Nuclear symmetry energy and crustal torsional oscillations in neutron stars

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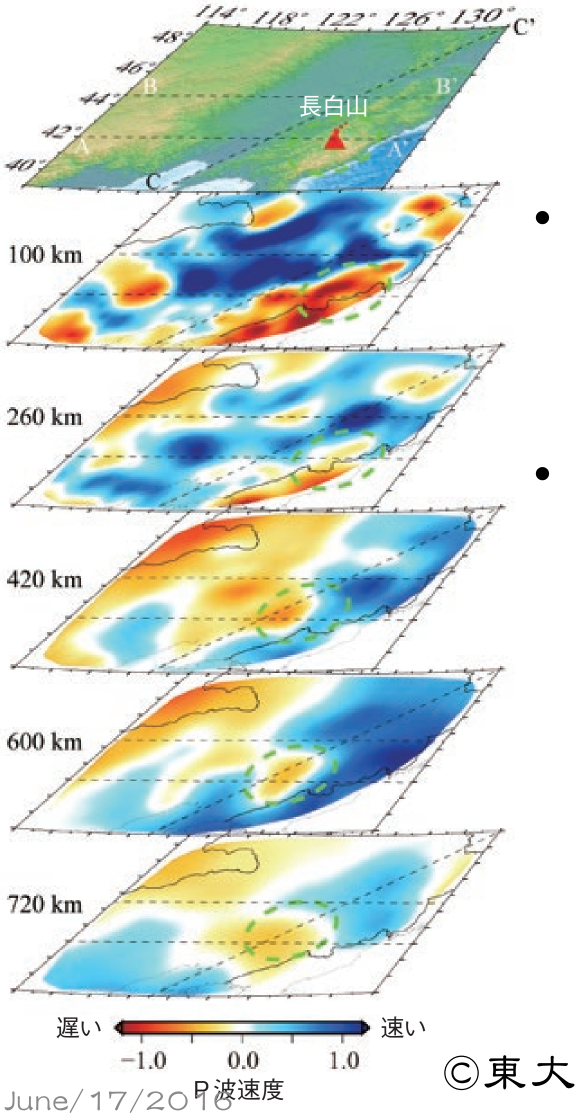
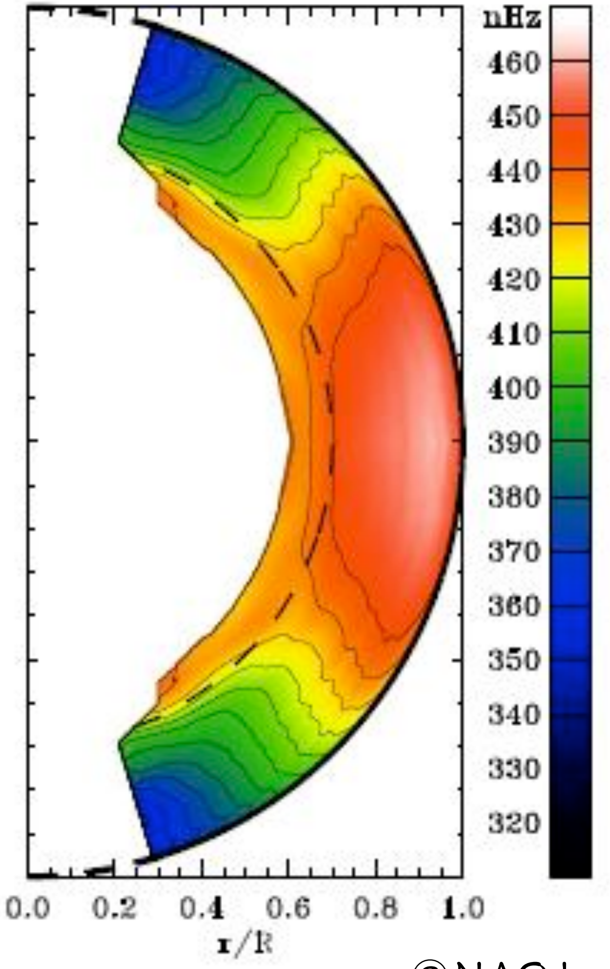
Seismology, Helioseismology



- Seismic waves tell us information inside the Earth (seismology)

- The interior of the Sun can be probed through the wave pattern on the surface (helioseismology)



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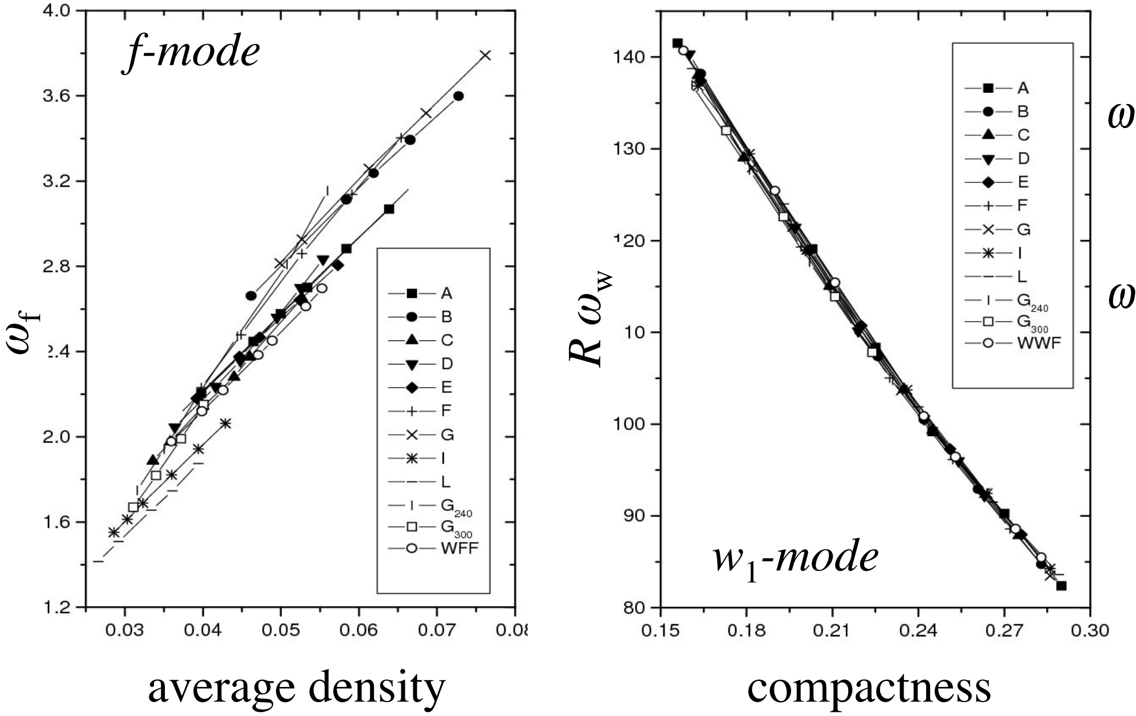
Asteroseismology in Neutron Stars

- via the observations of GW frequencies, one might be able to see the properties of NSs



Andersson & Kokkotas (1998)

$$\omega_f \approx 0.78 + 1.64\left[\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10\mathrm{km}}{R}\right)^3\right]^{1/2}$$

$$\omega_w \approx \left(\frac{10\mathrm{km}}{R}\right)\left[20.92 - 9.14\left(\frac{M}{1.4M_\odot}\right)\left(\frac{10\mathrm{km}}{R}\right)\right]$$

↓

determination of (M, R)

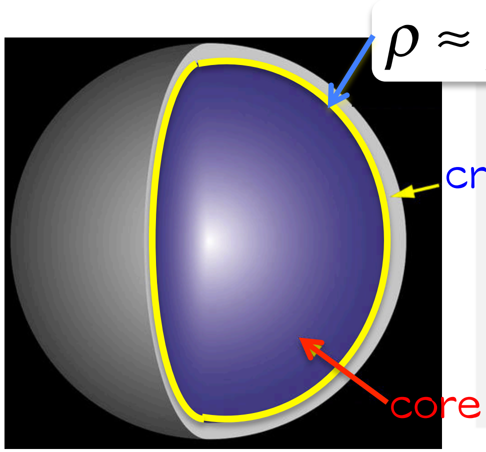
↓

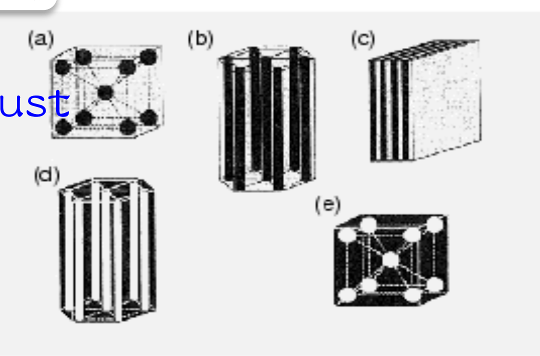
constraint on EOS

neutron stars

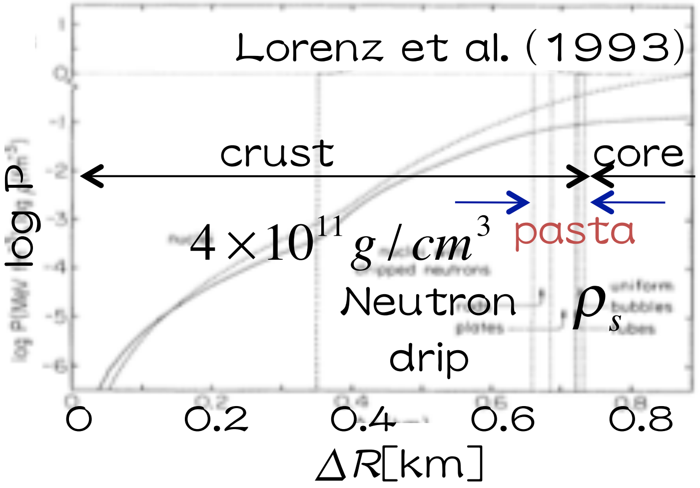
- Structure of NS
  - solid layer (crust)
  - nonuniform structure (pasta)
  - fluid core (uniform matter)
- Crust thickness $\lesssim$ 1km
  - strongly associated with nuclear saturation properties
- Constraint on EOS via observations of neutron stars
  - stellar mass and radius
  - stellar oscillations (& emitted GWs)

“(GW) asteroseismology”





Oyamatsu (1993)

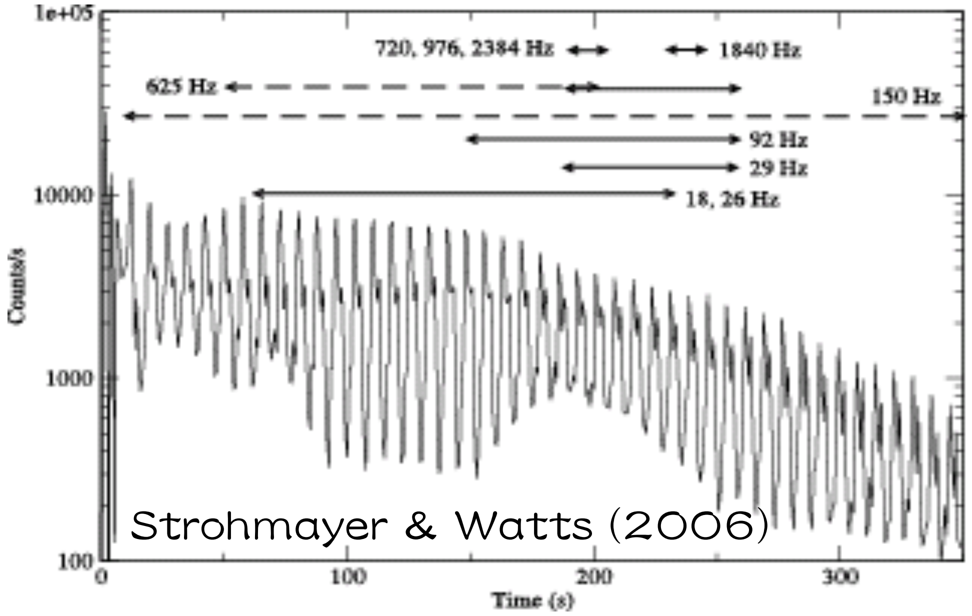


QPOs in SGRs

- Quasi-periodic oscillations (QPOs) in afterglow of giant flares from soft-gamma repeaters (SGRs)
  - SGR 0526-66 (5th/3/1979) : 43 Hz
  - SGR 1900+14 (27th/8/1998) : 28, 54, 84, 155 Hz
  - SGR 1806-20 (27th/12/2004) : 18, 26, 30, 92.5, 150, 626.5, 1837 Hz

  (Barat+ 1983, Israel+ 05, Strohmayer & Watts 05, Watts & Strohmayer 06)

  - additional QPO is found : 57Hz (Huppenkothen et al. 2014)



Strohmayer & Watts (2006)

- Crustal torsional oscillation ?
- Magnetic oscillations ?
- Asteroseismology
  → stellar properties
  (M, R, B, EOS ...)

