Disentangling Quantum Transport Equation for Chiral fermion

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JHG, Z.T. Liang, Q. Wang, X.N. Wang, arXiv:1802.06216

Outline

- **Introduction**
- **Entangle quantum transport equation**
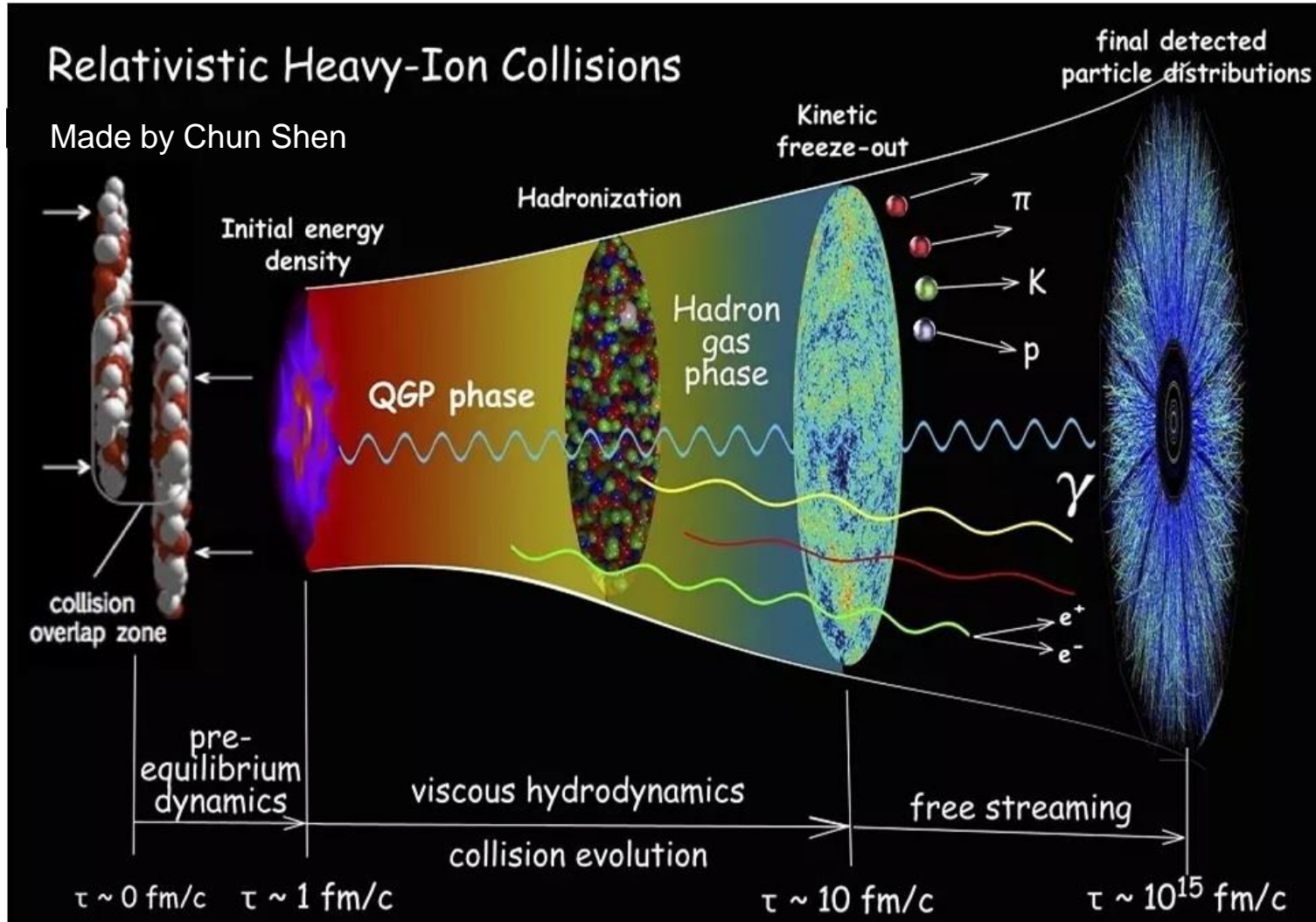- **Chiral kinetic equation and Chiral anomaly**
- **Summary**

