An asymmetry between matter and antimatter is observed
Kuzmin, Shaposhnikov ('85)

Instantons and Sphaleron

$$Q_w = \frac{g^2}{8\pi^2}\int d^4x\,\vec{E}_a\cdot\vec{B}_a = 0, \pm 1, \pm 2, \ldots.$$

Stable under smooth deformations
Change topological charge vacuum



Instantons: Configuration with finite action. Tunneling through barrier

Suppression of rate at T=0, 't Hooft ('76), Pisarski and Yaffe ('80)

Sphaleron: Configuration with finite energy. Go over barrier.

Only possible at finite temperature, rate not suppressed, look for it in QGP!

Manton ('83), Manton and Klinkhamer ('84), McLerran, Mottola and Shaposhnikov ('88)

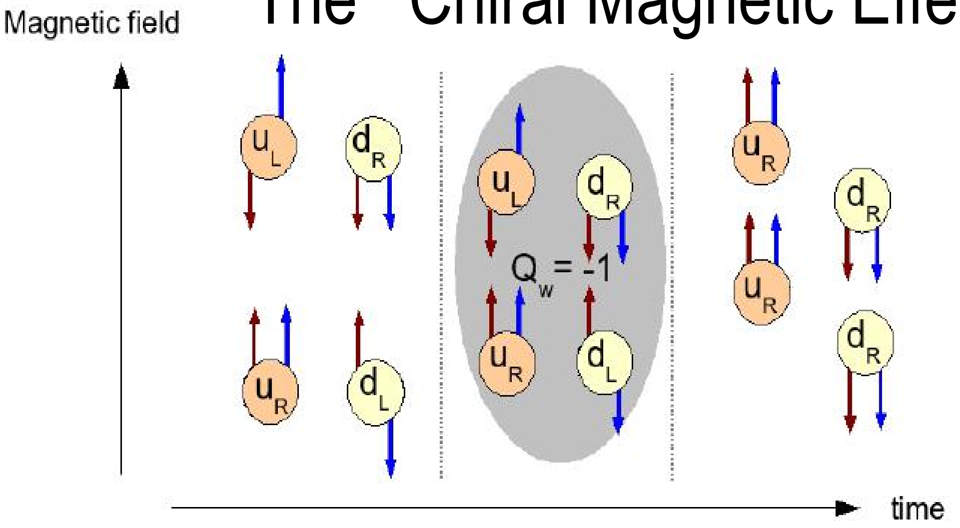
$$\frac{dN_t^{\pm}}{d^3x\,dt} \sim 385\,\alpha_S^5\,T^4$$

Bödeker, Moore and Rummukainen ('00),
several transitions per fm^{-3} per fm/c

Sheng-Qin Feng, CTGU, iHIC 2018, Tsinghua University, Beijing, April 7-11, 2018

D. Kharzeev, L. D. Mclerran, H. J. Warringa, NPA 803, 227 (2008)

The Chiral Magnetic Effect



1. Due to very large magnetic field, the up and down quarks in the lowest landau level and can only move along the direction of the magnetic field. Initially there are as many left-handed as right-handed quarks.

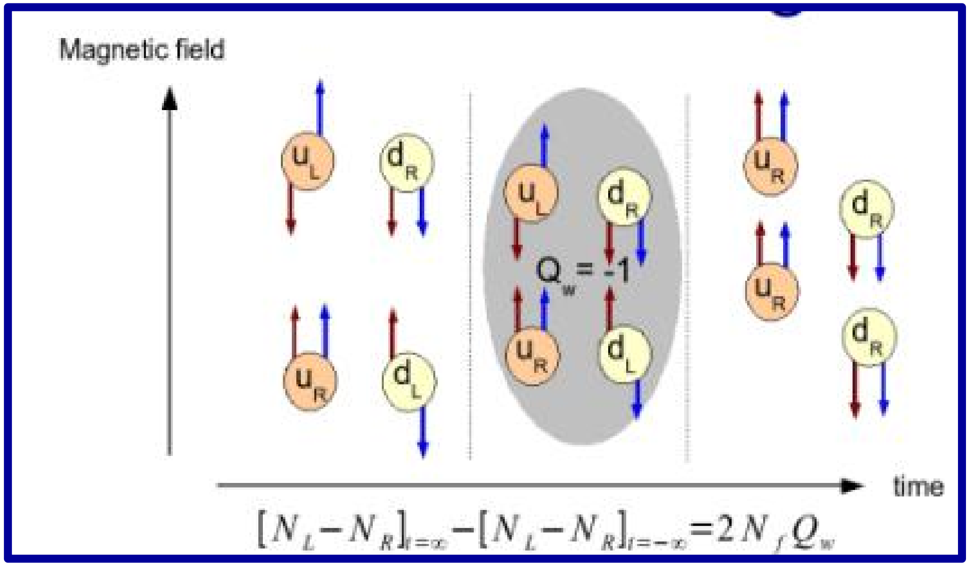
2. The quarks interact with a gauge configuration with non-zero Q_w . Assuming Q_w= -1, this will convert a left-handed up/down quark into a right-handed up/down quark by reversing the direction of momentum.

3. The right-handed up quarks will move upwards, the right-handed down quarks will move downwards. A charge of q = 2e will be created between two sides of a plane perpendicular the magnetic fields

In finite volume this causes separation of positive from negative charge

In presence of magnetic field this induces an Electromagnetic Current

The Chiral Magnetic Effect



$$[N_L - N_R]_{t=\infty} - [N_L - N_R]_{t=-\infty} = 2 N_f Q_w$$

Charge difference:

$$Q = 2 Q_w \sum_f |q_f|$$

Same sign for antiparticles!

Toplogical charge changing transitions induces chirality

In finite volume this causes separation of positive from negative charge

Reasonable polarization of quarks requires: $eB \sim \frac{1}{\rho^2} \sim \alpha_s^2 T^2 \sim 10^3 - 10^4 \text{ MeV}^2$

D. Kharzeev, L. D. Mclerran, H. J. Warringa, NPA 803, 227 (2008)

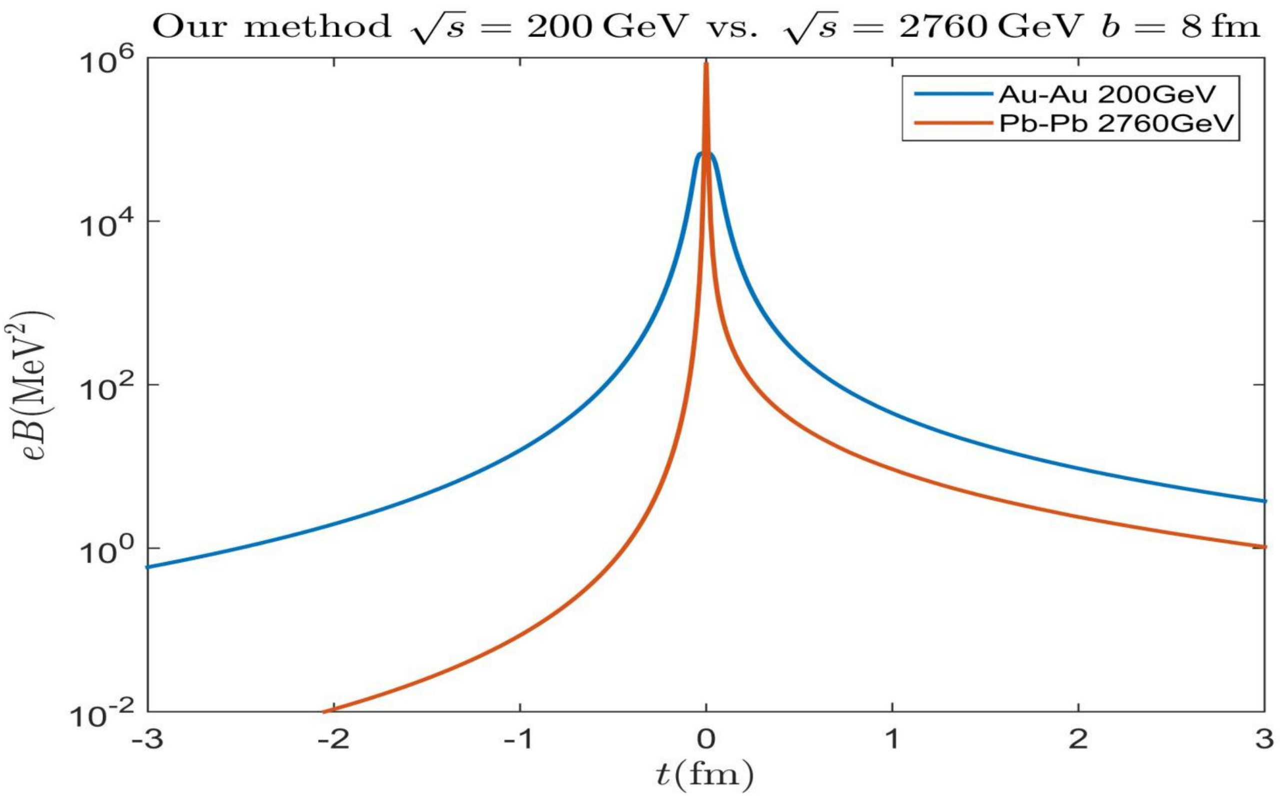
2. Magnetic field features in heavy ion collisions

Y. J. Mo, S. Q. Feng and Y. F. Shi, Phys. Rev. C 024901 (2013)

Y. Zhong, C. B. Yang, X. Cai and S. Q. Feng, Adv. High Energy Phys. 2014, (2014) 193039

Y. Zhong, C. B. Yang, X. Cai and S. Q. Feng, Chinese Physics C, 39, (2015) 104105

Magnetic fields change with time in heavy-ion collisions



At LHC, magnetic fields falls off much more rapidly with time than at RHIC, Chiral Magnetic Effect is early time dynamics

$$eB(\tau = 0.1\,\mathrm{fm}) = 10^{3} \sim 10^{4}\,\mathbf{MeV}^{2}(200\,\mathbf{GeV})$$