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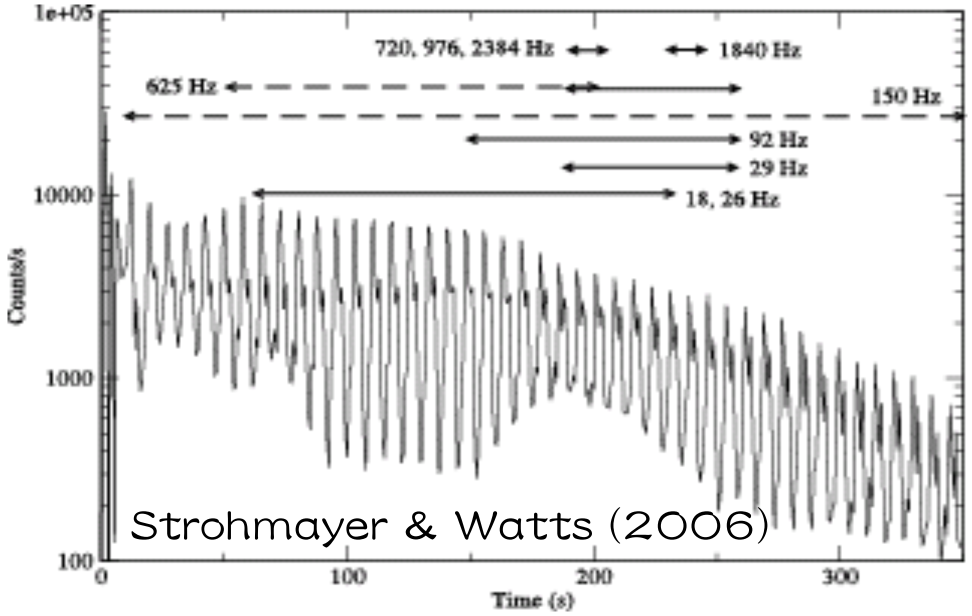

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- Crustal torsional oscillation ?
- Magnetic oscillations ?
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  → stellar properties
  (M, R, B, EOS ...)

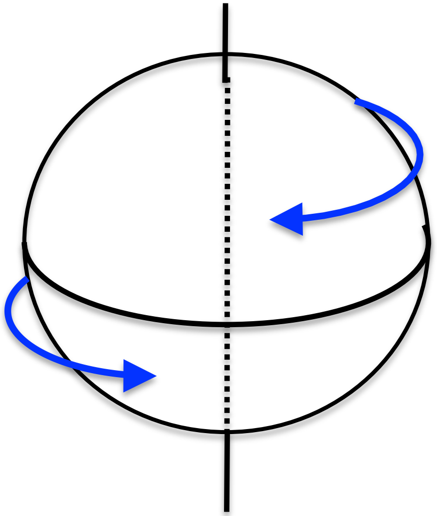
torsional oscillations

- axial parity oscillations
  - incompressible
  - no density perturbations
- in Newtonian case (Hansen & Cioffi 1980)

$$ {}_{\ell}t_0 \sim \frac{\sqrt{\ell(\ell+1)\mu/\rho}}{2\pi R} \sim 16\sqrt{\ell(\ell+1)}\ \text{Hz} \qquad {}_{\ell}t_n \sim \frac{\sqrt{\mu/\rho}}{2\Delta r} \sim 500 \times n\ \text{Hz} $$

  - μ : shear modulus
  - frequencies ∝ shear velocity $v_s = \sqrt{\mu/\rho}$
  - overtones depend on crust thickness
- one can consider torsional oscillations independently of core EOS
- effect of magnetic field
  - frequencies become larger

(Sotani+07, Gabler+12,13)



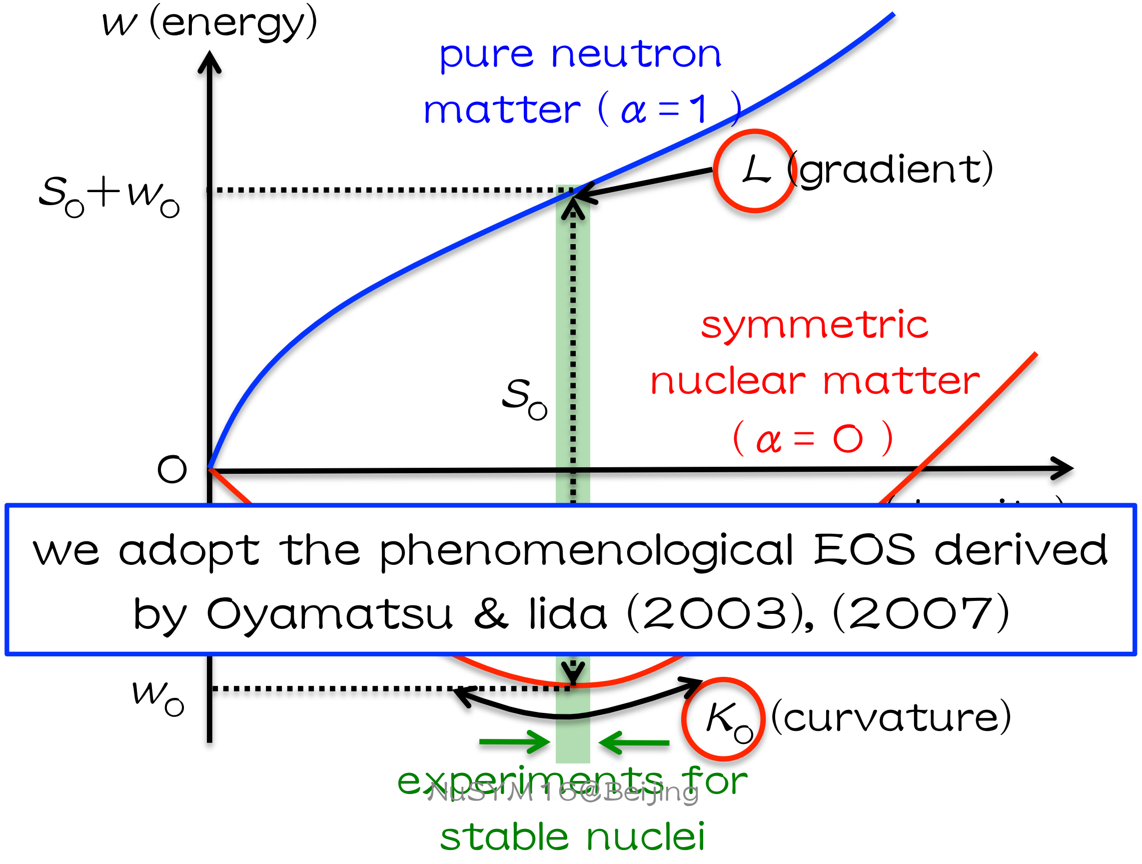
EOS near the saturation point

- Bulk energy per nucleon near the saturation point of symmetric nuclear matter at zero temperature;

incompressibility

symmetry parameter

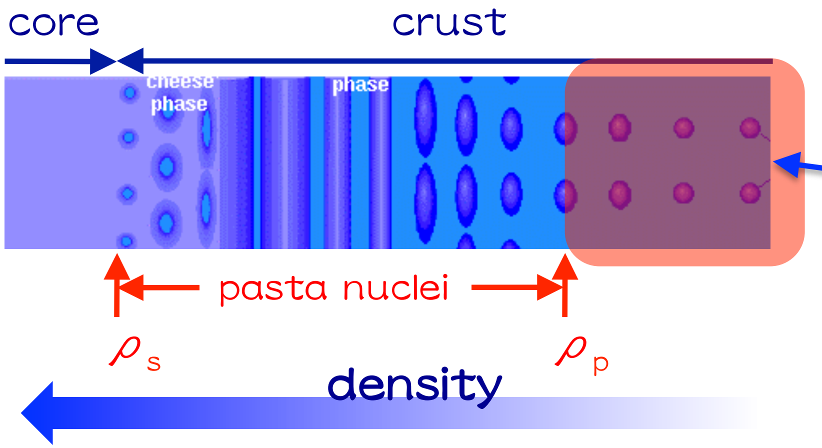
$$w = w_0 + \frac{\boxed{K_0}}{18n_0^2}(n-n_0)^2 + \left[S_0 + \frac{\boxed{L}}{3n_0}(n-n_0)\right]\alpha^2$$



we adopt the phenomenological EOS derived by Oyamatsu & Iida (2003), (2007)

in (our) previous works

- EOS for core region is still uncertain.
- To prepare the crust region, we integrate from $r=R$.
  - M, R : parameters for stellar properties
  - L, K_0 : parameters for curst EOS (Oyamatsu & Iida (2003), (2007))
    - → For $L \gtrsim$ 100MeV, pasta structure almost disappears
- In crust region, torsional oscillations are calculated.
  - considering the shear only in spherical nuclei.
  - frequency of fundamental oscillation $\propto v_s$ ($v_s^2 \sim \mu / H$)
  - calculated frequencies could be lower limit



for bcc lattice

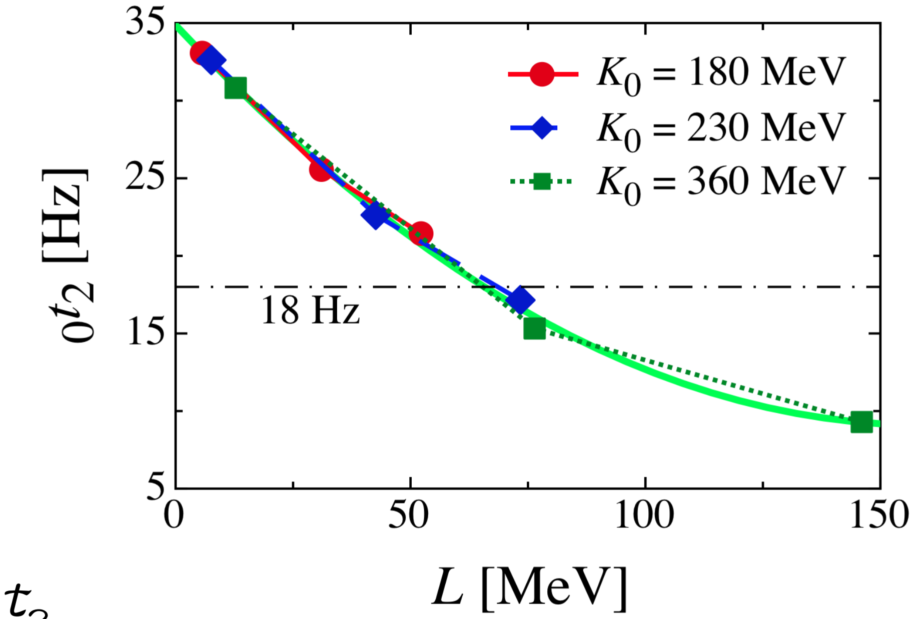
$$\boxed{\mu = 0.1194\frac{n_i(Ze)^2}{a}}$$

n_i : ion number density
Z : charge of nuclei
a : Wigner-Seitz radius

frequencies of crustal oscillations ${}_0t_l$

HS+2012a

- For M=1.4$M_\odot$ & R=12km, calculated frequencies ${}_0t_2$
- ${}_0t_2$ is almost independent of the value of K_0
- For R=10~14 km and $M/M_\odot$=1.4~1.8, similar dependence on K_0
- One can write fitting line
- Focus on L dependence of ${}_0t_2$
- ${}_0t_2$ becomes smaller with larger R and M.
- ${}_0t_l$ can also be expressed as a function of L.