Heavy-Ion Collisons and QCD

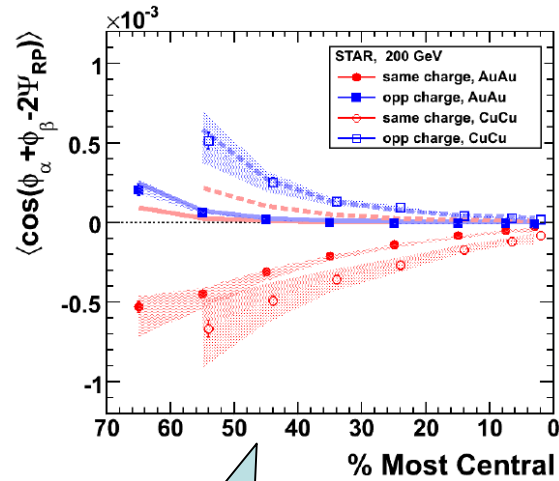K. Fukushima and T. Hatsuda Rept.Prog.Phys. 74 (2011) 014001
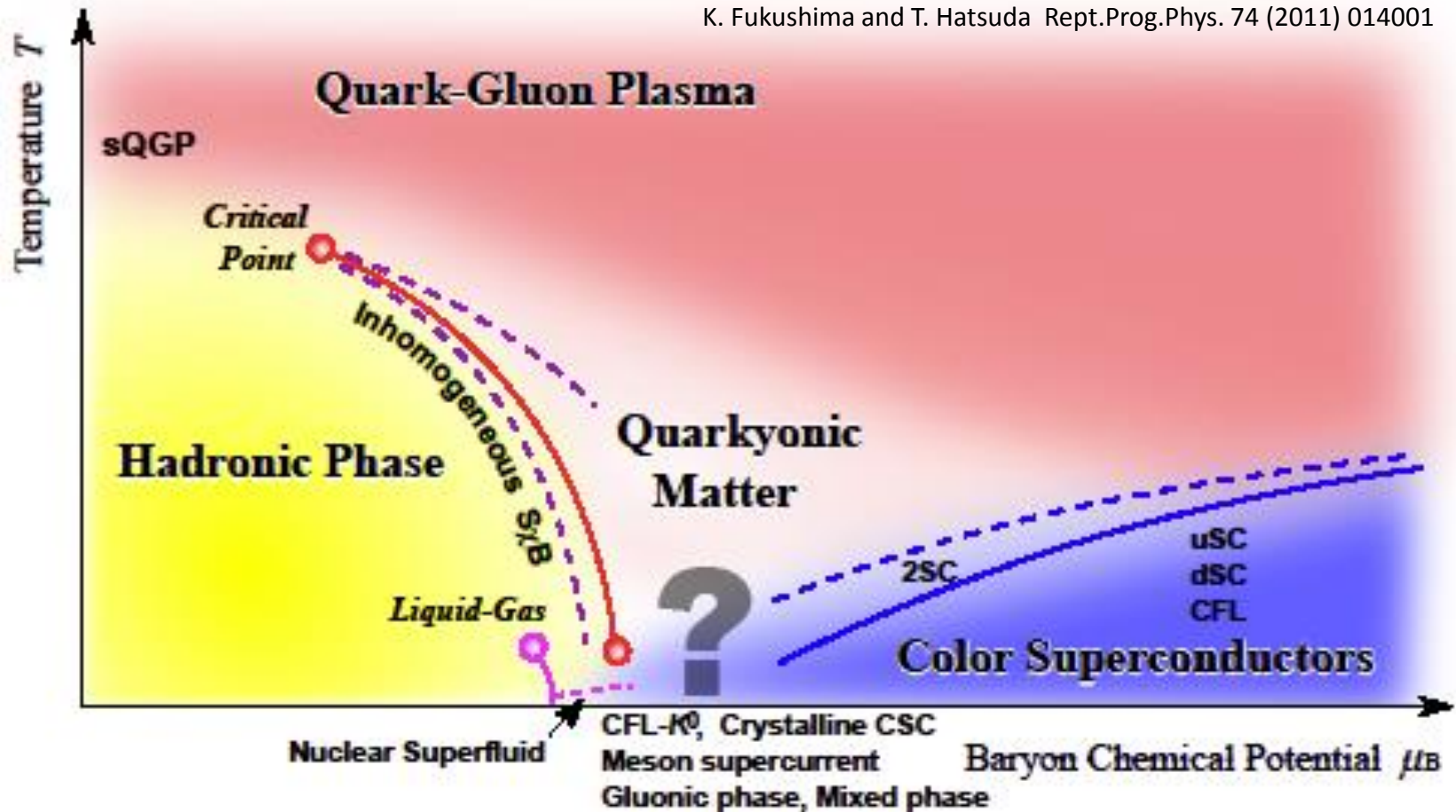
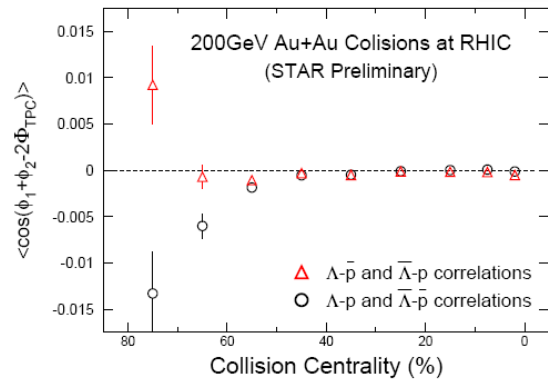**RHIC@BNL, ALICE@CERN,   FAIR@GSI, NICA@DUBNA, HIAF@IMPCAS**