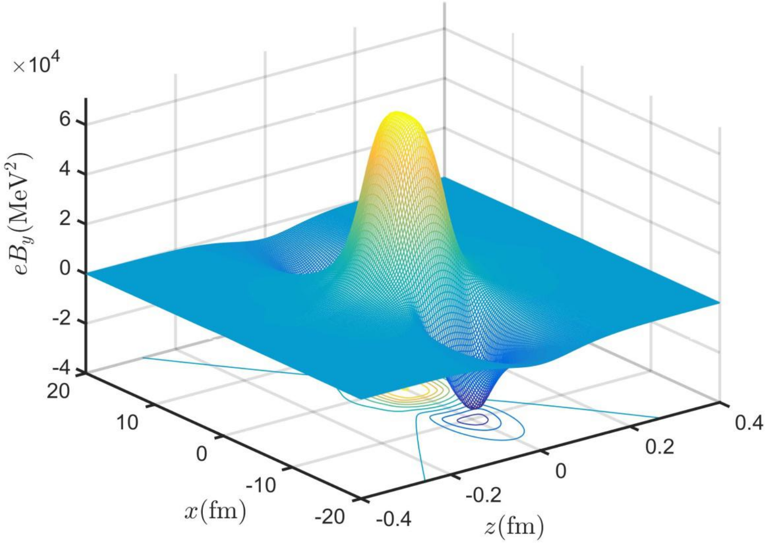
$$eB(\tau = 0.1\,\mathrm{fm}) = 10^{1} \sim 10^{2}\,\mathbf{MeV}^{2}(2760\,\mathbf{GeV})$$

Y. J. Mo, S. Q. Feng and Y. F. Shi, Phys. Rev. C 024901 (2013)

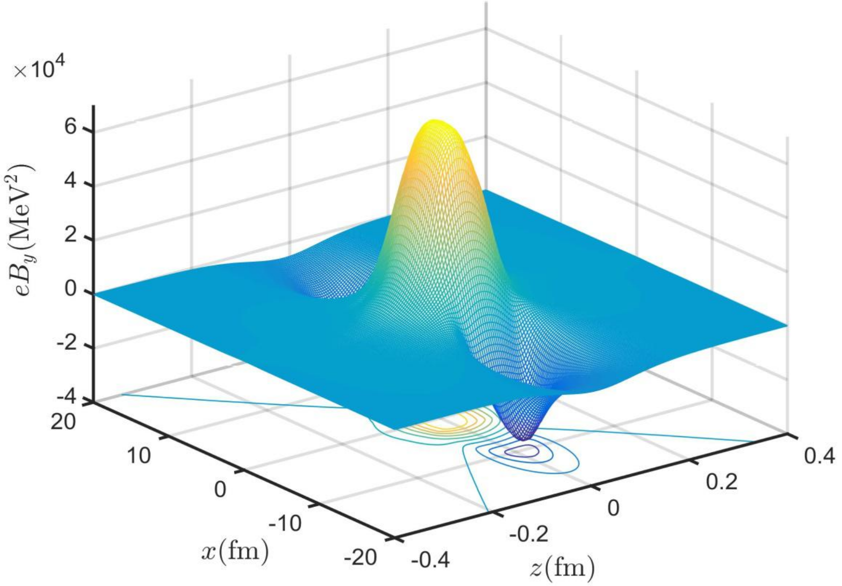
Y. J. Mo, S. Q. Feng and Y. F. Shi, Phys. Rev. C 024901 (2013)

1. Magnetic field is not homogeneous field
2. Magnetic field varies with time especically in the z direction.

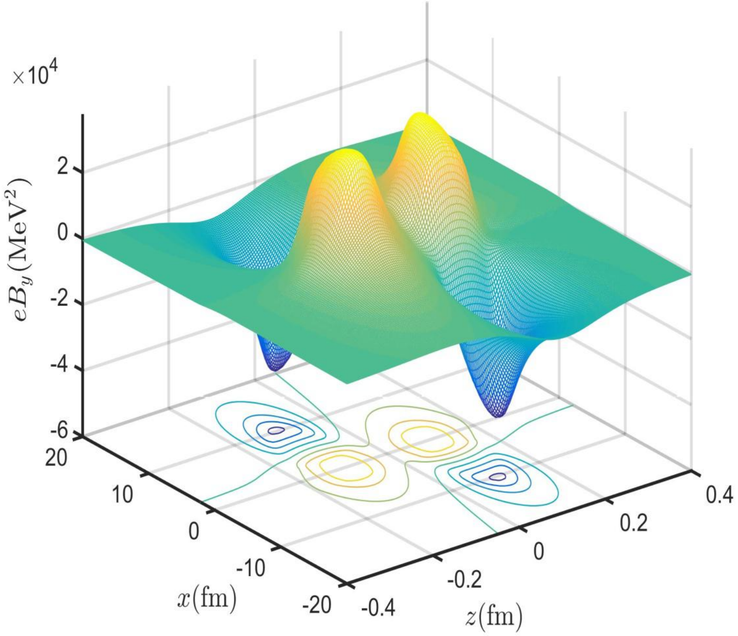
Au-Au $\sqrt{s} = 200\,\mathrm{GeV}$ $b = 8\,\mathrm{fm}$ $t = 0.001\,\mathrm{fm}$

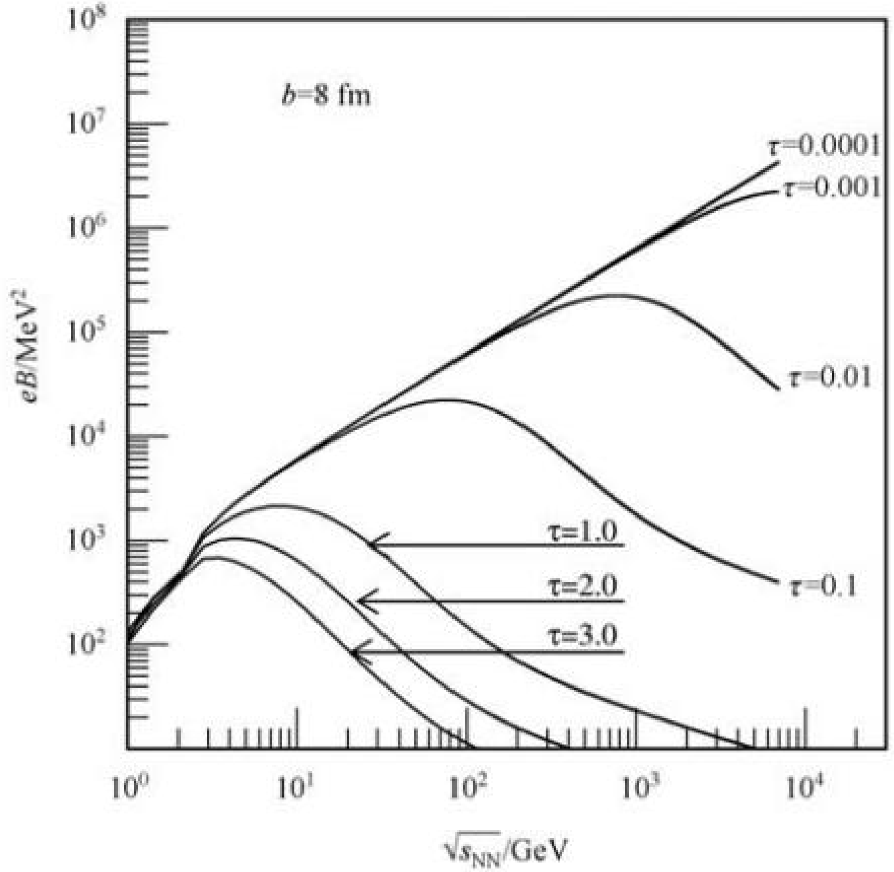


Au-Au $\sqrt{s} = 200\,\mathrm{GeV}$ $b = 8\,\mathrm{fm}$ $t = 0.01\,\mathrm{fm}$