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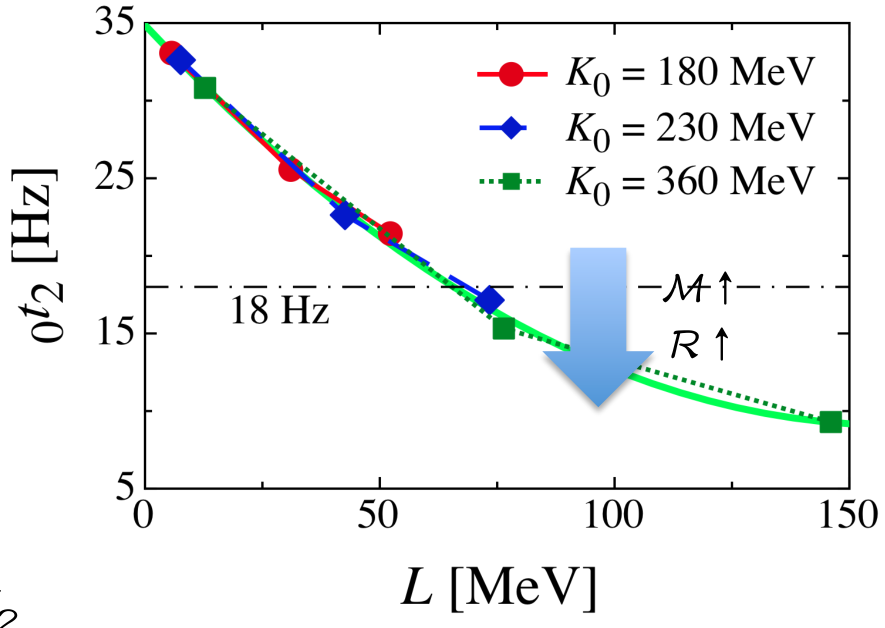

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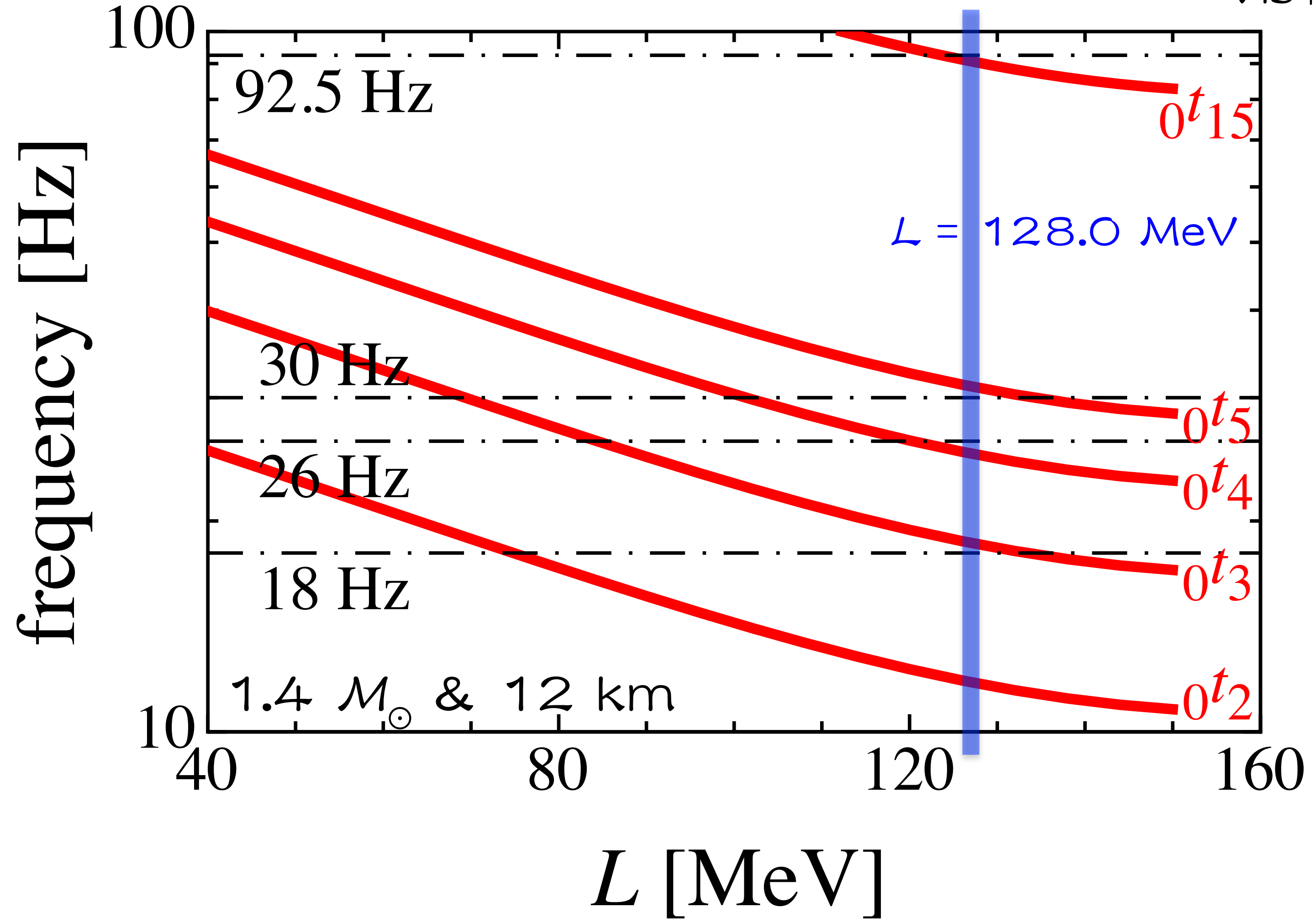
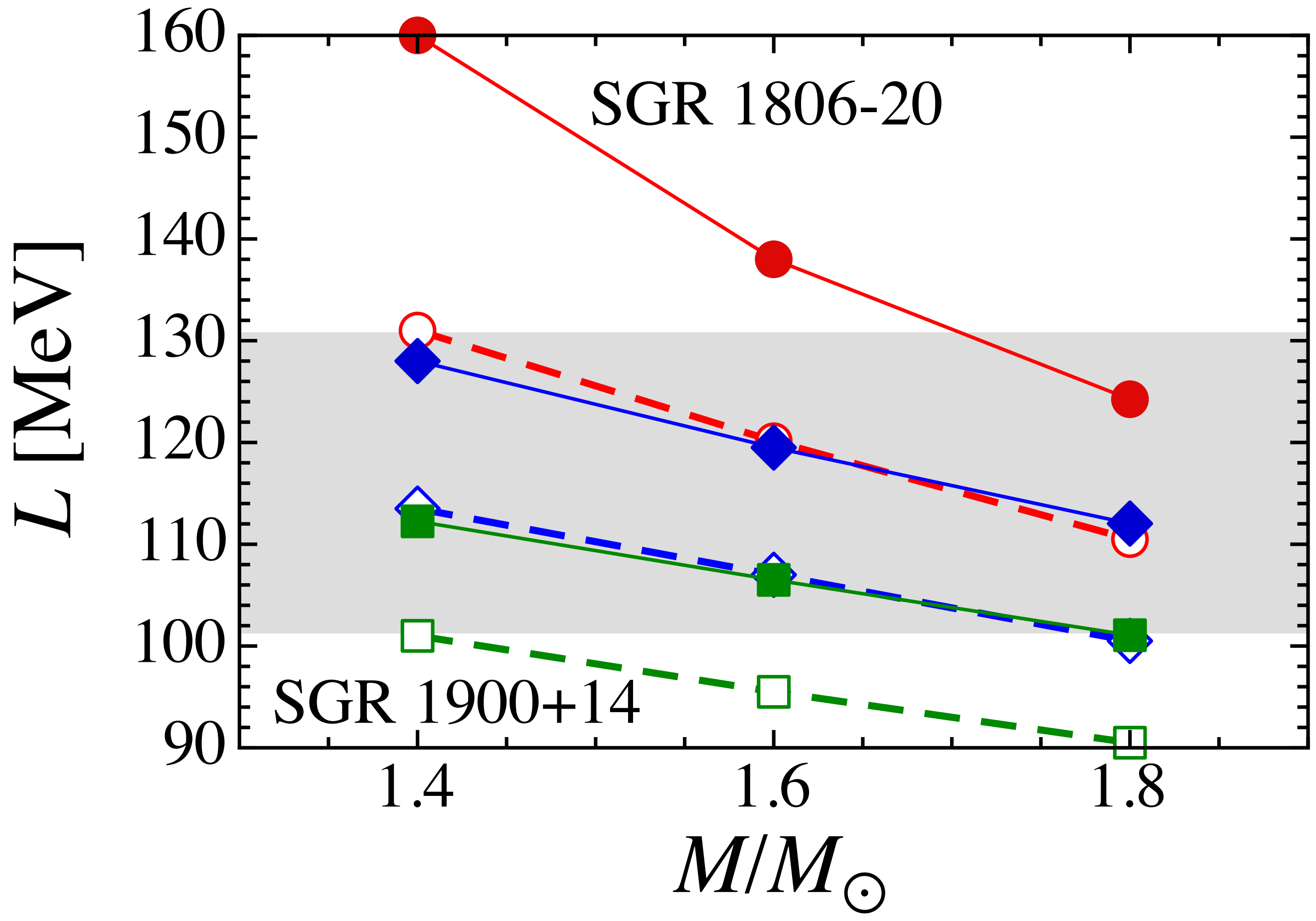
identification of SGR 1806-20