Non-Equilibrium Effect in HIC

Quantum Effect in HIC

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Quantum Transport Theory

Non-equilibrium Effect **Quantum Transport Theory** Quantum Effect
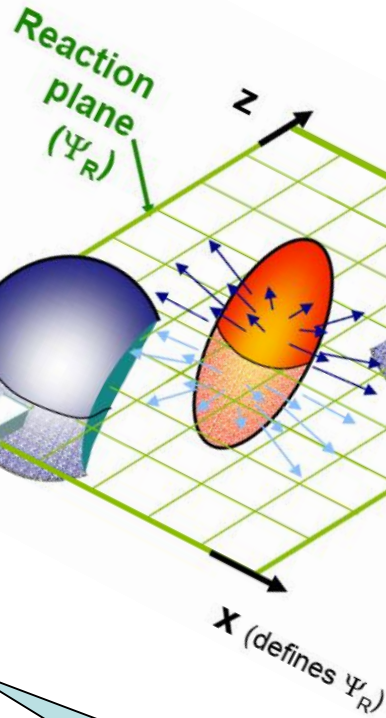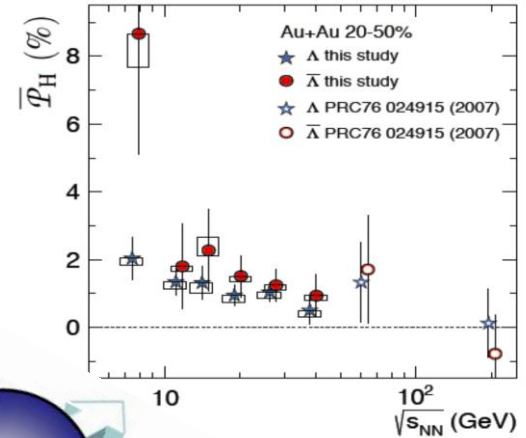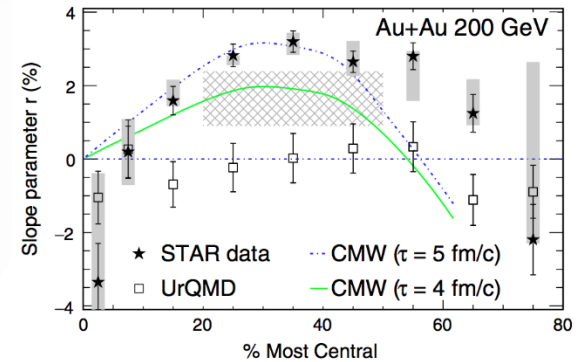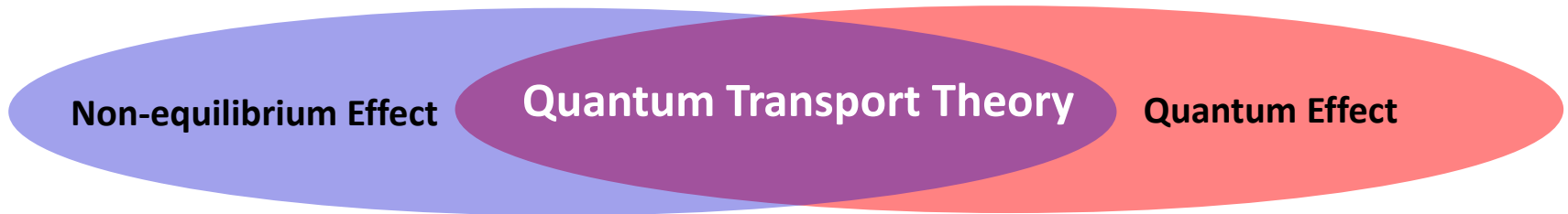
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Recent Study on Wigner Equation

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- ……

Classical Transport Theory

Distribution function in phase-space:

$$f(t, \vec{x}, \vec{p})$$

The classical Boltzmann equation with the background EM fields

$$p^\mu \left(\partial_\mu - F_{\mu\nu} \partial_p^\nu \right) f(t, \vec{x}, \vec{p}) \;=\; \mathcal{C}[f]$$

→ collision term

Conserved current:

$$j^\mu(x) \;\equiv\; \int \frac{d^3p}{2E_p} p^\mu f(t, \vec{x}, \vec{p})$$

Energy-momentum tensor:

$$T^{\mu\nu}(x) \equiv \int \frac{d^3p}{2E_p} p^\mu p^\nu f(t, \vec{x}, \vec{p})$$

Classical Transport Theory:

One distribution function, one equation! **Simple and Intuitive!**

Quantum Transport Theory: Wigner Functions

Quantum transport theory for spin-1/2 fermion with Abelian gauge field

Wigner operator:

$$\hat{W}_{\alpha\beta}(x,p) = \int \frac{d^4y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}_\beta(x_2) U(x_2,x_1) \psi_\alpha(x_1)$$

$$x \equiv \frac{x_2 + x_1}{2}$$
$$y \equiv x_2 - x_1$$

Gauge link: $U(x_2,x_1) \equiv \mathcal{P}\mathrm{Exp}\left(-ie \int_{x_1}^{x_2} dx^\mu A_\mu(x)\right)$ Straight line: $x_2 \bullet\!\!-\!\!-\!\!-\!\!\bullet x_1$

Particle density at x with p:

$$\hat{W}(x,p) = \bar{\psi}(x)\delta^4(p - \hat{\pi}(x))\psi(x)$$

Kinetic Momentum: $\quad \hat{\pi}_\mu = \hat{p}_\mu - eA_\mu(x)$

$$\begin{pmatrix} W_{11} & W_{12} & W_{13} & W_{14} \\ W_{21} & W_{22} & W_{23} & W_{24} \\ W_{31} & W_{32} & W_{33} & W_{34} \\ W_{41} & W_{42} & W_{43} & W_{44} \end{pmatrix}$$

Wigner functions: $\quad W_{\alpha\beta}(x,p) = <:\hat{W}(x,p)_{\alpha\beta}:>$

16 independent components due to: $\quad \hat{W}^\dagger(x,p) = \gamma^0 \hat{W}(x,p)\gamma^0$

D.Vasak, M.Gyulassy, H. Elze Annals Phys. 173 (1987)

Covariant Wigner Equations

Wigner Equations: D.Vasak, M.Gyulassy, H. Elze Annals Phys. 173 (1987)

$$\left[m - \gamma \cdot \left(p_\mu + \frac{1}{2} i \partial_\mu \right) \right] \hat{W}(x,p)$$

$$= ie \frac{\partial}{\partial p_\mu} \int \frac{d^4 y}{(2\pi)^4} e^{-ip\cdot y} \bar{\psi}(x_1) \otimes \mathcal{P} \left[\int_0^1 ds \, (1-s) \, F_{\mu\nu} \left(x - \frac{1}{2}y + sy \right) U(A; x_1, x_2) \right] \gamma^\nu \psi(x_2)$$

Background EM Fields

$$(\gamma \cdot K - m) \, W(x,p) = 0$$

$$K^\mu \equiv \Pi^\mu + \frac{1}{2} i G^\mu \qquad \Pi^\mu \equiv p^\mu - \frac{1}{2} j_1 \left(\frac{1}{2}\Delta \right) F^{\mu\nu} \partial_\nu^p \qquad G^\mu \equiv \partial_x^\mu - j_0 \left(\frac{1}{2}\Delta \right) F^{\mu\nu} \partial_\nu^p$$

j_0, j_1 :Spherical Bessel Functions $\Delta \equiv \partial^p \cdot \partial_x$ ∂_x only act on EM fields

16 independent functions and 32 coupled equations:
More complicated, abstract and highly constrained!

Decoupled Equations under Chiral Limit

Wigner functions spinor decomposition:

$$W = \frac{1}{4}\left[\mathscr{F} + i\gamma^5\mathscr{P} + \gamma^\mu\mathscr{V}_\mu + \gamma^5\gamma^\mu\mathscr{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathscr{S}_{\mu\nu}\right]$$

$$\Pi^\mu\mathscr{V}_\mu = m\mathscr{F}, \quad G^\mu\mathscr{A}_\mu = -2m\mathscr{P},$$

$$\Pi_\mu\mathscr{F} + \frac{1}{2}G^\nu\mathscr{S}_{\mu\nu} = m\mathscr{V}_\mu,$$

$$G_\mu\mathscr{P} - \epsilon_{\mu\nu\rho\sigma}\Pi^\nu\mathscr{S}^{\rho\sigma} = 2m\mathscr{A}_\mu,$$

$$\frac{1}{2}\left(G_\mu\mathscr{V}_\nu - G_\nu\mathscr{V}_\mu\right) - \epsilon_{\mu\nu\rho\sigma}\Pi^\rho\mathscr{A}^\sigma = m\mathscr{S}_{\mu\nu}.$$

$$G^\mu\mathscr{V}_\mu = 0, \quad \Pi^\mu\mathscr{A}_\mu = 0,$$

$$\frac{1}{2}G_\mu\mathscr{F} - \Pi^\nu\mathscr{S}_{\mu\nu} = 0,$$

$$\Pi_\mu\mathscr{P} + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}G^\nu\mathscr{S}^{\rho\sigma} = 0,$$

$$\left(\Pi_\mu\mathscr{V}_\nu - \Pi_\nu\mathscr{V}_\mu\right) + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^\rho\mathscr{A}^\sigma = 0.$$