Au-Au $\sqrt{s} = 200\,\mathrm{GeV}$ $b = 8\,\mathrm{fm}$ $t = 0.1\,\mathrm{fm}$





Y. Zhong, C. B. Yang, X. Cai and S. Q. Feng,
Adv. High Energy Phys. 2014 (2014) 193039

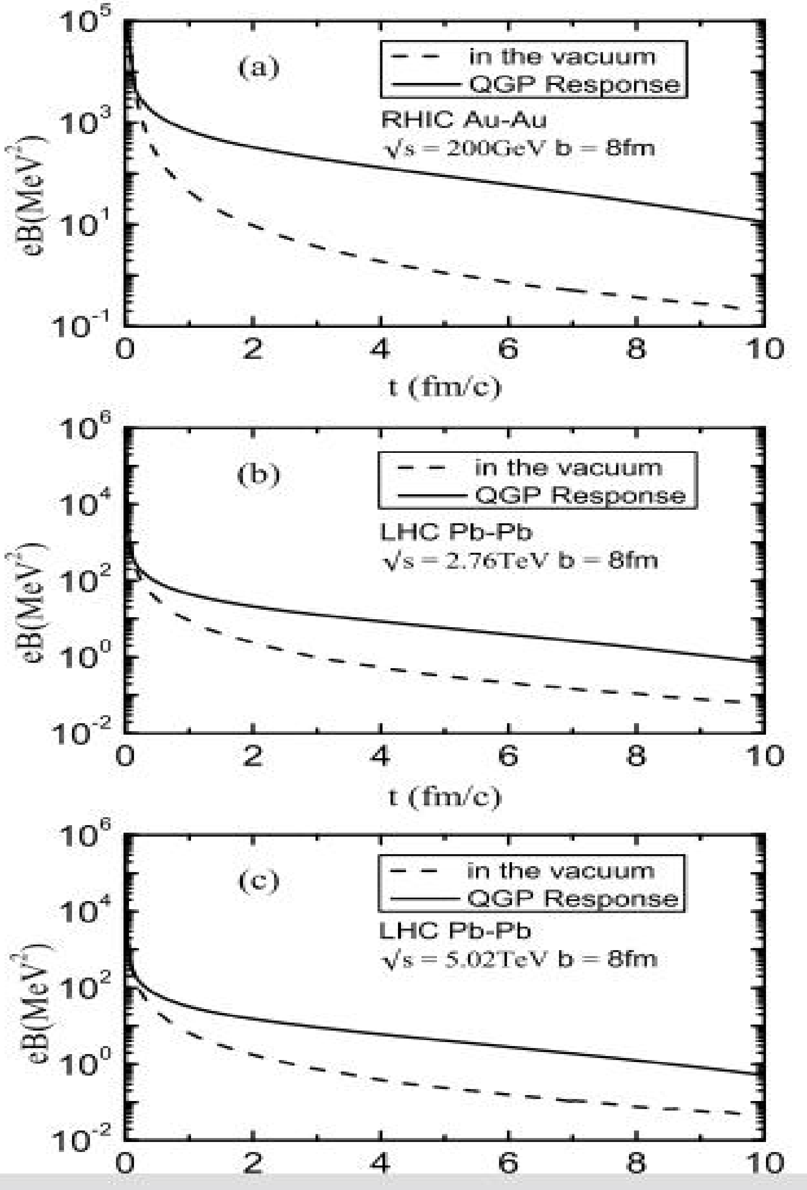
Magnetic field varies with
collision energy at different time.

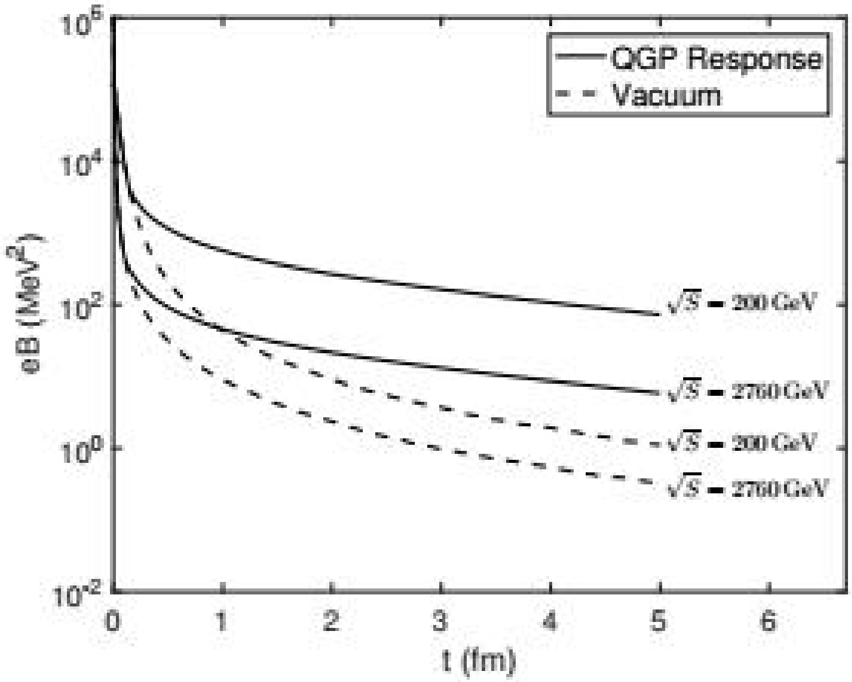
It is found that at different time, the velocity of magnetic field approach zero with time is different. The larger of the time, the larger of the velocity of magnetic field approach zero.

It is found that the maximum value of magnetic field decrease with the increase of time.

Coupled Maxwell +hydro: W. T. Deng, and X. G. Huang, Phys. Rev. C 85, 044907 (2012).

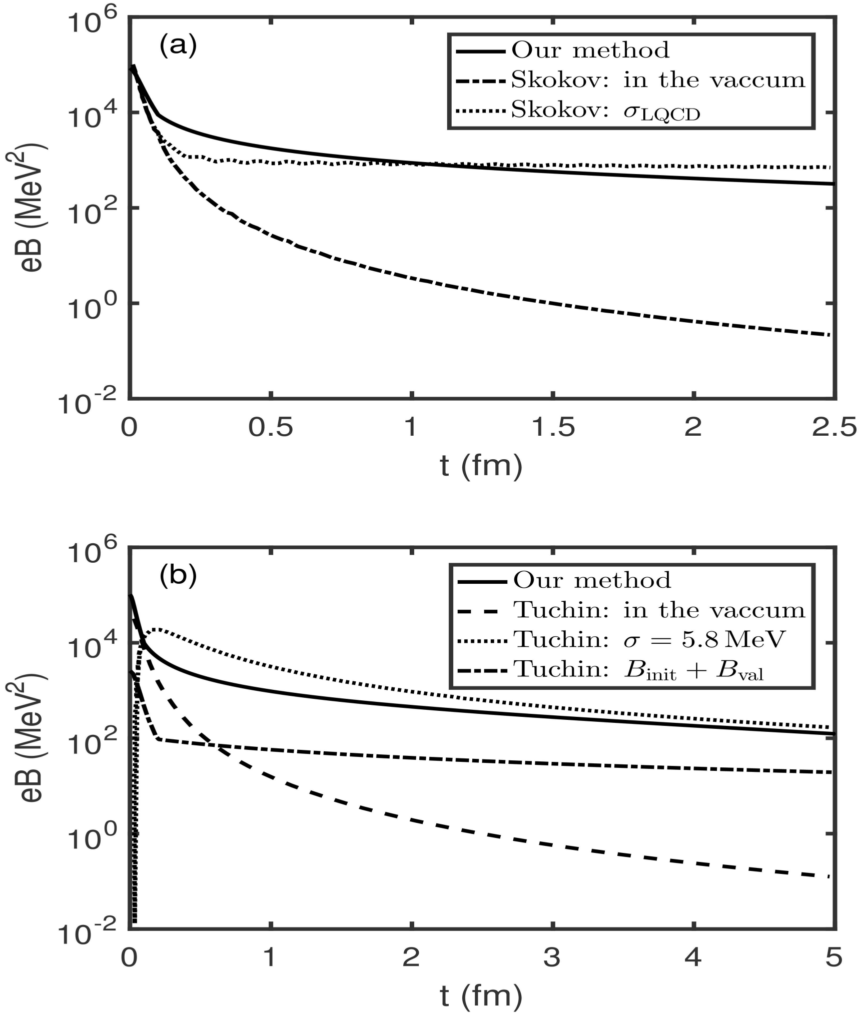
Consider the response of magnetic field with QGP medium





It is found that the magnetic field considering the response will last longer than in the vacuum. Magnetic field last longer than LHC

S.Q. Feng., Xin Ai, et al, Chinese Physics C42, (2018) 054102



1. L. McLerran, V. Skokov, Nucl. Phys. A929, 184 (2014).
2. K. Tuchin, Phys. Rev. C 93, 014905 (2016).

SQ Feng, Xin Ai et al., Chinese Physics C, 42, (2018)054102

3. Charge Separation in heavy-ion collisions

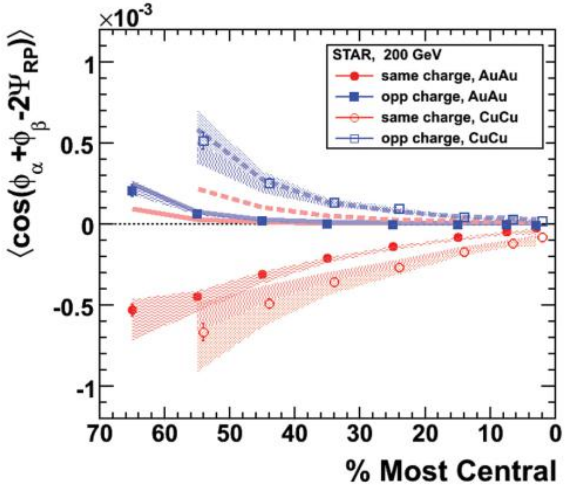
SQ Feng, Xin Ai, Lei Pei, Fei Sun., Chinese Physics C, 42, 054102 (2018)

Selected for a Viewpoint in *Physics*

PRL **103,** 251601 (2009) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2009

Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

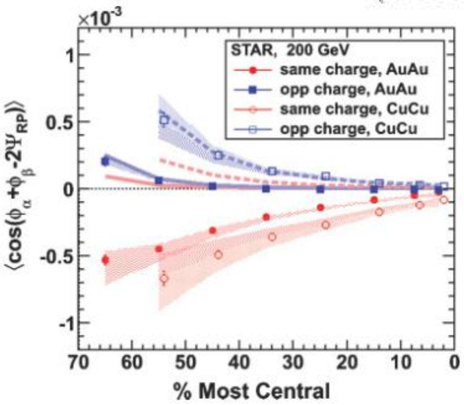
(STAR Collaboration)

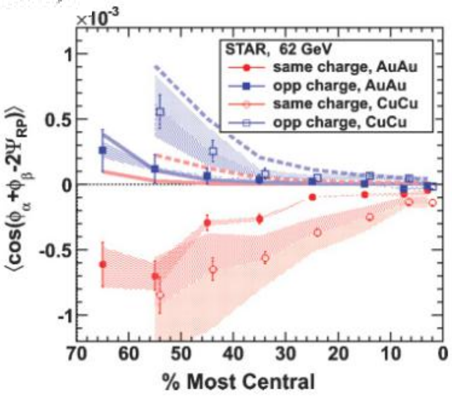


PHYSICAL REVIEW C **81**, 054908 (2010)

Observation of charge-dependent azimuthal correlations and possible local strong parity violation in heavy-ion collisions

(STAR Collaboration)