HS+ 2013a



allowed region for L

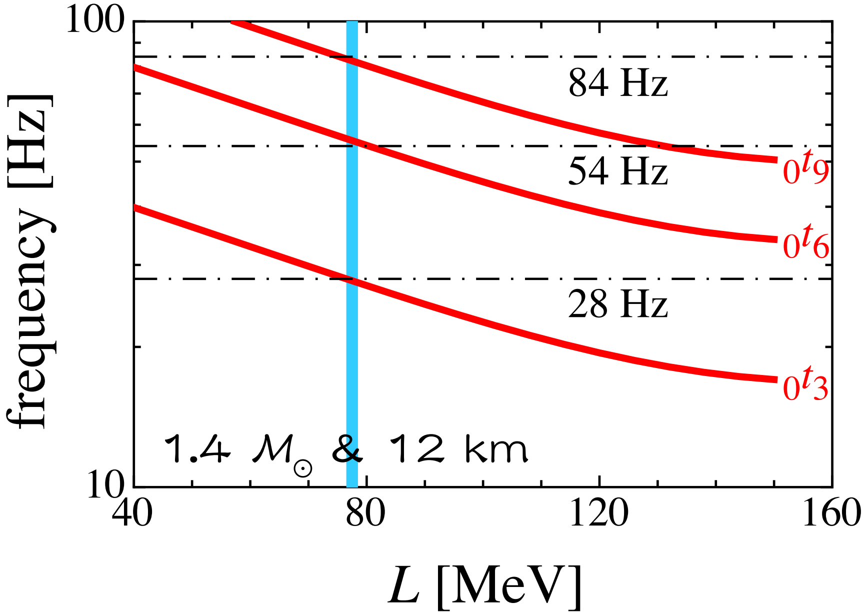


➡ $101.1 \text{ MeV} \leq L \leq 131.0 \text{ MeV}$

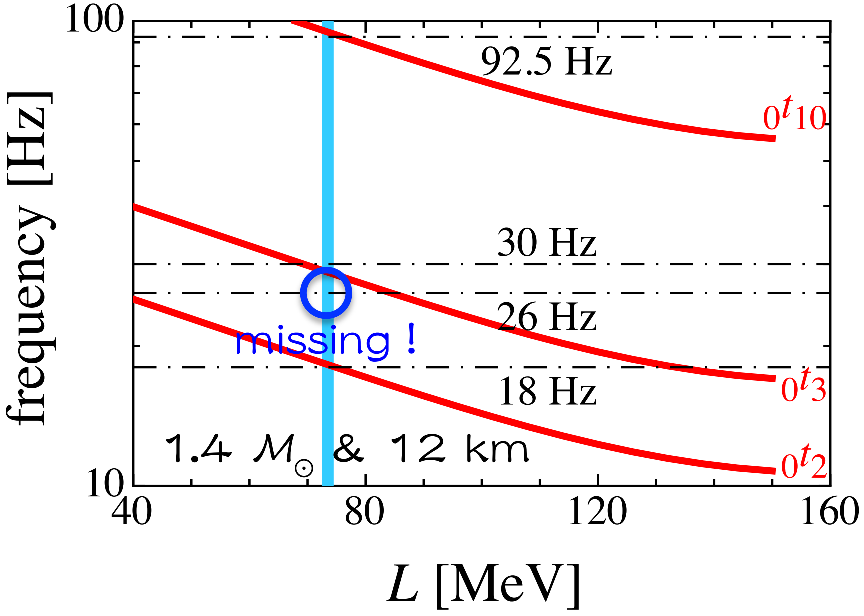
alternative possibility

instead of previous correspondence, i.e., $l = 4, 8, 13$ for SGR 1900+14, and $l = 3, 4, 5, 15$ for SGR 1806-20, we may consider alternative possibility as

SGR 1900+14

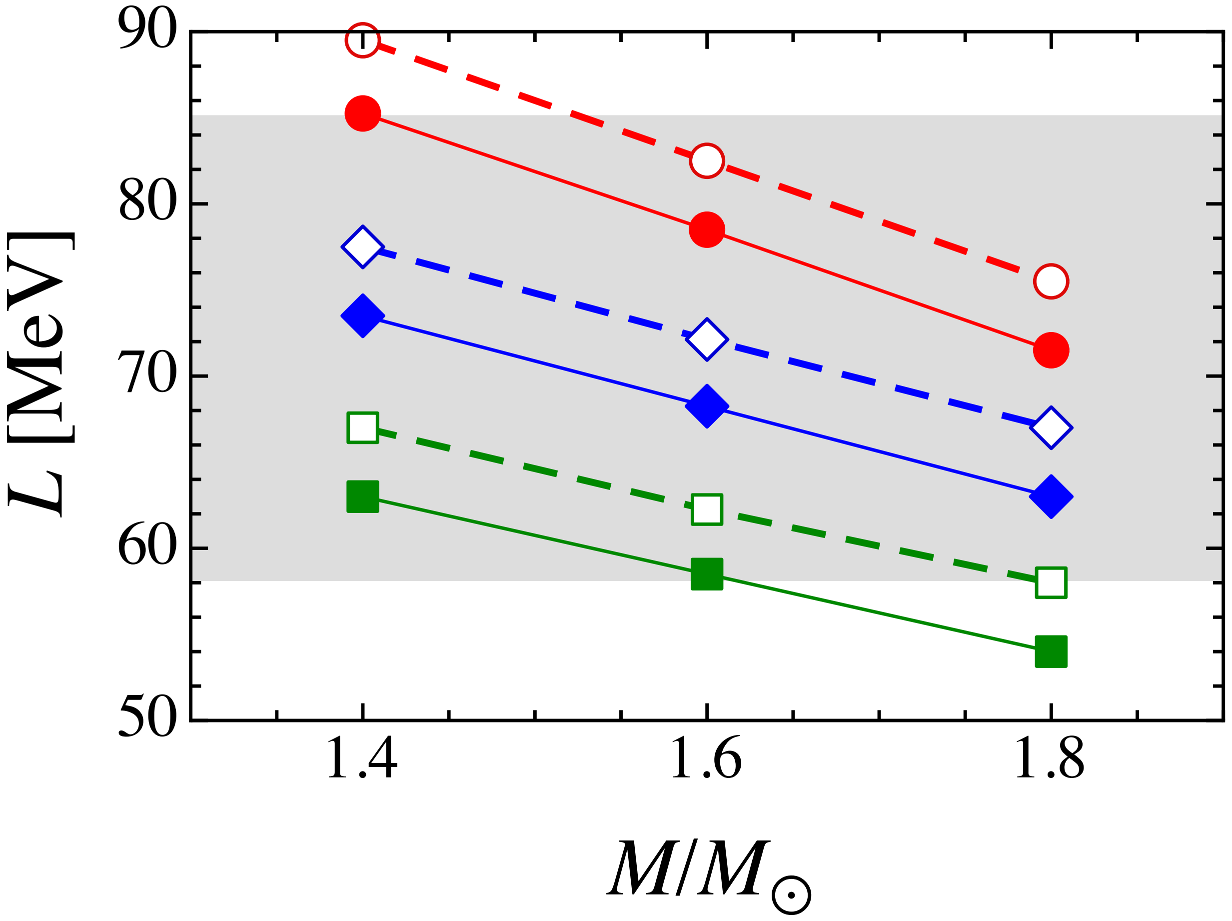


SGR 1806-20



26 Hz QPO observed in SGR 1806-20 remains a complete puzzle !!

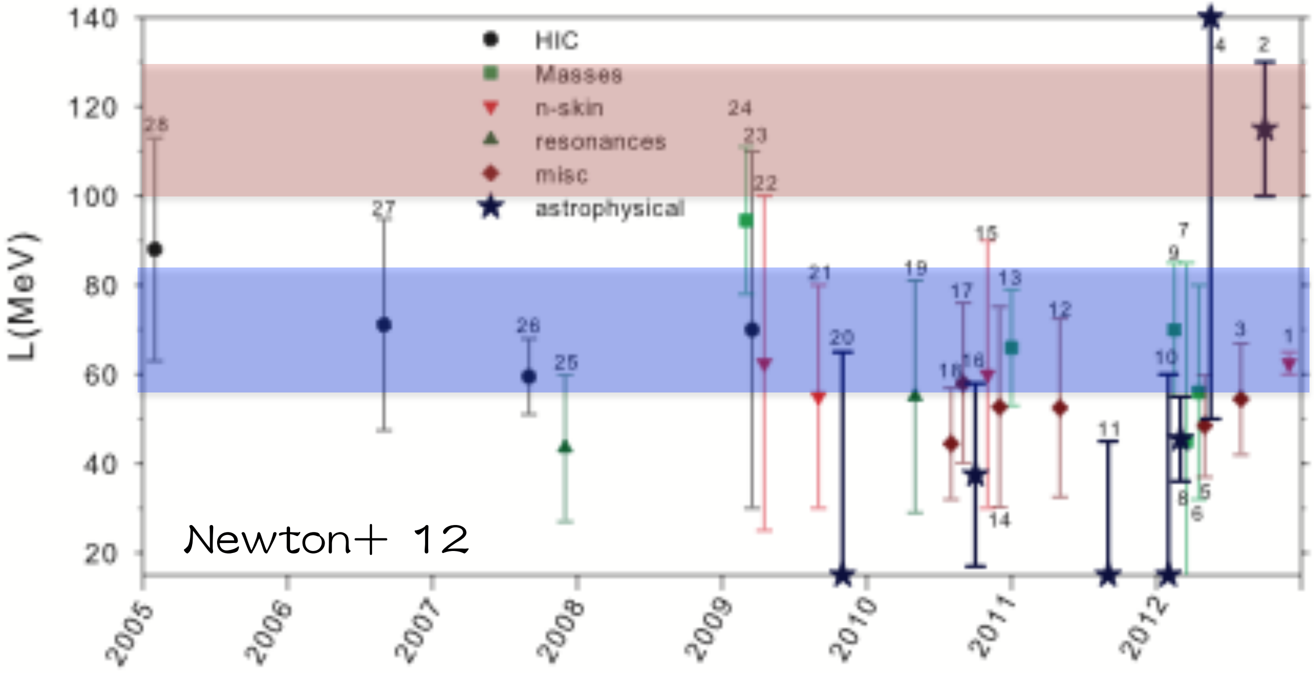
alternative allowed region for L



➡ $58.0 \text{ MeV} \leq L \leq 85.3 \text{ MeV}$

other constraints on L

- other constraints suggests $L \sim 60 \pm 20$ MeV ?
  - this means that alternative correspondences may be favored ??
  - if so, one has to prepare <u>another oscillation mechanism</u>…



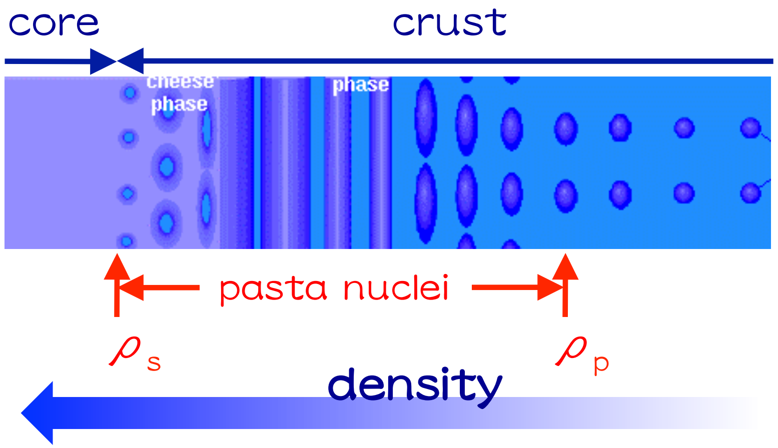
Newton+ 12

all QPOs come from crustal oscillations

missing 26Hz

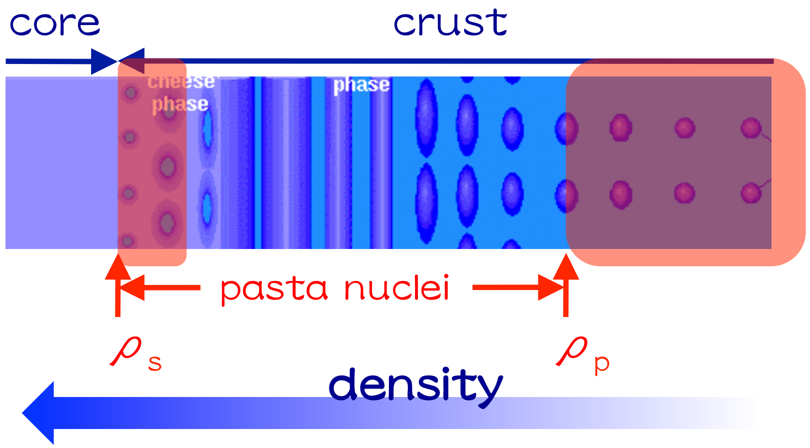
as a possibility of 26Hz…

- we consider the oscillations in the pasta structure
- shear modulus in pasta phase
  - slab phase: shear is the 3rd order of displacement (Landau)
    → in the linear perturbation, oscillations in slab are negligible