Chiral limit   *m*=0

$$\Pi^\mu\mathscr{V}_\mu = 0, \quad \Pi^\mu\mathscr{A}_\mu = 0,$$

$$G^\mu\mathscr{V}_\mu = 0, \quad G^\mu\mathscr{A}_\mu = 0,$$

$$\epsilon_{\mu\nu\rho\sigma}G^\rho\mathscr{A}^\sigma = -2\left(\Pi_\mu\mathscr{V}_\nu - \Pi_\nu\mathscr{V}_\mu\right),$$

$$\epsilon_{\mu\nu\rho\sigma}G^\rho\mathscr{V}^\sigma = -2\left(\Pi_\mu\mathscr{A}_\nu - \Pi_\nu\mathscr{A}_\mu\right).$$

$$\Pi_\mu\mathscr{F} + \frac{1}{2}G^\nu\mathscr{S}_{\mu\nu} = 0,$$

$$\frac{1}{2}G_\mu\mathscr{F} - \Pi^\nu\mathscr{S}_{\mu\nu} = 0,$$

$$-G_\mu\mathscr{P} + \epsilon_{\mu\nu\rho\sigma}\Pi^\nu\mathscr{S}^{\rho\sigma} = 0,$$

$$\Pi_\mu\mathscr{P} + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}G^\nu\mathscr{S}^{\rho\sigma} = 0.$$

8 functions & 16 equations   8 functions & 16 equations
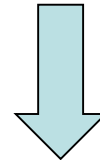
Decoupled Equations under Chiral Limit

Our interest:

$$j^\mu = \int d^4p \, \mathscr{V}^\mu, \quad j_5^\mu = \int d^4p \, \mathscr{A}^\mu, \quad T^{\mu\nu} = \int d^4p \, p^\mu \mathscr{V}^\nu$$

We focus on:

$$\Pi^\mu \mathscr{V}_\mu = 0, \quad \Pi^\mu \mathscr{A}_\mu = 0,$$
$$G^\mu \mathscr{V}_\mu = 0, \quad G^\mu \mathscr{A}_\mu = 0,$$
$$\epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{A}^\sigma = -2 \left(\Pi_\mu \mathscr{V}_\nu - \Pi_\nu \mathscr{V}_\mu \right),$$
$$\epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{V}^\sigma = -2 \left(\Pi_\mu \mathscr{A}_\nu - \Pi_\nu \mathscr{A}_\mu \right).$$

Right/Left ($s = \pm 1$)
$$\mathscr{J}_s^\mu \equiv \frac{1}{2} \left(\mathscr{V}_\mu + s \mathscr{A}_\mu \right)$$

$$\Pi_\mu \mathscr{J}_s^\mu = 0, \quad G_\mu \mathscr{J}_s^\mu = 0, \quad \epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{J}_s^\sigma = -2s \left(\Pi_\mu \mathscr{J}_{s\nu} - \Pi_\nu \mathscr{J}_{s\mu} \right)$$

4 independent Wigner functions and 8 coupled equations

S.Ochs, U. Heinz Annals Phys. 266 (1998), JHG, Q. Wang Phys.Lett. B749 (2015)

$\hbar$ Expansion in Wigner Equations

4D Covariant Wigner equations in background EM field:

$$\Pi_\mu \mathscr{J}^\mu \;=\; 0, \quad G_\mu \mathscr{J}^\mu = 0,$$
$$\hbar \epsilon_{\mu\nu\rho\sigma} G^\rho \mathscr{J}^\sigma \;=\; -2s\left(\Pi_\mu \mathscr{J}_\nu - \Pi_\nu \mathscr{J}_\mu\right)$$

Wigner functions, G^μ, Π^μ operators expand in $\hbar$:

$$\mathscr{J}^\mu \;=\; \sum_{k=0}^\infty \hbar^k \mathscr{J}^\mu_{(k)} \quad G^\mu = \sum_{k=0}^\infty \hbar^k G^\mu_{(k)} \quad \Pi^\mu = \sum_{k=0}^\infty \hbar^k \Pi^\mu_{(k)}$$

k=0:

$$G^\mu_{(0)} \;\equiv\; \partial_x^\mu - F^{\mu\nu}\partial_\nu^p$$
$$\Pi^\mu_{(0)} \;\equiv\; p^\mu$$

k>0:

$$G^\mu_{(k)} \;\equiv\; \frac{\left[1+(-1)^k\right](-1)^{\frac{k+2}{2}}}{2^k(k+1)!}(\partial_x \cdot \partial_p)^k F^{\mu\nu}\partial_\nu^p$$
$$\Pi^\mu_{(k)} \;\equiv\; \frac{\left[1+(-1)^k\right]k(-1)^{\frac{k}{2}}}{2^k(k+1)!}(\partial_x \cdot \partial_p)^{k-1} F^{\mu\nu}\partial_\nu^p,$$

For odd number *k*, $G^\mu_{(k)}, \Pi^\mu_{(k)}$ vanish.   JHG,Z.T. Liang, Q. Wang, X.N. Wang, arXiv:1802.06216

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3D vector forms for Wigner equations