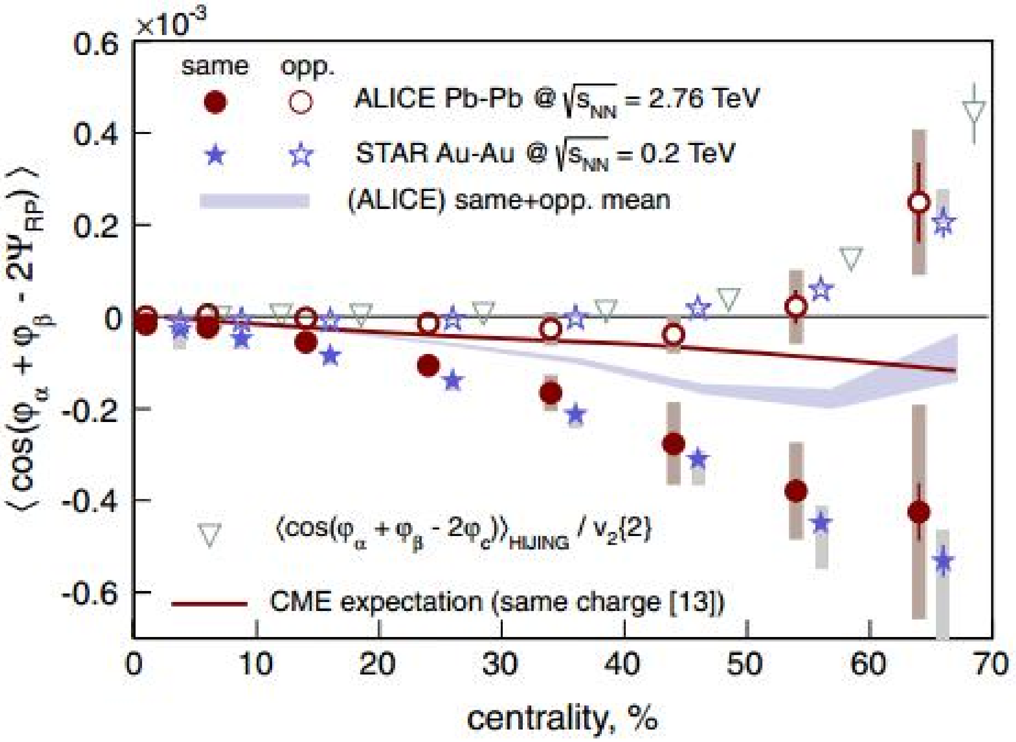


PRL **110,** 012301 (2013) PHYSICAL REVIEW LETTERS week ending 4 JANUARY 2013

Charge separation relative to the reaction plane in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

B. Abelev *et al.*[*]
(ALICE Collaboration)

(Received 5 July 2012; published 2 January 2013)



Calculate the charge difference between both sides of reaction plane

$$< \Delta_+\Delta_- >= -2 \ {}^{t_f}_{ti} \ dt \quad d^3x \ \frac{dN_t}{d^3xdt}[\xi_+(\mathrm{x}_\perp)\xi_-(\mathrm{x}_\perp)](\sum_f q_f^2 eB\rho)^2$$

$$\langle \Delta_\pm^2 \rangle = 2\int_{t_i}^{t_f} \mathrm{d}t \int_V \mathrm{d}^3x \ \frac{\mathrm{d}N_t}{\mathrm{d}^3x\mathrm{d}t} \ [\xi_+^2(x_\perp)+\xi_-^2(x_\perp)] \ (\sum_f q_f^2 eB\rho)^2$$

Time & Volume integral
Overlap region

Rate of
Transitions

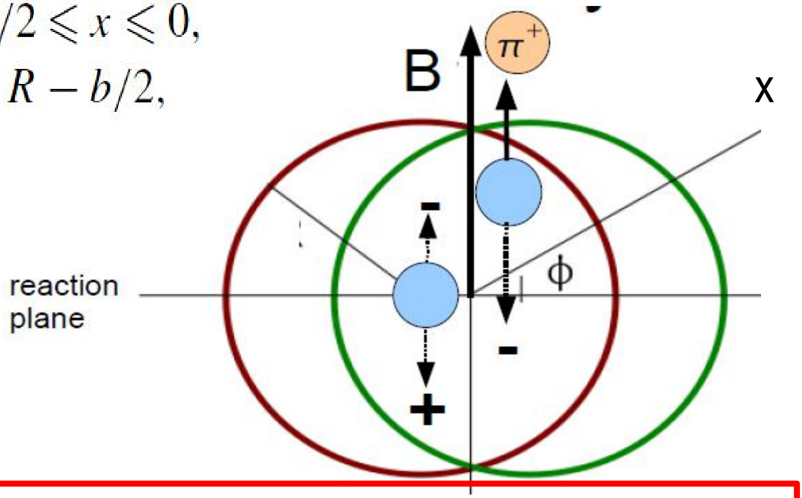
Screening
Functions

Square of Change
Charge difference

$$\xi_\pm(x_\perp) = \exp(-|y_\pm(x) - y|/\lambda)$$

$$y_+(x) = -y_-(x) = \begin{cases} \sqrt{R^2-(x-b/2)^2} & -R+b/2 \leqslant x \leqslant 0, \\ \sqrt{R^2-(x+b/2)^2} & 0 \leqslant x \leqslant R-b/2, \end{cases}$$

The Chiral Magnetic Effect is
a near the surface effect



PRL **113**, 052302 (2014) PHYSICAL REVIEW LETTERS week ending 1 AUGUST 2014

Beam-Energy Dependence of Charge Separation along the Magnetic Field in Au + Au Collisions at RHIC

three particle correlator *γ*, two particle correlator *δ* ,

$$\gamma \equiv \langle \cos(\phi_1 + \phi_2 - 2\Psi_{\mathrm{RP}}) \rangle = \kappa v_2 F - H,$$

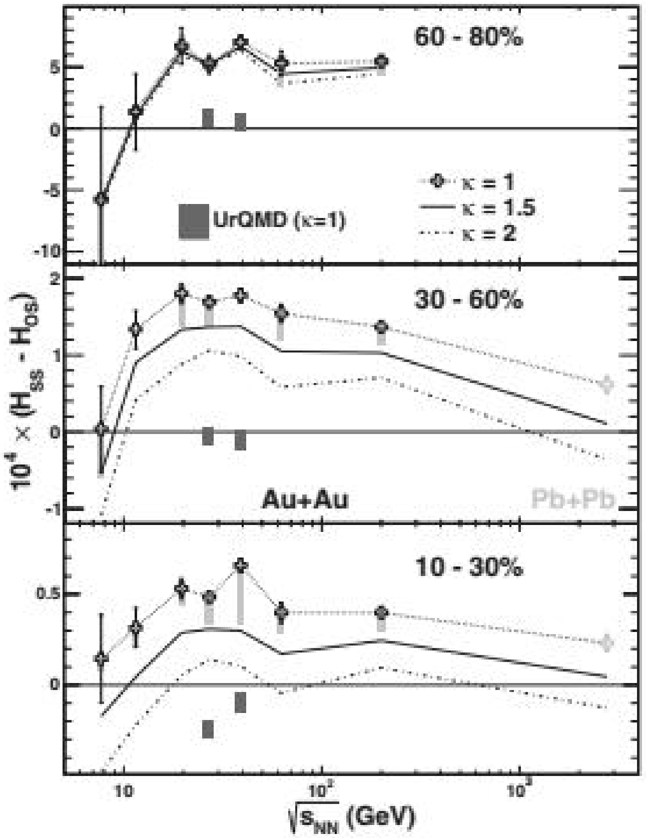
$$\delta \equiv \langle \cos(\phi_1 - \phi_2) \rangle = F + H,$$

where, H and F represent signal from CME and background，F mainly comes from elliptic flow.

$$H_{\alpha,\beta} = (\kappa v_2 \delta_{\alpha,\beta} - \gamma_{\alpha,\beta}) / (1 + \kappa v_2)$$

Star's (14) used: H_{SS}, H_{OS}

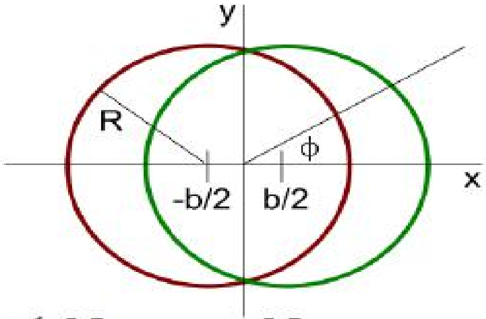
$H_{SS} - H_{OS}$ can subtract non-flow background



A. Bzdak, V. Koch, and J. Liao, Phys. Rev. C 83, 014905 (2011).
A. Bzdak, V. Koch, and J. Liao, Lect. Notes Phys. 871, 503 (2013)

$$a_{++} = a_{--} = \frac{1}{N_{+}^{2}} \frac{\pi^2}{16} \langle \Delta_{\pm}^{2} \rangle,$$

$$a_{+-} = a_{-+} = \frac{1}{N_{+}N_{-}} \frac{\pi^2}{16} \langle \Delta_{+} \Delta_{-} \rangle$$



ϕ : angle between particle and reaction plane

$$\frac{dN_{\pm}}{d\phi} = \frac{N_{\pm}}{2\pi} + a_{\pm} \sin\phi + v_{2} \cos 2\phi + \ldots.$$