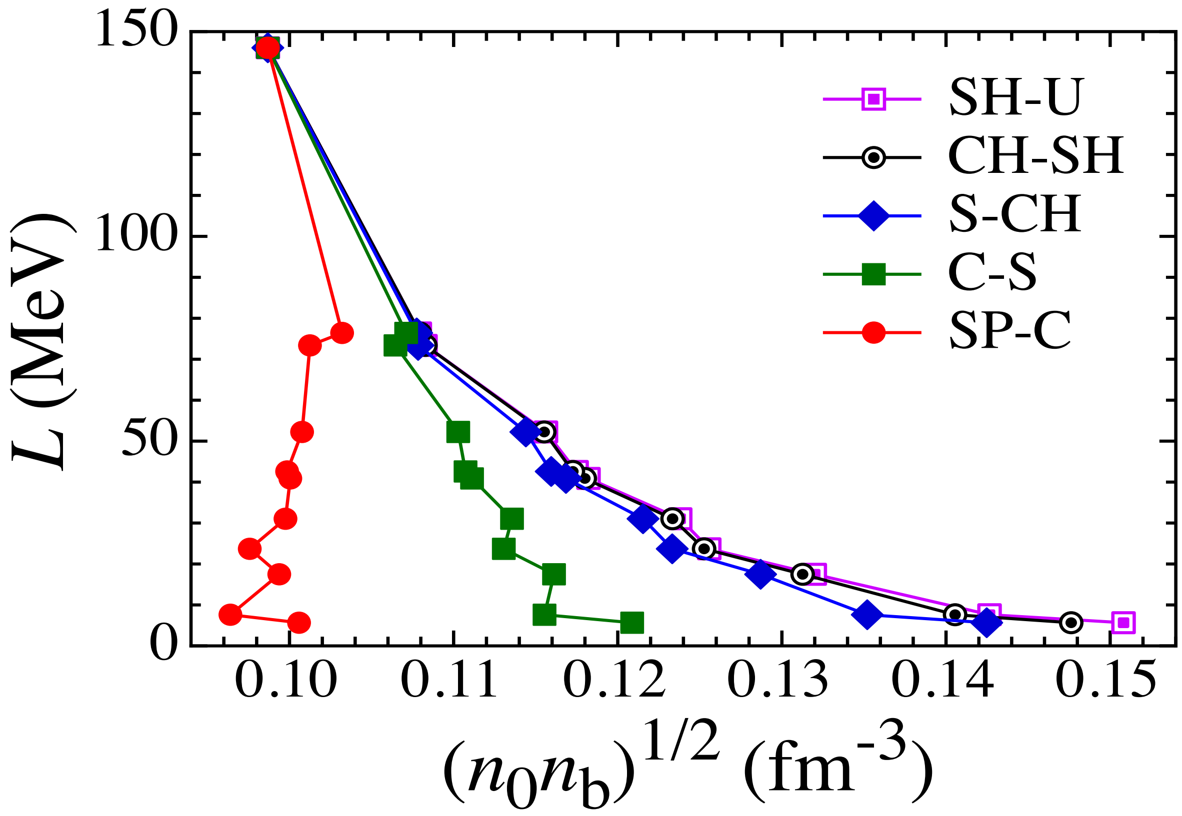
as a possibility of 26Hz...

- we consider the oscillations in the pasta structure
- shear modulus in pasta phase
  - slab phase: shear is the 3rd order of displacement (Landau)
    → in the linear perturbation, oscillations in slab are negligible
  - two independent oscillations can be excited in different regions:
    - oscillations in spherical and cylindrical nuclei
    - oscillations in bubble and cylindrical-hole nuclei
  - as a first step, we consider only oscillations in bubble phase



$$\boxed{\mu = 0.1194\frac{n_i(Ze)^2}{a}}$$

bubble structures in crust



- with larger L, small region of pasta phase
- for $L \gtrsim 75$MeV, bubble structure disappears

torsional oscillations in bubble phase

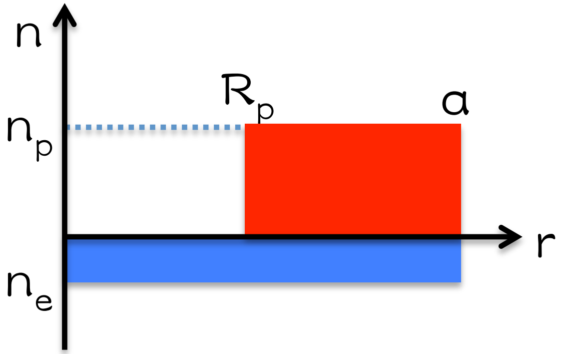
- effective charge density in bubble

$$n_Q = \underbrace{\boxed{-n_e}}_{\text{bubble}} - \underbrace{\boxed{\left(n_p - n_e\right)}}_{\text{background}} = -n_p$$

- effective charge in bubble

$$Z_{bubble} = n_Q \times V_{bubble}$$

$$\boxed{\mu = 0.1194 \frac{n_i (Ze)^2}{a}}$$

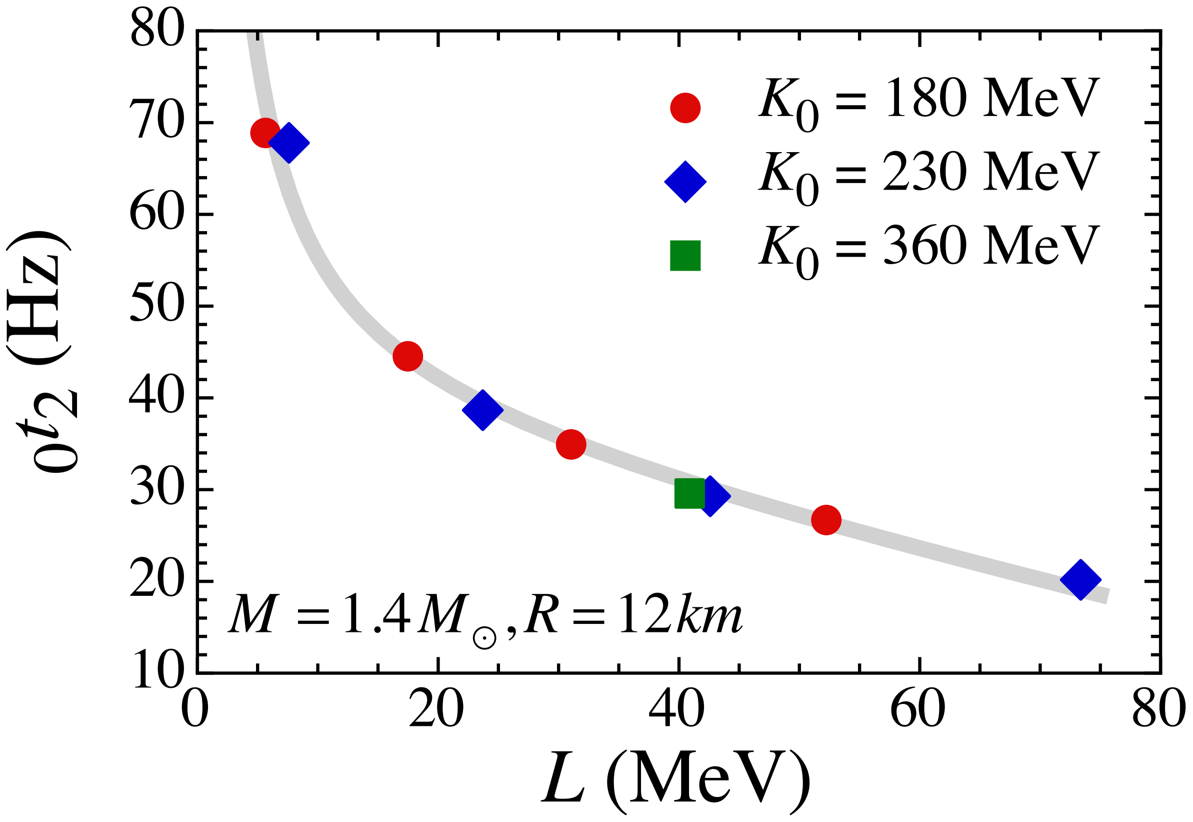


- enthalpy contributing to oscillations (entrainment rate?)
  - maximum: all matter elements contribute to oscillations (minimum frequency)
  - minimum: only matter elements inside the bubble contribute (maximum frequency)

$$f \propto v_s \qquad v_s = \frac{\mu}{\underbrace{\boxed{\varepsilon + P}}_{\text{enthalpy}}}$$

bubble oscillations 1

- in the case that all matter elements contribute to oscillations

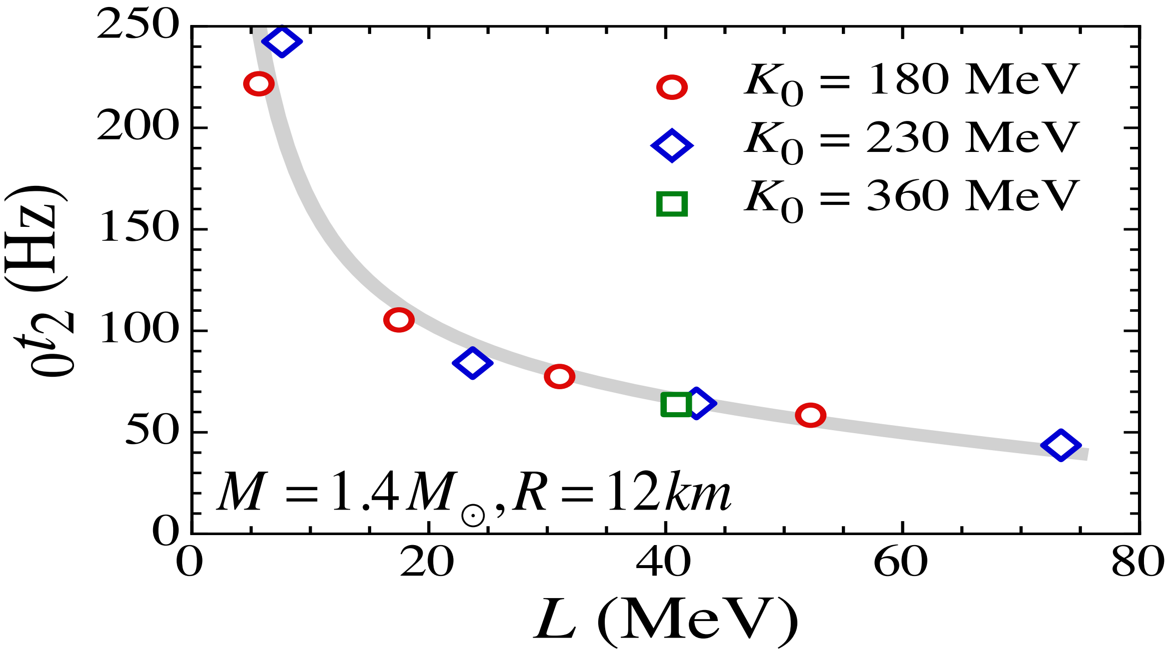


- the frequency is almost independent of the value of K_0

$_0t_2 = 205.5 / L + 37.73 - 0.2922 L$

bubble oscillations 2

- in the case that only matter elements inside the bubble contribute to oscillations