4D covariant Wigner equation at $O(\hbar^n)$: JHG, Z.T. Liang, Q. Wang, X.N. Wang, arXiv:1802.06216

$$\sum_{k=0}^{n} \Pi_\mu^{(k)} \mathscr{J}_{(n-k)}^\mu = 0, \quad \sum_{k=0}^{n} G_\mu^{(k)} \mathscr{J}_{(n-k)}^\mu = 0,$$

$$\epsilon_{\mu\nu\rho\sigma} \sum_{k=0}^{n} G_{(k)}^\rho \mathscr{J}_{(n-k)}^\sigma = -2s \sum_{k=0}^{n} \left(\Pi_\mu^{(k)} \mathscr{J}_\nu^{(n+1-k)} - \Pi_\nu^{(k)} \mathscr{J}_\mu^{(n+1-k)} \right)$$

3D vector Wigner equation at $O(\hbar^n)$: $\mathscr{J}^\mu = \left(\mathscr{J}^0, \vec{\mathscr{J}} \right)$ $G^\mu = \left(G^0, -\vec{G} \right), \quad \Pi^\mu = \left(\Pi^0, \vec{\Pi} \right)$

Evolution equations:

$$\sum_{k=0}^{n} \left[G_0^{(k)} \mathscr{J}_0^{(n-k)} + \vec{G}^{(k)} \cdot \vec{\mathscr{J}}^{(n-k)} \right] = 0, \qquad G_0^{(0)} = \partial_t + \vec{E} \cdot \vec{\nabla}_p$$

$$\sum_{k=0}^{n} \left[G_0^{(k)} \vec{\mathscr{J}}^{(n-k)} + \vec{G}^{(k)} \mathscr{J}_0^{(n-k)} \right] = 2s \sum_{k=0}^{n+1} \vec{\Pi}_{(k)} \times \vec{\mathscr{J}}^{(n-k+1)}$$

Constraint equations:

$$\sum_{k=0}^{n} \left[\Pi_0^{(k)} \mathscr{J}_0^{(n-k)} - \vec{\Pi}^{(k)} \cdot \vec{\mathscr{J}}^{(n-k)} \right] = 0,$$

$$\sum_{k=0}^{n} \vec{G}^{(k)} \times \vec{\mathscr{J}}^{(n-k)} = -2s \sum_{k=0}^{n+1} \left[\vec{\Pi}_{(k)} \mathscr{J}_0^{(n-k+1)} - \Pi_{(k)}^0 \vec{\mathscr{J}}^{(n-k+1)} \right]$$

Wigner Equations at $O(\hbar^0)$

Wigner equations at $O(\hbar^0)$ in 3D vector forms:

$$G_0^{(0)} = \partial_t + \vec{E} \cdot \vec{\nabla}_p$$

E0   Evolution equation $\mathscr{I}_0^{(0)}$:

$$G_0^{(0)} \mathscr{I}_0^{(0)} + \vec{G}^{(0)} \cdot \vec{\mathscr{J}}^{(0)} = 0$$

Ei   Evolution equation $\vec{\mathscr{J}}^{(0)}$:

$$G_0^{(0)} \vec{\mathscr{J}}^{(0)} + \vec{G}^{(0)} \mathscr{I}_0^{(0)} = -2s\vec{p} \times \vec{\mathscr{J}}^{(1)}$$

C1   Constraint equation:

$$p_0 \mathscr{I}_0^{(0)} - \vec{p} \cdot \vec{\mathscr{J}}^{(0)} = 0,$$

C2   Constraint equation:

$$0 = -2s\left[\vec{p}\mathscr{I}_0^{(0)} - p^0 \vec{\mathscr{J}}^{(0)}\right] \Rightarrow \vec{\mathscr{J}}^{(0)} = \frac{\vec{p}}{p_0} \mathscr{I}_0^{(0)}$$

$$\vec{G}^{(0)} \times \vec{\mathscr{J}}^{(0)} = -2s\left[\vec{p}\mathscr{I}_0^{(1)} - p^0 \vec{\mathscr{J}}^{(1)}\right] \Rightarrow \vec{\mathscr{J}}^{(1)} = \frac{\vec{p}}{p_0} \mathscr{I}_0^{(1)} + \frac{s}{2}\vec{G}^{(0)} \times \vec{\mathscr{J}}^{(0)}$$

Remarkable property: **E0** $+$ **C1** $+$ **C2** $\Longrightarrow$ **Ei**

Only $\mathscr{I}_0^{(0)}$ is independent!   **Only** **E0** **&** **C1** **are necessary!**

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Wigner Equations at $O(\hbar^n)$