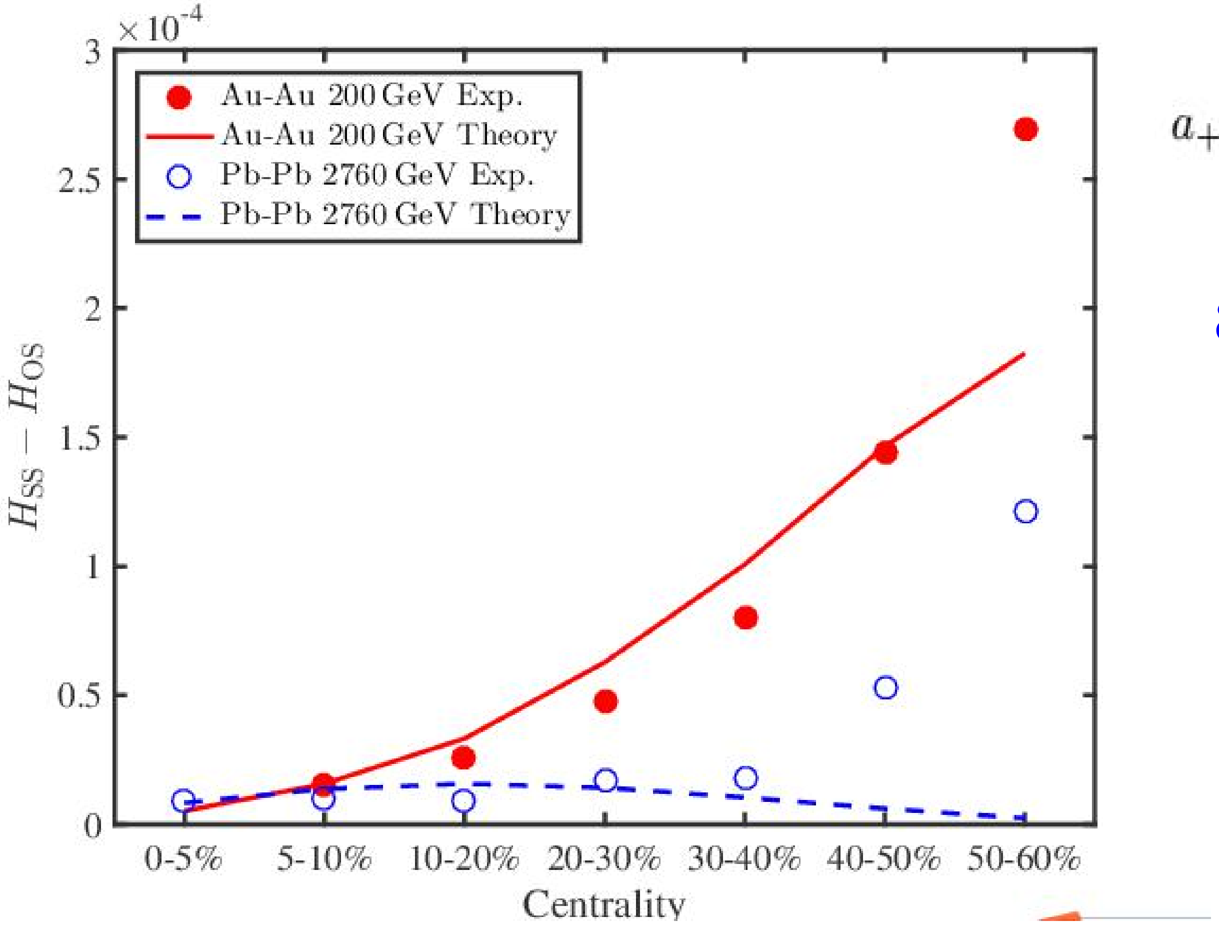
Voloshin 04

Average over many equivalent events (to cancel statistical fluctuations) can give us

$\langle a_{+}^{2} \rangle \sim \langle \Delta_{+}^{2} \rangle$ Pref. emission positive on one side

$\langle a_{-}^{2} \rangle \sim \langle \Delta_{-}^{2} \rangle$ Pref. emission negative on one side

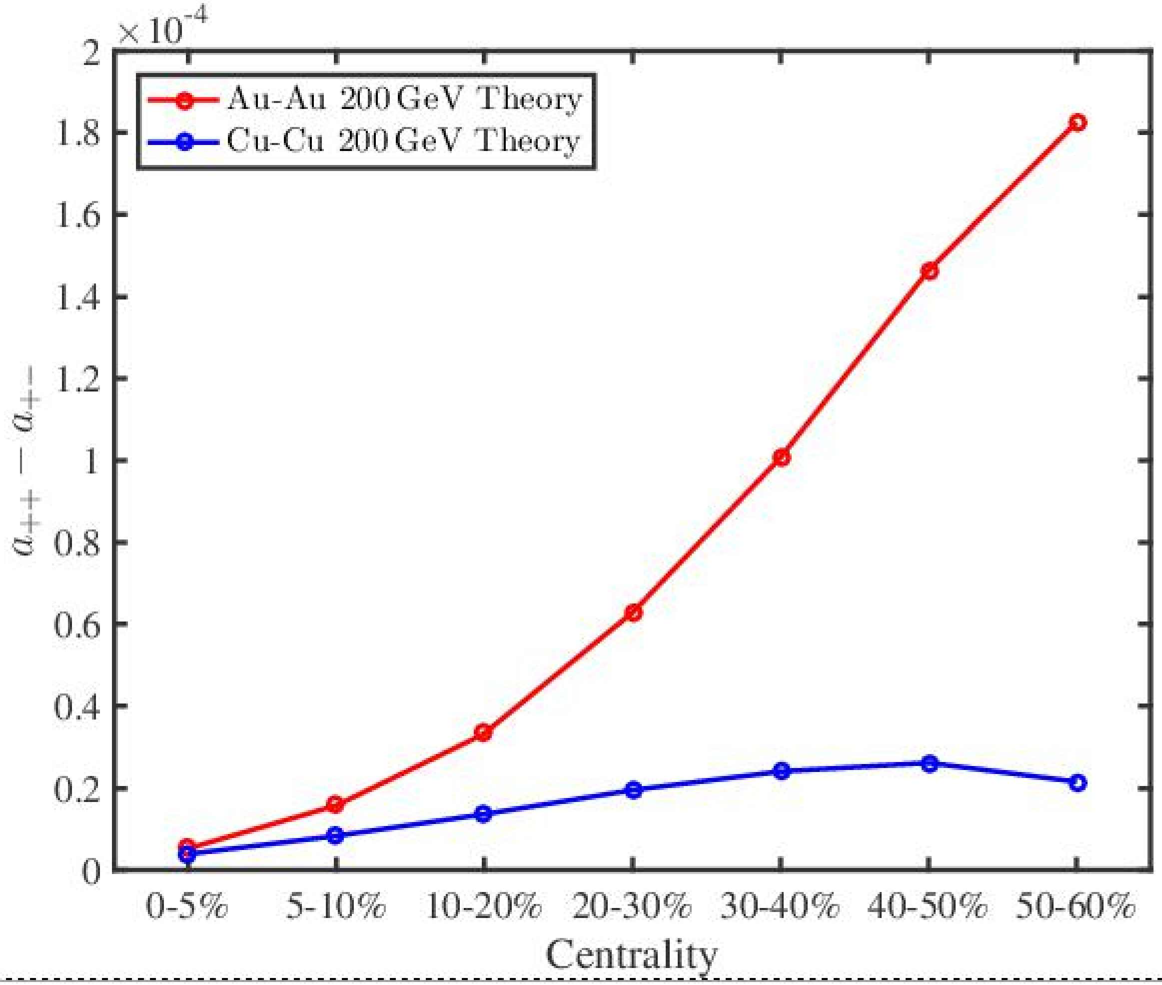
$\langle a_{+} a_{-} \rangle \sim \langle \Delta_{+} \Delta_{-} \rangle$ Correlations between positive on one and negative on other side



$$a_{++} = a_{--} = \frac{1}{N_{+}^{2}} \frac{\pi^2}{16} \langle \Delta_{\pm}^{2} \rangle,$$

$$a_{+-} = a_{-+} = \frac{1}{N_{+}N_{-}} \frac{\pi^2}{16} \langle \Delta_{+}\Delta_{-} \rangle$$

$$\mathbf{a}_{++} - \mathbf{a}_{+-} = H_{ss} - H_{os}$$


×10^-4
Au-Au 200 GeV Theory
Cu-Cu 200 GeV Theory
$a_{++} - a_{+-}$
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0-5%
5-10%
10-20%
20-30%
30-40%
40-50%
50-60%
Centrality

4. Chiral Electromagnetic Current

Duan She, Sheng-Qin Feng, Yang Zhong and Zhong-Bao Yin, Eur. Phys. J. A54, (2018) 48

Relativistic heavy ion collisions produce a magnetic field that varies with time. It is a very interesting research topic to calculate the chiral electromagnetic cruuent by considering the response of a nonzero chiral system to a time varying magnetic field.