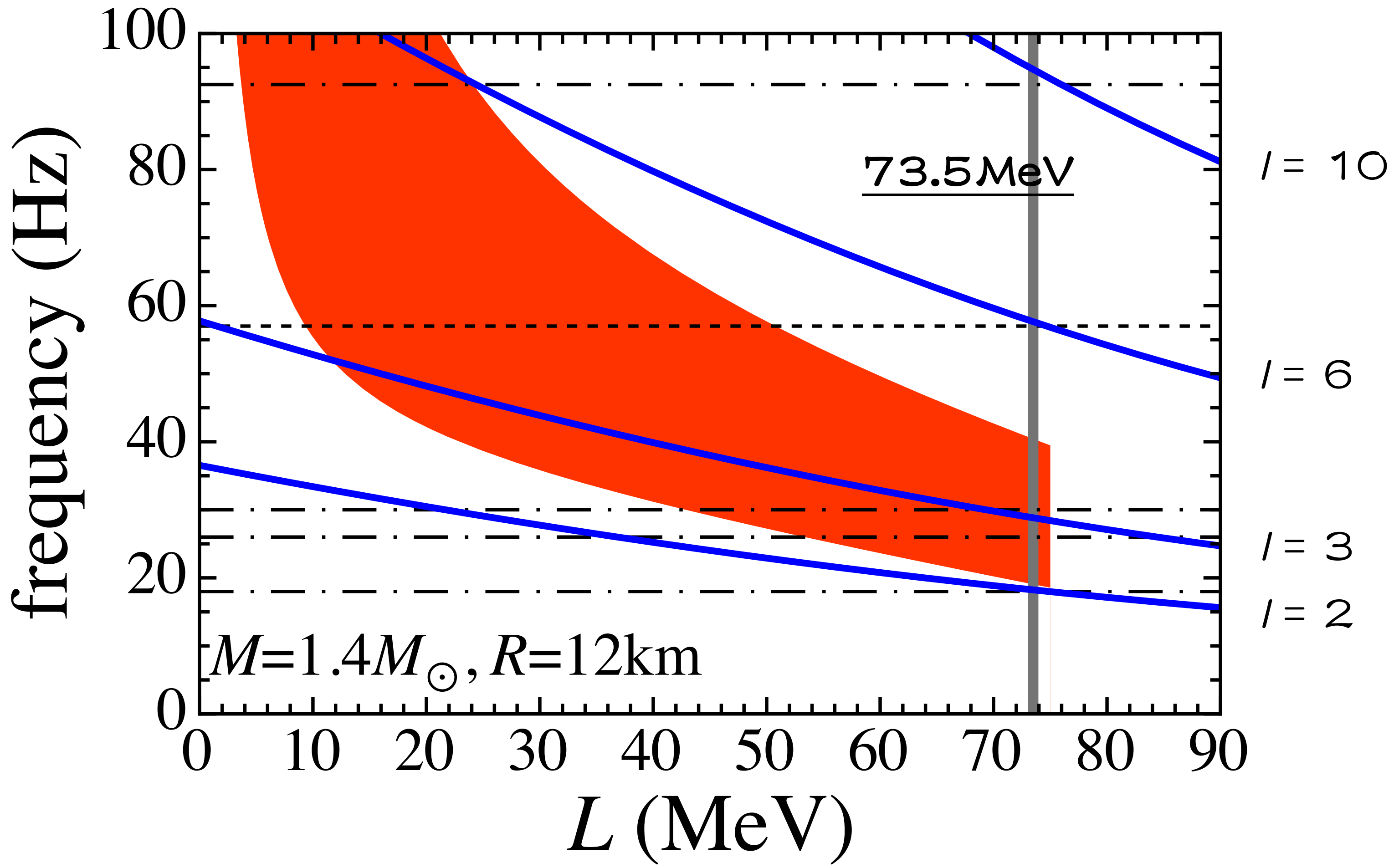


- again, the frequencies are almost independent of K_0

$$_0t_2 = 1100 / L + 57.39 - 0.4345 L$$

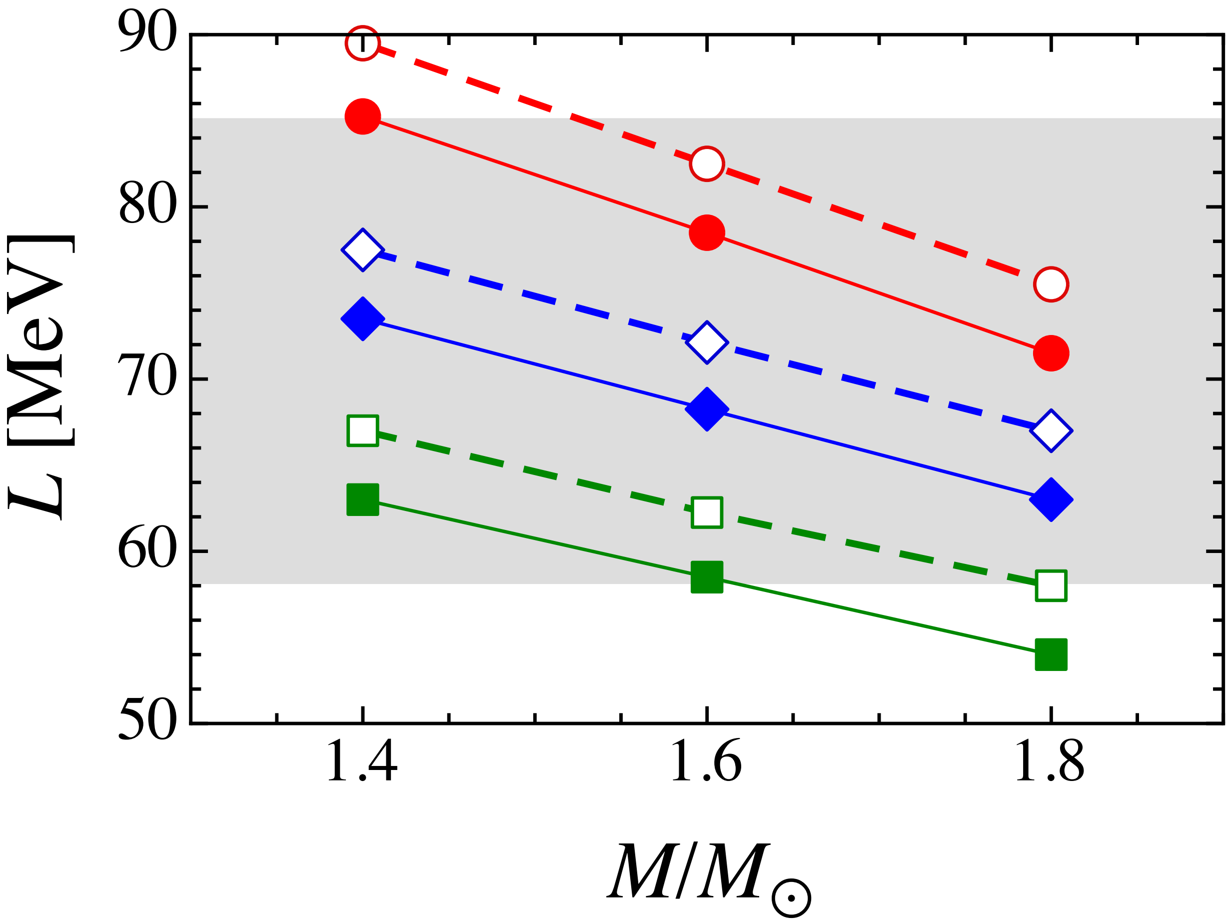
- frequencies strongly depend on the entrainment rate

comparison with QPOs

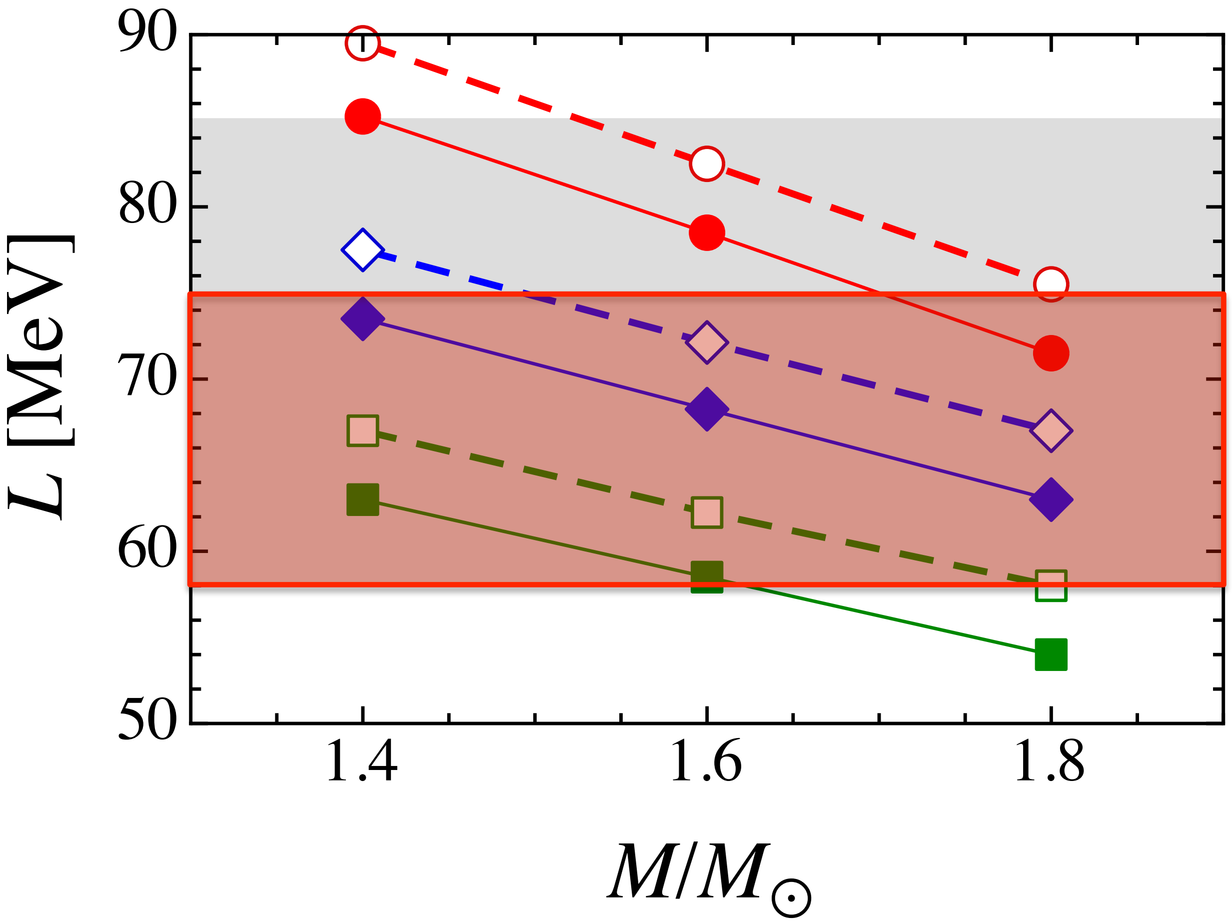


- oscillation in bubble might be possible to correspond to 26Hz QPO, depending on the entrainment rate.