With the help of mathematical induction, we can prove that only the time component $\mathscr{J}_0$ of Wigner functions $\mathscr{J}^\mu$ is independent up to any $O(\hbar^n)$, Space components $\vec{\mathscr{J}}$ can be given by the iterative relation:

$$\vec{\mathscr{J}}^{(n+1)} = \frac{\vec{p}}{p_0}\mathscr{J}_0^{(n+1)} + \frac{s}{2p_0}\sum_{k=0}^{n}\vec{G}^{(k)}\times\vec{\mathscr{J}}^{(n-k)} + \frac{1}{p_0}\sum_{k=1}^{n+1}\left[\vec{\Pi}_{(k)}\mathscr{J}_0^{(n-k+1)} - \Pi_{(k)}^0\vec{\mathscr{J}}^{(n-k+1)}\right]$$

$$\vec{\mathscr{J}}^{(0)} = \frac{\vec{p}}{p_0}\mathscr{J}_0^{(0)} \quad\Longrightarrow\quad \vec{\mathscr{J}}^{(n+1)} = \frac{\vec{p}}{p_0}\mathscr{J}_0^{(n+1)} + \vec{C}\left[\mathscr{J}_0^{(n)}, \mathscr{J}_0^{(n-1)}, \cdots, \mathscr{J}_0^{(0)}\right]$$

Evolution equation for $\mathscr{J}_0^{(n)}$:

$$\sum_{k=0}^{n}\left[G_0^{(k)}\mathscr{J}_0^{(n-k)} + \vec{G}^{(k)}\cdot\vec{\mathscr{J}}^{(n-k)}\right] = 0$$

Constraint equation for $\mathscr{J}_0^{(n)}$:

$$\sum_{k=0}^{n}\left[\Pi_0^{(k)}\mathscr{J}_0^{(n-k)} - \vec{\Pi}^{(k)}\cdot\vec{\mathscr{J}}^{(n-k)}\right] = 0$$

At any $O(\hbar^n)$, one distribution function and two equations.

Peel the Last Constraint Equation

From the constraint equation

$$\sum_{k=0}^{n} \left[\Pi_0^{(k)} \mathscr{J}_0^{(n-k)} - \vec{\Pi}^{(k)} \cdot \vec{\mathscr{J}}^{(n-k)} \right] = 0$$

$O(\hbar^0)$:

$$\mathscr{J}_0^{(0)} = p_0 f_p^{(0)} \delta \left(p^2 \right),$$

$O(\hbar^1)$:

$$\mathscr{J}_0^{(1)} = p_0 f_p^{(1)} \delta \left(p^2 \right) + s \left(\vec{B} \cdot \vec{p} \right) f_p^{(0)} \delta' \left(p^2 \right)$$

$O(\hbar^2)$:

$$\mathscr{J}_0^{(2)} = p_0 f_p^{(2)} \delta \left(p^2 \right) + s \left(\vec{B} \cdot \vec{p} \right) f_p^{(1)} \delta' \left(p^2 \right) + \frac{1}{2p_0} \left(\vec{B} \cdot \vec{p} \right)^2 f_p^{(0)} \delta'' \left(p^2 \right)$$

$$+ \frac{1}{p^2} \vec{p} \cdot \left(\vec{\Pi}^{(2)} \mathscr{J}_0^{(0)} - \Pi_0^{(2)} \vec{\mathscr{J}}^{(0)} \right) - \frac{1}{p^2} p_0 \left(\Pi_0^{(2)} \mathscr{J}_0^{(0)} - \vec{\Pi}_0^{(2)} \cdot \vec{\mathscr{J}}^{(0)} \right)$$

$$+ \frac{1}{4p^2} \vec{p} \cdot \left[\vec{G}^{(0)} \times \left(\frac{1}{p_0} \vec{G}^{(0)} \times \frac{\vec{p} \mathscr{J}_0^{(0)}}{p_0} \right) \right]$$

Now $f_p^{(0)}, f_p^{(1)}, f_p^{(2)}, \cdots, f_p^{(n)}, \cdots$ are free functions.

$$f_p = f_p^{(0)} + \hbar f_p^{(1)} + \hbar^2 f_p^{(2)} + \cdots$$

The analog of distribution functions in classical transport theory

Chiral kinetic equations in 4D up to $O(\hbar^2)$

Boltzmann-like equation in 4D ($p_0, \vec{p}$)

$O(\hbar^0):$

$$p^\mu G_\mu^{(0)}\left[f_p(x,p)\delta\left(p^2\right)\right] = 0 \qquad G_\mu^{(0)} \equiv \partial_\mu^x - F_{\mu\nu}\partial_p^\nu$$

$O(\hbar^0) + O(\hbar^1):$

$$\left[p^\mu G_\mu^{(0)}+\frac{s\hbar}{2}\vec{G}^{(0)}\cdot\left(\frac{1}{p_0}\vec{G}^{(0)}\times\vec{p}\right)\right]\left[f_p(x,p)\delta\left(p^2+\frac{s\hbar\vec{B}\cdot\vec{p}}{p_0}\right)\right] = 0$$

$$p_0 = |\vec{p}|\left(1-\hbar\vec{B}\cdot\vec{\Omega}_p\right), \quad \vec{\Omega}_p \equiv \frac{s\widehat{\vec{p}}}{2\vec{p}^2}$$