Using Kubo formation， current vector equals retarded corrector factor：

$$\left\langle j^{\mu}(\mathrm{x})\right\rangle = \int d^{4}x\ \Pi_{R}^{\mu\nu}\left(x, x\ \right)A_{\nu}(\mathrm{x}\ )$$

Where current vector $j^{\mu}(x)=e\bar{\psi}(x)\gamma^{\mu}\psi(x)$ and retarded corrector facror as follows：

$$\Pi_{R}^{\mu\nu}\left(x, x\ \right) = i\left\langle[j^{\mu}(\mathrm{x}), j^{\nu}(\mathrm{x}\ )]\right\rangle\theta(\mathrm{t}-\mathrm{t}\ )$$

Chiral Magnetic Conductivity (CMC)

Select vector field: $A_x = A_z = 0\ ,\ A_y \neq 0$

Then: $B_z(x) = \partial_x A_y(x) \quad \tilde{B}_z(p) = ip^1 \tilde{A}^2(p)$

The induced vector current in the magnetic field direction:

$$\langle j_z(x) \rangle = \sigma_\chi(p) \tilde{B}_z(p) e^{-ipx}$$

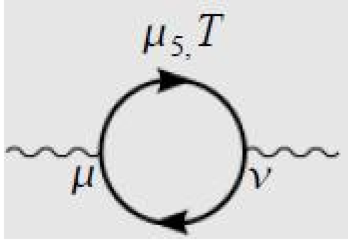
$$\sigma_\chi(p) = \frac{1}{ip^1} \tilde{\Pi}_R^{23}(p) = \frac{1}{2ip^i} \tilde{\Pi}_R^{jk}(p) \varepsilon^{ijk}$$

Chiral magnetic conductivity(CMC):

$$\sigma_\chi(p) = \frac{1}{ip^i} G_R^i(p) \qquad G_R^i(p) = \frac{1}{2} \varepsilon^{ijk} \tilde{\Pi}_R^{jk}(p)$$

Retarded Correlator

Retarded correlator can be given by Euclidean correlator:



$$G_R^i(p_0, p) = G_E^i(\omega_n, p)\Big|_{i\omega_n \to p_0 + i\varepsilon}$$

At very high temperatures， Euclidean correlator:

$$G_E^i(p) = \frac{e^2}{2\beta}\sum_{\tilde{\omega}_m}\int\frac{d^3q}{(2\pi)^3}\varepsilon^{ijk} tr \ \ \gamma^k S(Q)\gamma^j S(P+Q)$$

$$G_R^i(p) = \frac{ie^2}{16\pi^2}\frac{p^i}{p}\frac{p^2 - p_0^2}{p^2}\int_0^\infty dq f(q)\sum_{t=\pm}(2q + tp_0)\times\log\left[\frac{(p_0 + i\varepsilon + tq)^2 - (q+p)^2}{(p_0 + i\varepsilon + tq)^2 - (q-p)^2}\right]$$

$$f(q) \equiv \sum_{s=\pm} s[\tilde{n}(q - \mu_s) - \tilde{n}(q + \mu_s)]$$

Sheng-Qin Feng, CTGU, iHIC 2018, Tsinghua University, Beijing, April 7-11, 2018

Duan She, Sheng-Qin Feng, Yang Zhong and Zhong-Bao Yin, Eur. Phys. J. A54, (2018) 48

Real and Imaginary Part of CMC

In the limit of magnetic field of homogeneous magnetic field:

$$\sigma_\chi(\omega)=\frac{e^2}{3\pi}\omega\delta(\omega)\mu_5+\frac{e^2\omega|\omega|}{96\pi}\sum_{s,t=\pm} st[\frac{d}{dx}n(x+t\mu_s)]\Big|_{x=|\omega|/2}$$

At zero temperature:

$$\sigma_\chi(\omega)=\frac{e^2}{3\pi}\omega\delta(\omega)\mu_5-\frac{e^2\omega^2}{96\pi}\sum_{s,t=\pm} st\delta(\omega/2+t\mu_s)$$

For high temperature:

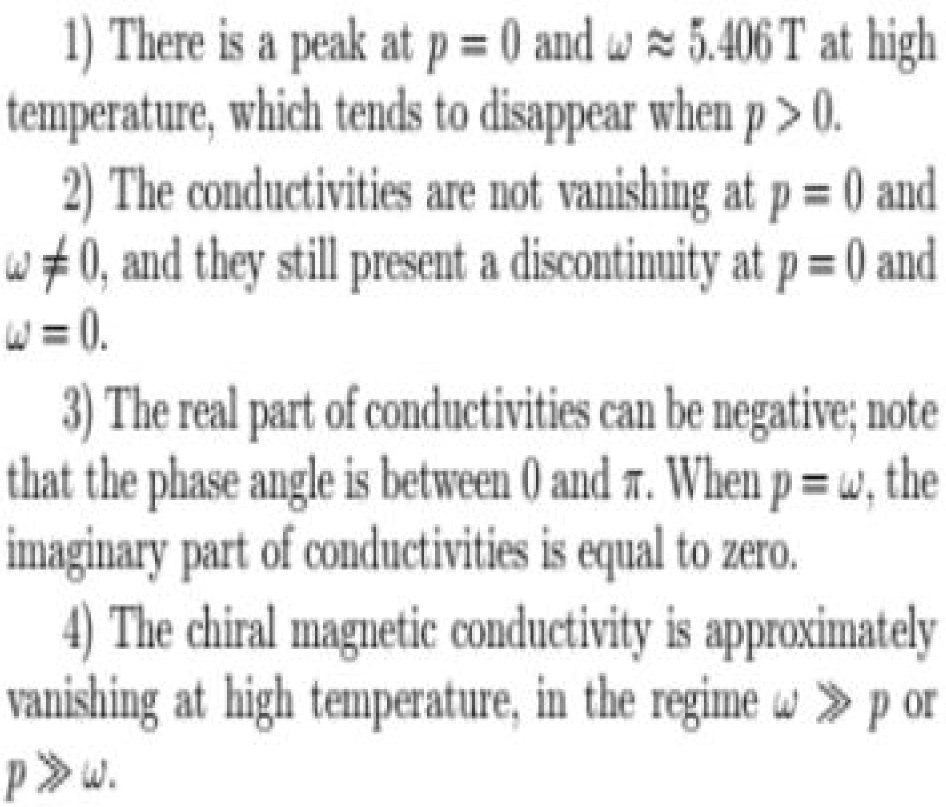
$$\sigma_\chi(\omega)=\frac{e^2}{3\pi}\omega\delta(\omega)\mu_5+\frac{e^2\omega|\omega|}{24\pi T^2}\tilde{n}(|\omega|/2)^3\times[e^{|\omega|/T}-e^{|\omega|/(2T)}]\mu_5$$

By Kramers-Kroning realtion:

$$\sigma'_\chi(\omega)=\frac{1}{\pi}\int_{-\infty}^{\infty}dq_0\frac{\sigma_\chi(q_0)}{q_0-\omega}$$

$$\sigma_\chi(\omega)=-\frac{1}{\pi}\int_{-\infty}^{\infty}dq_0\frac{\sigma'_\chi(q_0)}{q_0-\omega}$$

CMC at high temperture

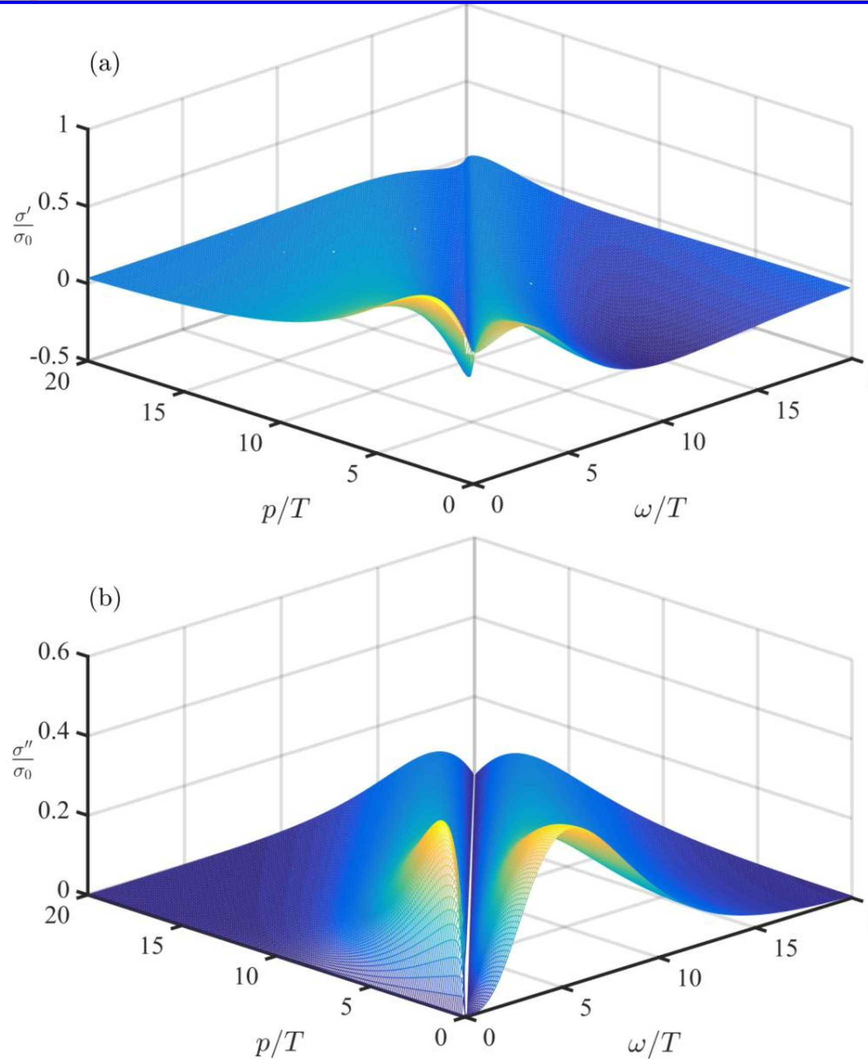


1) There is a peak at $p = 0$ and $\omega \approx 5.406\,T$ at high temperature, which tends to disappear when $p > 0$.

2) The conductivities are not vanishing at $p = 0$ and $\omega \neq 0$, and they still present a discontinuity at $p = 0$ and $\omega = 0$.

3) The real part of conductivities can be negative; note that the phase angle is between 0 and π. When $p = \omega$, the imaginary part of conductivities is equal to zero.

4) The chiral magnetic conductivity is approximately vanishing at high temperature, in the regime $\omega \gg p$ or $p \gg \omega$.