constraint on L



constraint on *L*



➡ $58.0\ \text{MeV} \leq L \lesssim 75\ \text{MeV}$

conclusion

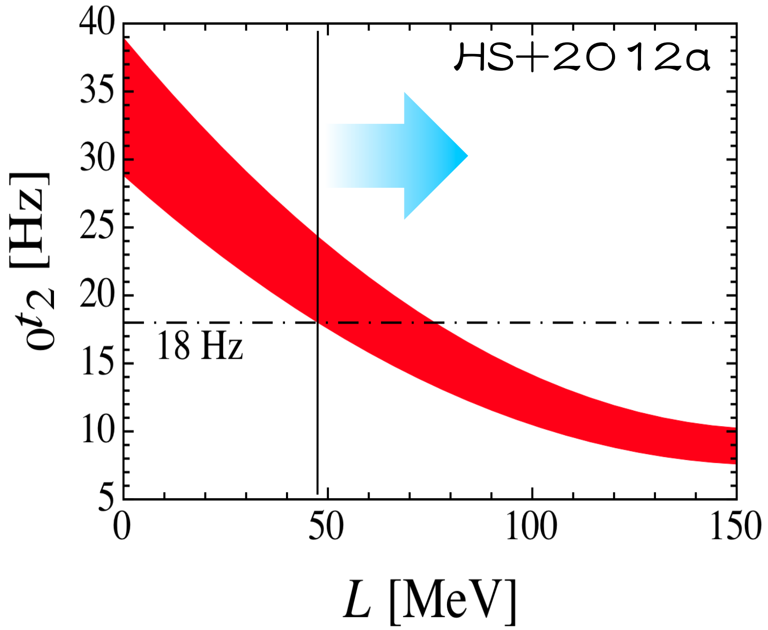
- neutron stars are good candidates to examine the physics under the extreme state.
  - QPOs in SGRs may be good examples to adopt the asteroseismology
- constraint on L can be obtained as
  - $100 \lesssim L \lesssim 130$ MeV, if all QPOs come from torsional oscillations
  - $58 \lesssim L \lesssim 85$ MeV, if QPOs except for 26 Hz QPO coms from torsional oscillation
- as another possibility to produce the 26 Hz, we consider the torsional oscillations in bubble structure
  - frequencies strongly depend on the entrainment rate
  - even so, the frequency might be possible to correspond to 26 Hz with the suitable value of L to explain the other QPOs by torsional oscillations in the region composed of spherical nuclei
  - the constraint on L should be modified as

    $$58 \lesssim L \lesssim 75 \text{ MeV},$$

    in order that the bubble structure should exist

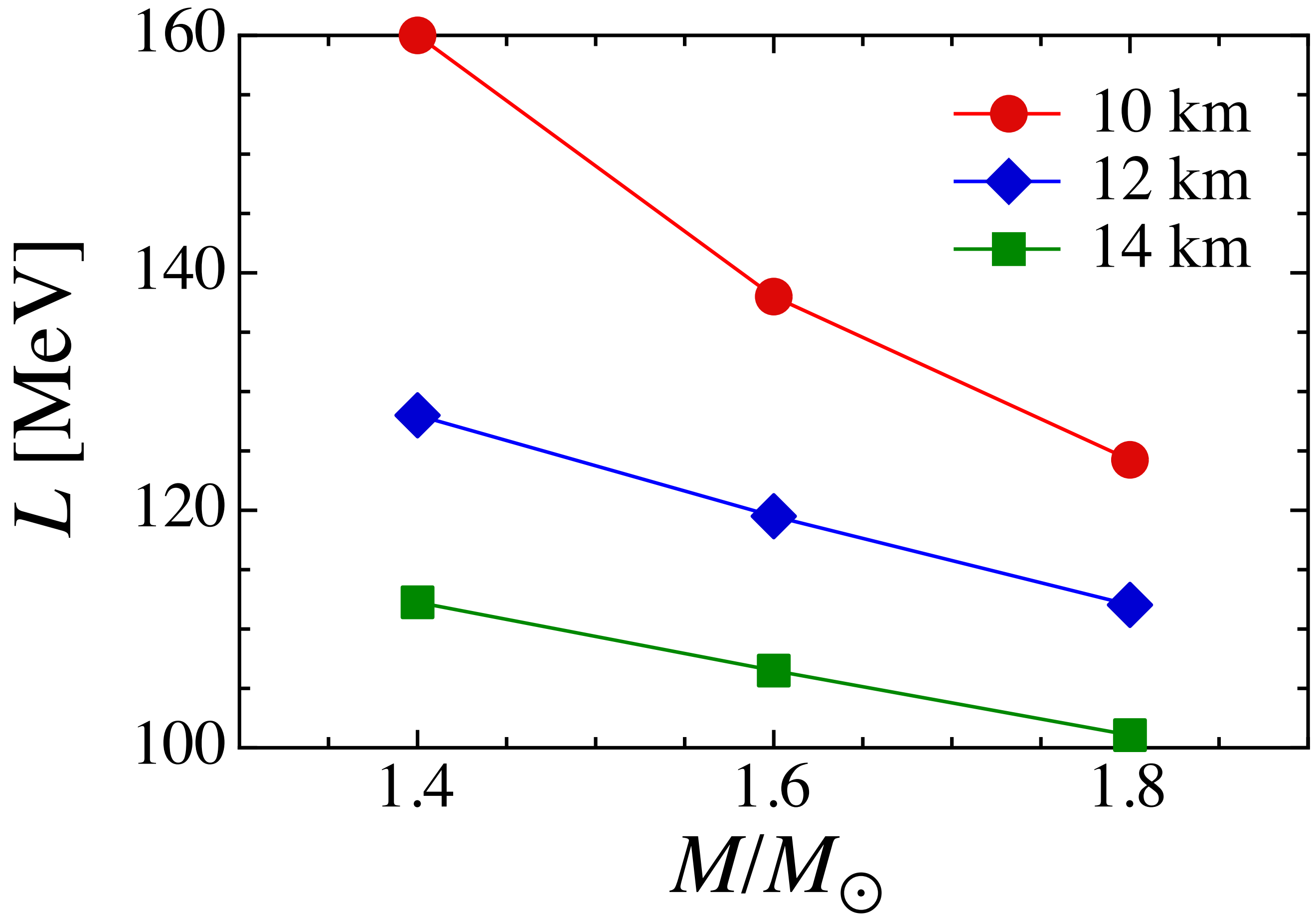
Constraint on L

- For R=10km~14km & $M/M_{\odot}$=1.4~1.8, ${}_0t_2$ are calculated
- Assuming that the observed QPOs would come from torsional oscillations
- ${}_0t_2$ is the smallest frequency among a lot of torsional oscillations
  - ${}_0t_2$ should be equal to or smaller than the smallest observed QPOs frequency



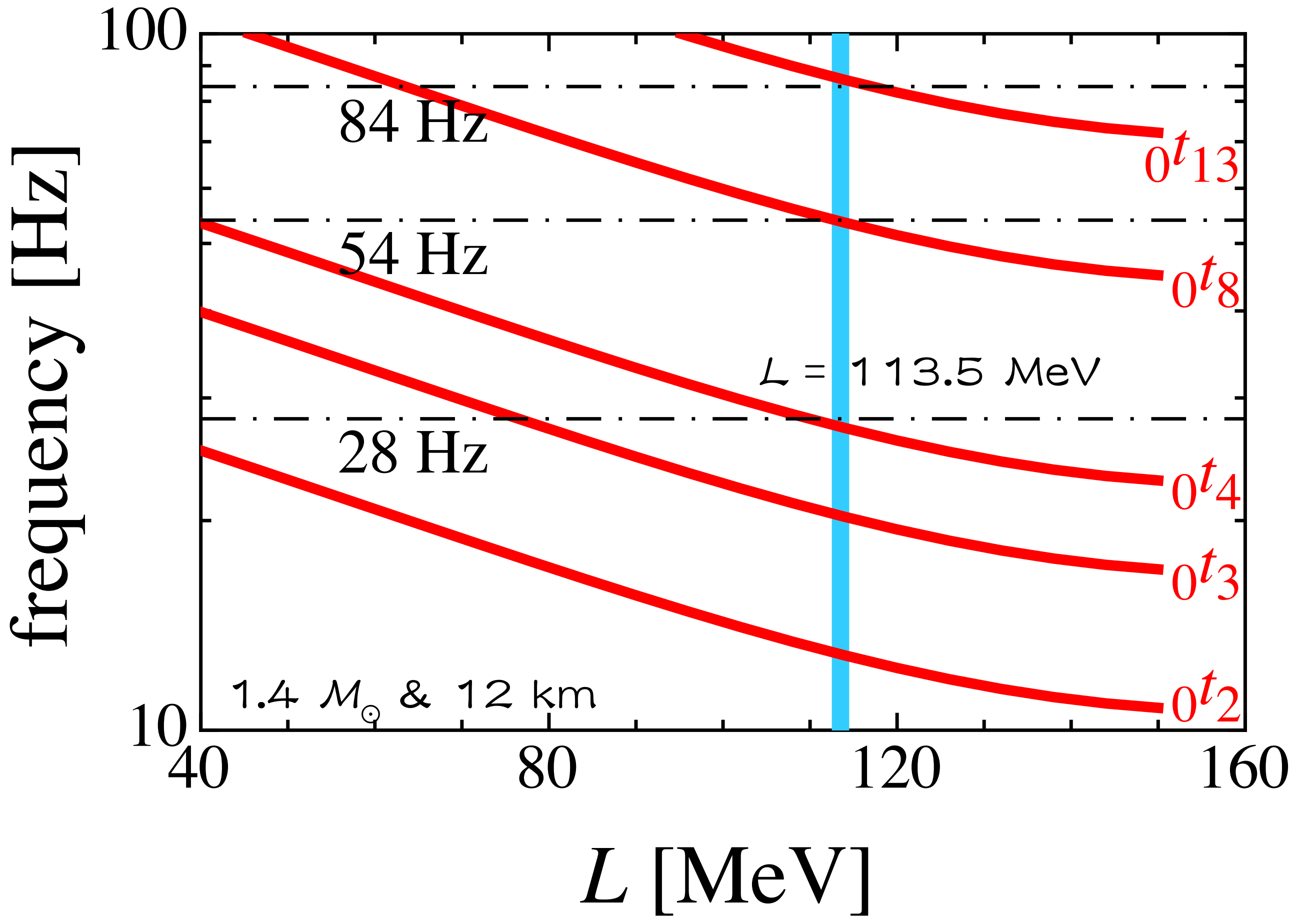
- Consequently, $L \gtrsim 50$ MeV.
  - For $L \gtrsim 50$ MeV, pasta region could be very narrow
  - Modification due to the pasta effect should be small
  - This is first constraint in the symmetry parameter with astronomical observations