$O(\hbar^0) + O(\hbar^1) + O(\hbar^2):$

$$\left[p^\mu G_\mu^{(0)}+\frac{s\hbar}{2}\vec{G}^{(0)}\cdot\left(\frac{1}{p_0}\vec{G}^{(0)}\times\vec{p}\right)\right]\left[f_p(x,p)\delta\left(p^2+\frac{s\hbar\vec{B}\cdot\vec{p}}{p_0}\right)\right] + \hbar^2 C(f) = 0$$

$$C(f) = \frac{1}{4}\left(p^\mu G_\mu^{(0)}\frac{\mathbf{p}}{p^2}+\mathbf{G}^{(0)}\right)\cdot\left[\frac{1}{p_0}\mathbf{G}^{(0)}\times\left(\frac{1}{p_0}\mathbf{G}^{(0)}\times\mathbf{p}f\delta\left(p^2\right)\right)+\frac{4}{p_0}\left(\Pi^{(2)}p_0-\Pi_0^{(2)}\mathbf{p}\right)f\delta\left(p^2\right)\right]$$

$$-p^\mu G_\mu^{(0)}\left[\frac{1}{p^2}\Pi_\nu^{(2)}p^\nu f\delta\left(p^2\right)\right] + G_\mu^{(2)}p^\mu f\delta\left(p^2\right)$$

Boltzmann-like equation in 3D $\vec{p}$ can be obtained by integrating over p_0 !

Wigner Equations in 3D vector forms

Chiral kinetic equation in 3D forms up to $O(\hbar^1)$ $\qquad \vec{v} = \left(1 + 2\hbar\vec{B}\cdot\vec{\Omega}_p\right)\hat{p} - \hbar\left(\hat{p}\cdot\vec{\Omega}_p\right)\vec{B}$

Large $|\vec{p}|$:

$$\left(1 + \hbar\vec{B}\cdot\vec{\Omega}_p\right)\partial_t f_p + \left[\vec{v} + \hbar\vec{E}\times\vec{\Omega}_p + \hbar\left(\hat{p}\cdot\vec{\Omega}_p\right)\vec{B}\right]\cdot\vec{\nabla}_x f_p$$
$$+ \left[\vec{E} + \vec{v}\times\vec{B} + \hbar|\vec{p}|\vec{\nabla}_x\left(\vec{B}\cdot\vec{\Omega}_p\right) + \hbar\left(\vec{E}\cdot\vec{B}\right)\vec{\Omega}_p\right]\cdot\vec{\nabla}_p f_p = 0$$

Small $|\vec{p}|$: New terms appear from infrared region of momentum

$$\hbar\left(\vec{E}\cdot\vec{B}\right)\left(\vec{\nabla}_p\cdot\vec{\Omega}_p\right)f_p - \frac{2s\hbar}{\Lambda}\left(\vec{E}\cdot\vec{p}\right)\left(\vec{B}\cdot\vec{p}\right)\delta'\left(\Lambda^2 - |\vec{p}|^2\right)f_p$$
$$\rightarrow 2\pi s\hbar\left(\vec{E}\cdot\vec{B}\right)\delta^3\left(\vec{p}\right)f_p - \frac{3s\hbar}{4}\left(\vec{E}\cdot\hat{p}\right)\left(\vec{B}\cdot\hat{p}\right)\delta''\left(|\vec{p}|\right)f_p$$

Where does the chiral anomaly come from ?

$$\partial_\mu j_s^\mu = \frac{\hbar}{2}\int d^3\vec{p}\left[\left(\vec{E}\cdot\vec{B}\right)\vec{\Omega}_p\cdot\vec{\nabla}_p f_p + \left(\vec{E}\cdot\vec{B}\right)\left(\vec{\nabla}_p\cdot\vec{\Omega}_p\right)f_p - \frac{2s}{\Lambda}\left(\vec{E}\cdot\vec{p}\right)\left(\vec{B}\cdot\vec{p}\right)\delta'\left(\Lambda^2 - |\vec{p}|^2\right)f_p\right]$$

previous term: (1)   New term: (2)   New term: (3)

(2)+(3)=0 > (1) contributes.   (1)+(2)$\sim\int d^3\vec{p}\,\vec{\nabla}\cdot\left(\vec{\Omega}_p f_p\right)$=0 > (3) contributes.

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Summary

- **Chiral systems in a background EM field can be described sufficiently by one distribution function and one Boltzmann-like equation up to any order of $\hbar$ in 4D.**

- **This property will significantly simplify the description and simulation of chiral effects in HIC and Dirac/Weyl semimetals**

- **We present chiral kinetic equations in 4D forms up to $O\left(\hbar^2\right)$.**

- **The chiral anomaly may come from a new source in chiral kinetic equations, in contrast to the well-known Berry curvature term.**

Thanks for your attention !