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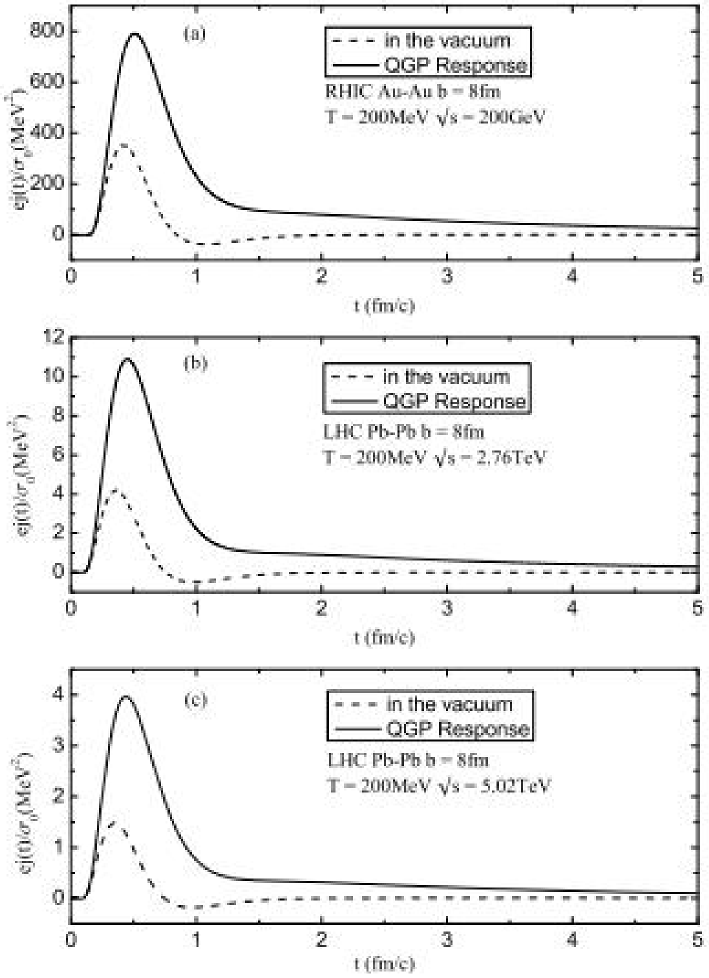
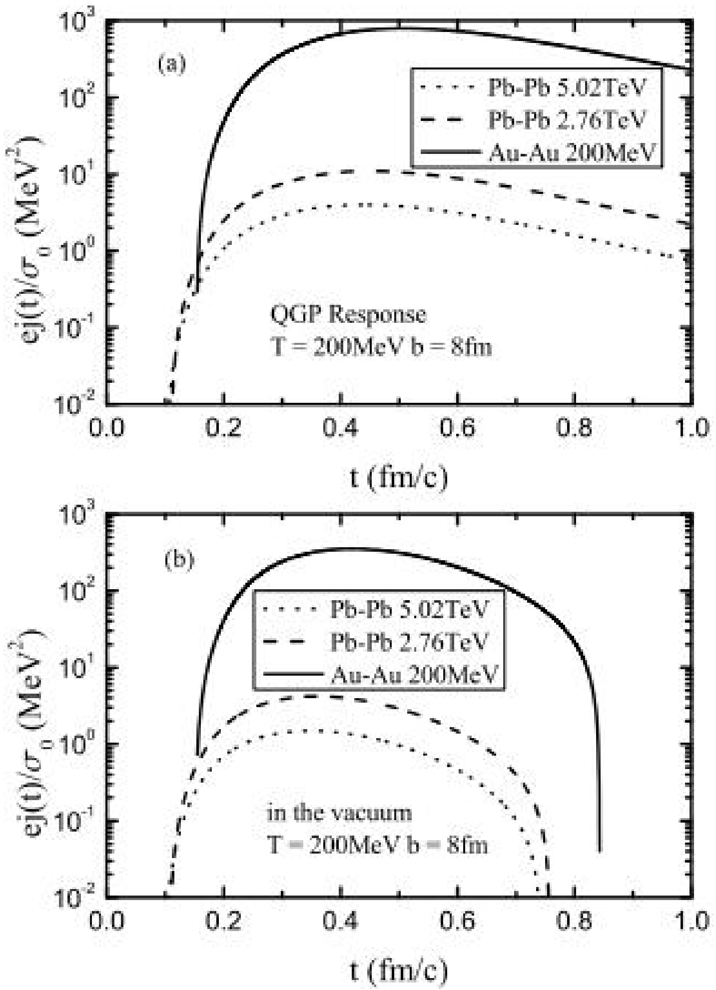
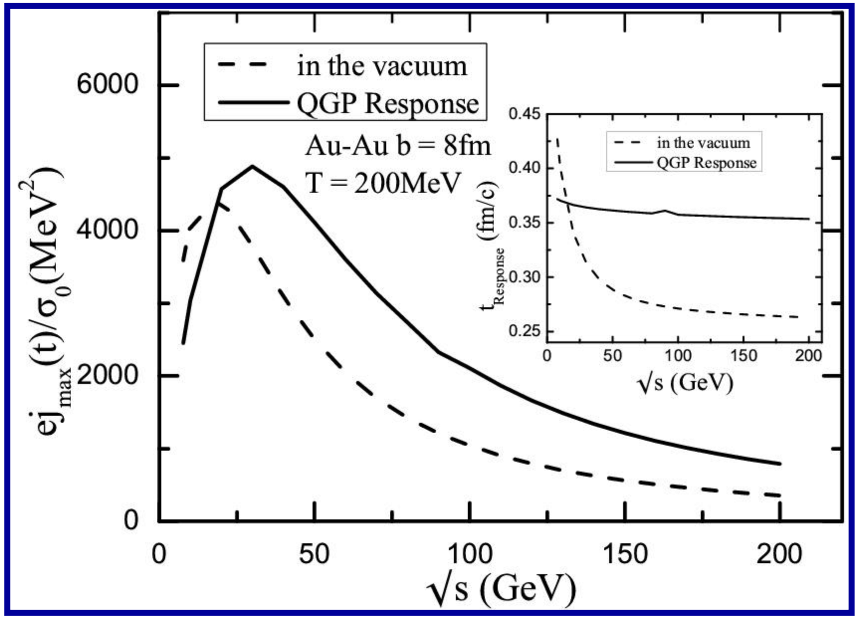
Chiral Electromagnetic Current in homogeneous magnetic field

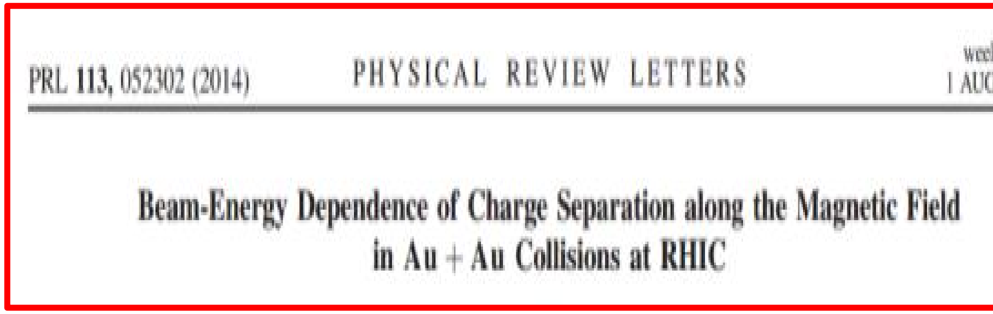
$$j(t) = \int_0^\infty \frac{d\omega}{\pi}\left[\sigma\ (\omega)\cos(\omega t) + \sigma\ \ (\omega)\sin(\omega t)\right]\tilde{B}(\omega)$$

$$\tilde{B}(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} B(t)$$





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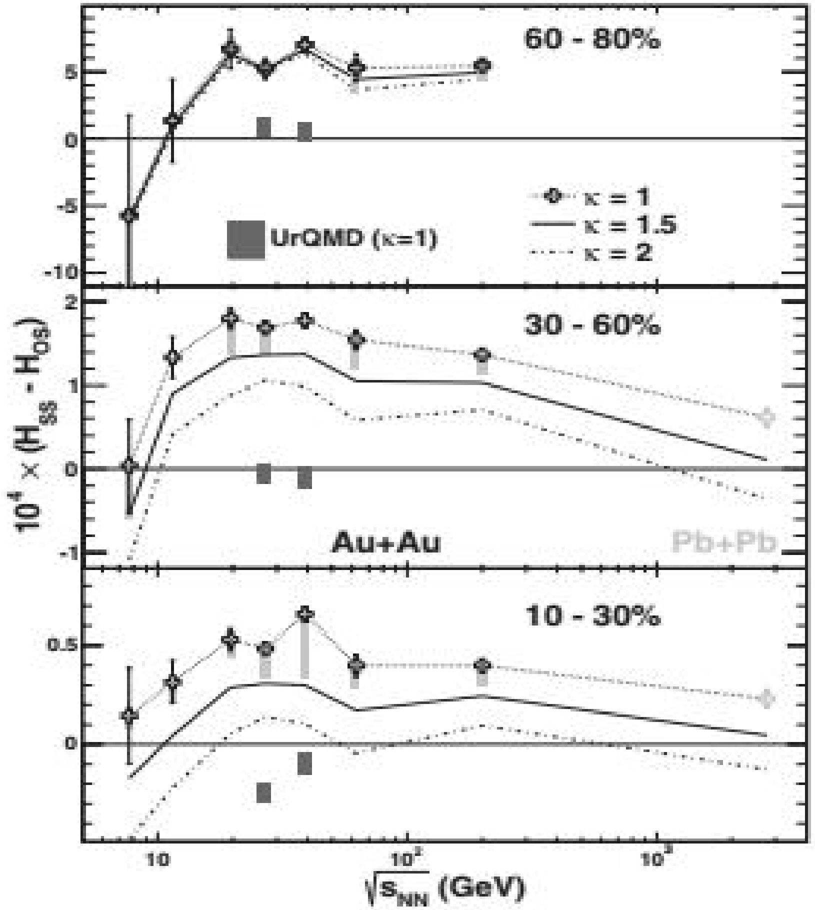
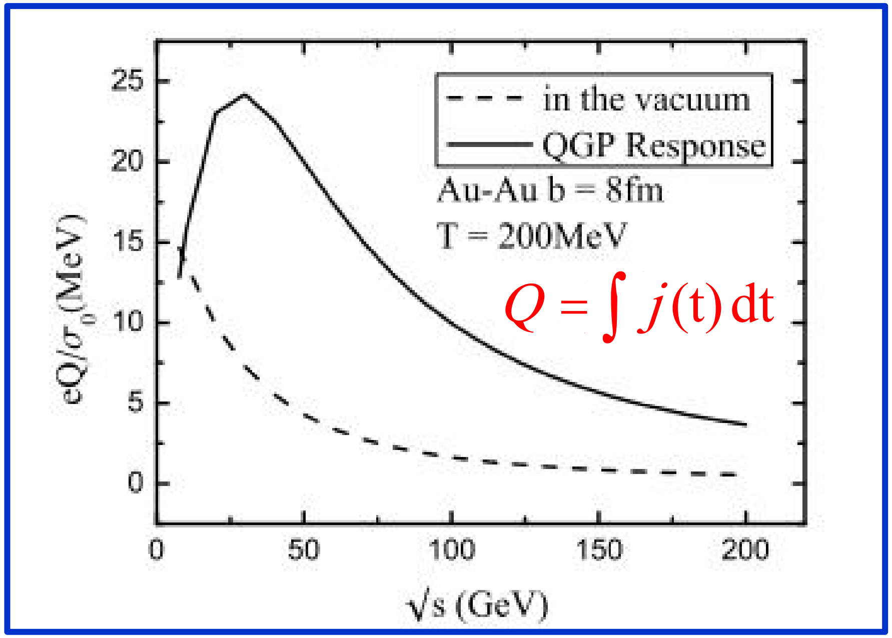
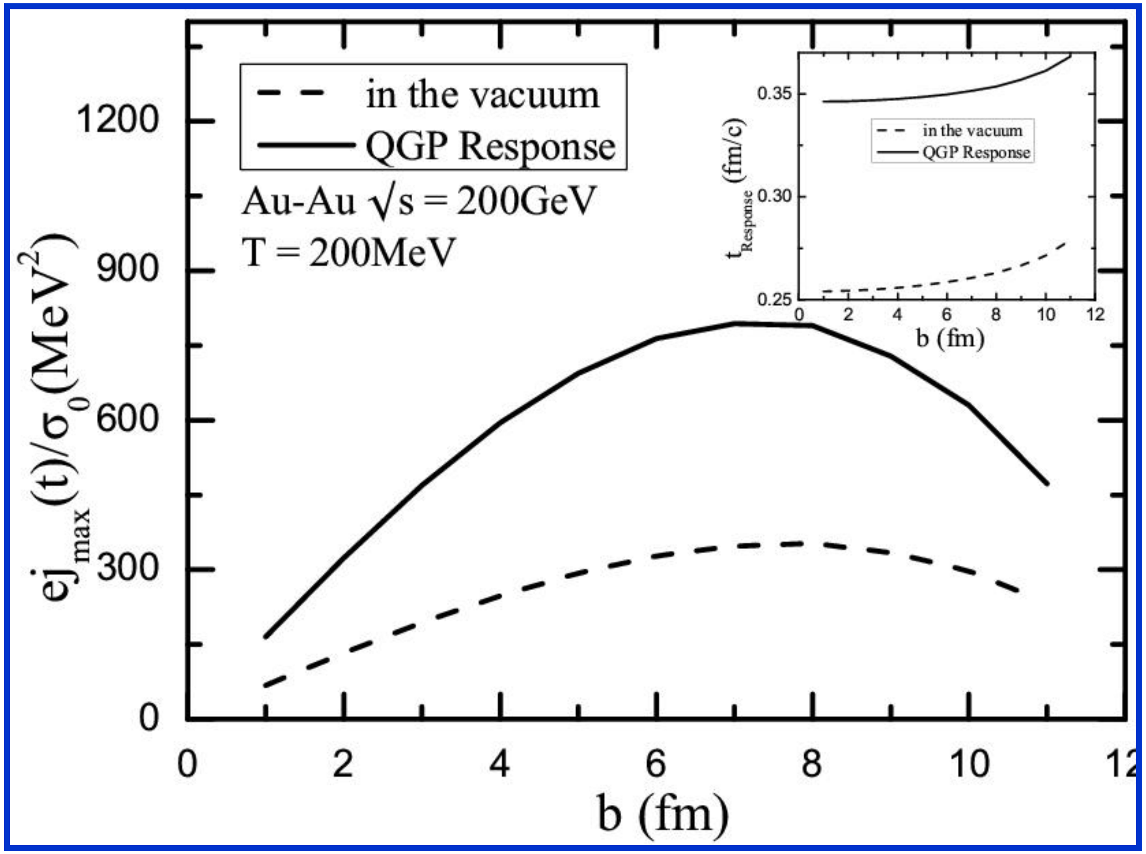
Beam-Energy Dependence of Charge Separation along the Magnetic Field in Au + Au Collisions at RHIC



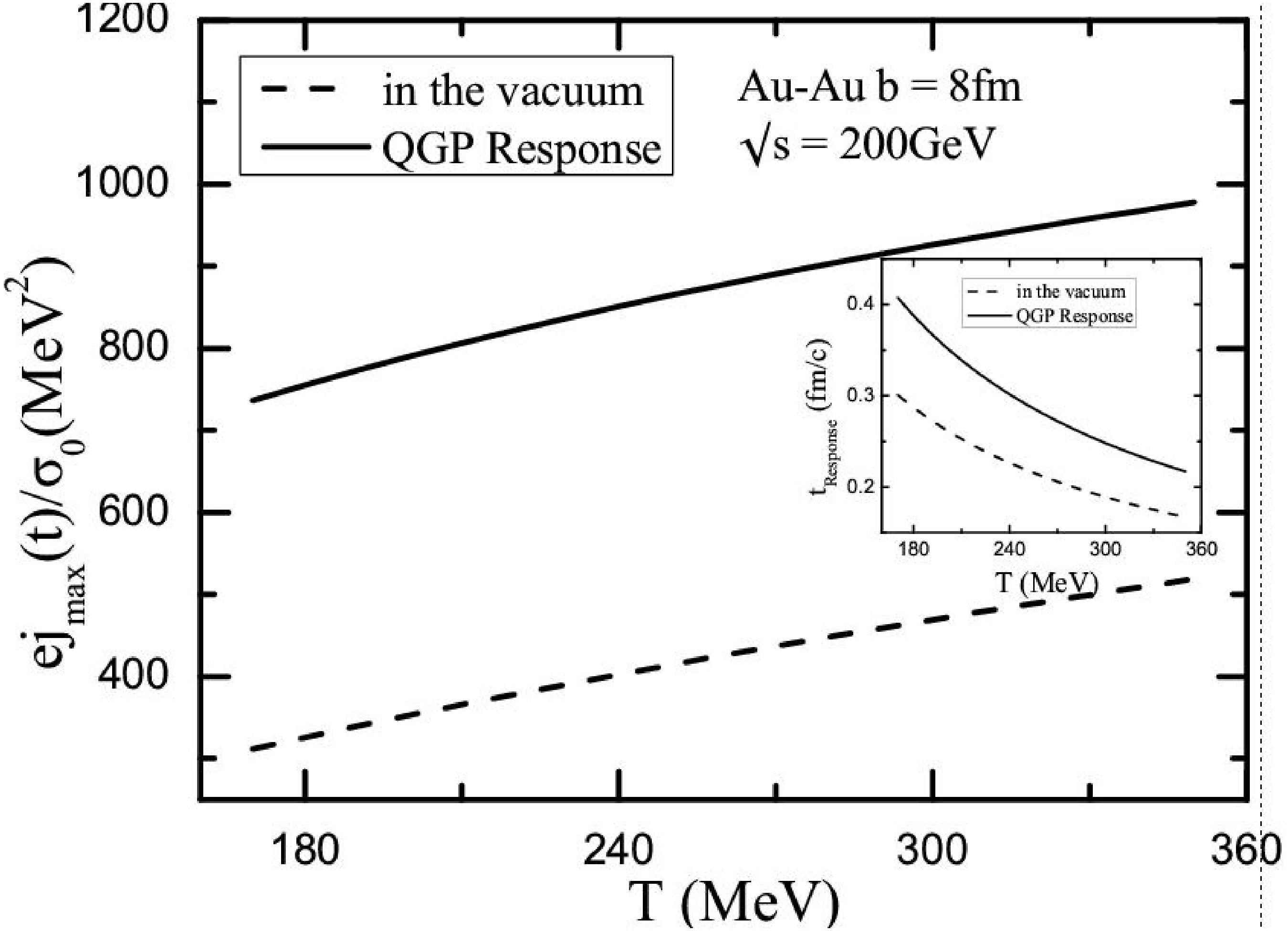


$$Q = \int j(t)\,\mathrm{d}t$$

Duan She, Sheng-Qin Feng, Yang Zhong and Zhong-Bao Yin, Eur. Phys. J. A54, (2018) 48



in the vacuum
QGP Response
Au-Au √s = 200GeV
T = 200MeV
$ej_{max}(t)/\sigma_0(MeV^2)$
1200
900
600
300
0
0
2
4
6
8
10
12
b (fm)
$t_{Response}$ (fm/c)
0.35
0.30
0.25
0
2
4
6
8
10
12
b (fm)
in the vacuum
QGP Response


in the vacuum
QGP Response
Au-Au b = 8fm
$\sqrt{s} = 200$GeV
$ej_{max}(t)/\sigma_0(MeV^2)$
1200
1000
800
600
400
180
240
300
360
T (MeV)
$t_{Response}$ (fm/c)
0.4
0.3
0.2
180
240
300
360
T (MeV)

Summary and Conclusions

1. The magnetic field are systematically studied, it is found that magnetic field play a key effect to chiral magnetic effect.
2. The magnetic field with the response of QGP medium will maintain a longer time than in a vacuum;
3. We study the charge separation signal of CME for RHIC and LHC energy region with different centrality. It is found that CME at RHIC is more obvious than at LHC.
4. We study the chiral electromagnetic current for RHIC and LHC energy region. It is found the maximum chiral magnetic current is around collision energy 30GeV.

Thank your attentions