constraint on L via SGR 1806-20



➡ 101.1 MeV ≤ L ≤ 160.0 MeV

identification of SGR 1900+14



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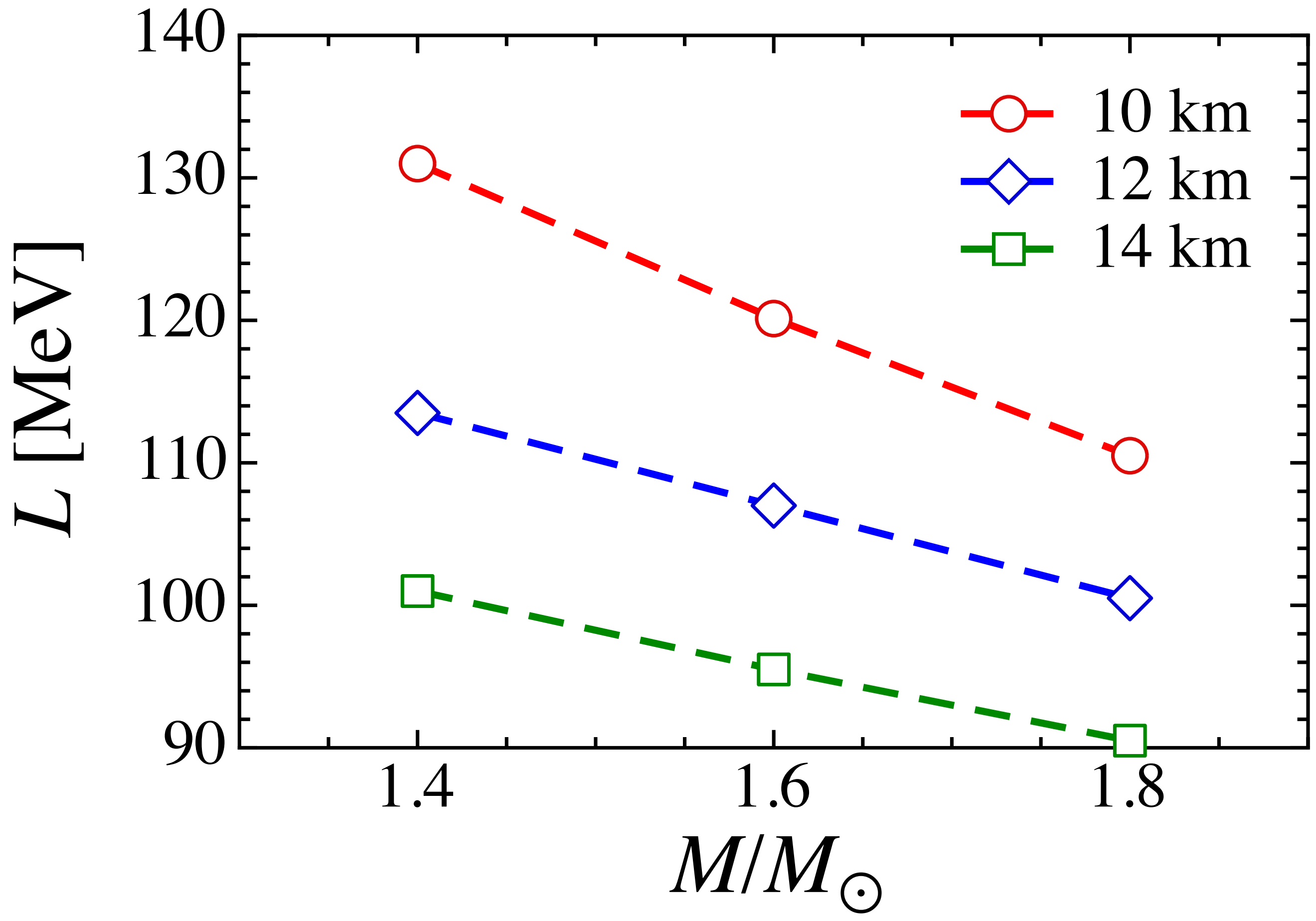
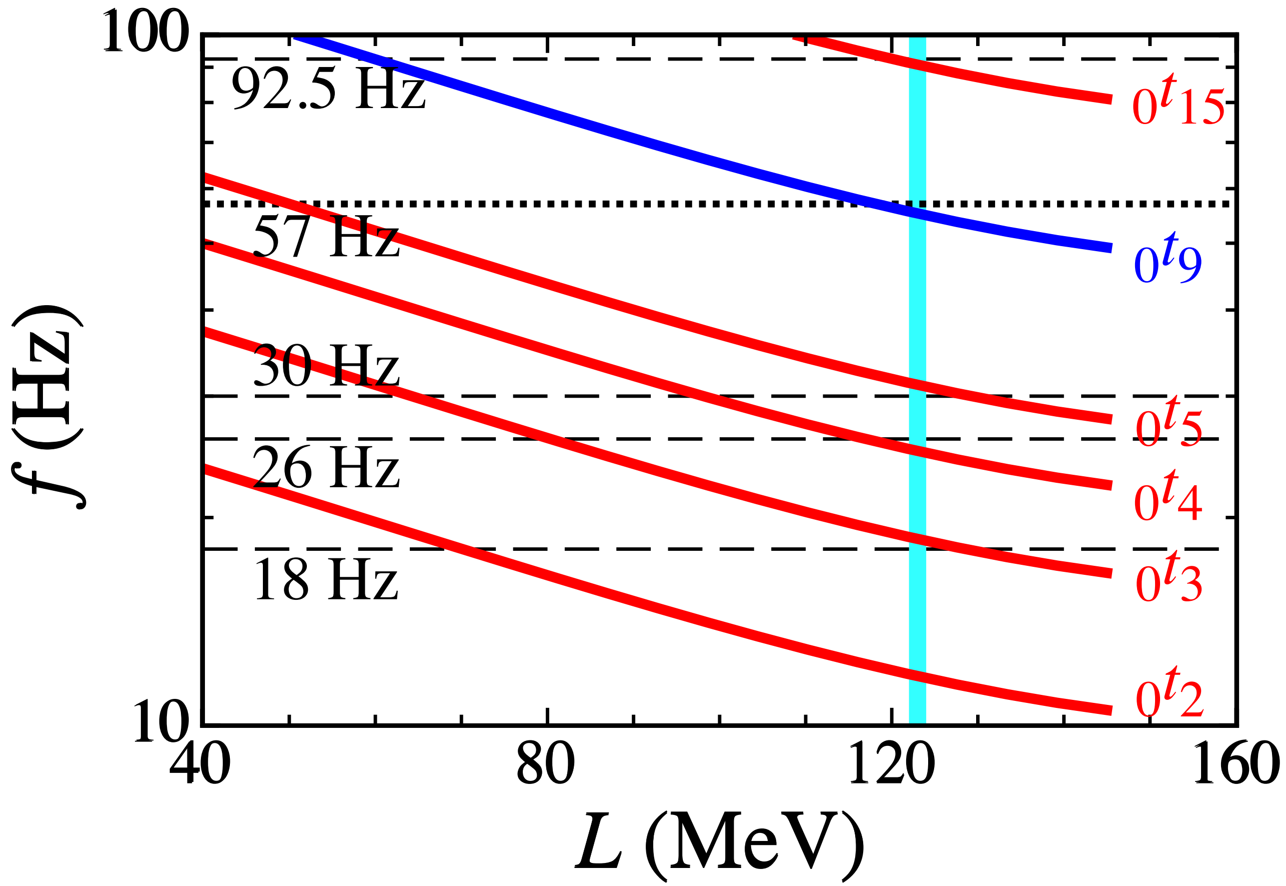


➡ 90.5 MeV ≤ L ≤ 131.0 MeV

Identifications of SGR 1806-20

- for R = 12 km and M = 1.4$M_\odot$



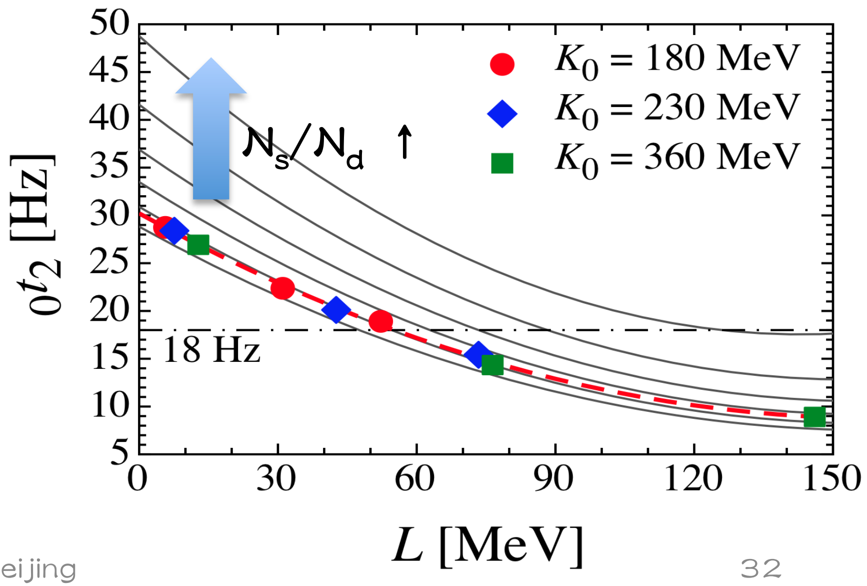
Effect of superfluidity

HS+2012b

- For $\rho \gtrsim 4 \times 10^{11}$ g cm^{-3}, neutron could drip from nuclei
- Some of dripped neutron play a role as superfluid
- Effective enthalpy affecting on the shear oscillations could be reduced
  - shear speed ($v_s^2 \sim \mu/H$) increases due to the effect of superfluidity

$$\mathcal{Y}'' + \left[\left(\frac{4}{r}+\Phi'-\Lambda'\right)+\frac{\mu'}{\mu}\right]\mathcal{Y}' + \left[\boxed{\frac{\epsilon+p}{\mu}}\omega^2 e^{-2\Phi} - \frac{(\ell+2)(\ell-1)}{r^2}\right]e^{2\Lambda}\mathcal{Y} = 0.$$

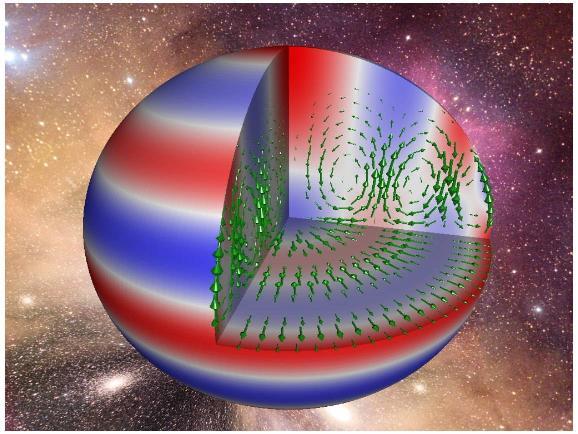
- ${}_0t_l$ could also increase due to the effect of superfluidity
- While, the fraction of superfluid neutron in dripped neutron is still unknown…
  - Chamel (2012): superfluid neutron are not so much (~10-30%?)
- ${}_0t_l$ with using a parameter of N_s/N_d for R=14km & M=1.8$M_\odot$



Oscillations & Instabilities

The most promising strategy for constraining the physics of neutron stars involves observing their "ringing" (oscillation modes)

- **f-mode :** scales with average density
- **p-modes:** probes the sound speed through out the star
- **g-modes :** sensitive to thermal/composition gradients
- **w-modes:** oscillations of spacetime itself.
- **s-modes:** Shear waves in the crust
- **Alfvèn modes:** due to magnetic field
- **i-modes:** inertial modes associated with rotation (r-mode)

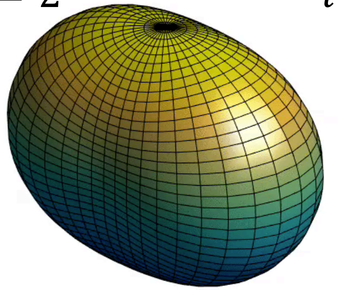


Typically **SMALL AMPLITUDE** oscillations –> weak emission of GWs

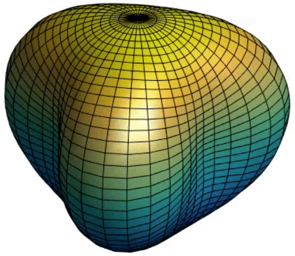
UNLESS

they become **unstable due to rotation** (r-mode & f-mode)

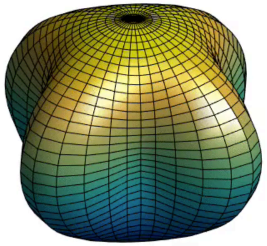
$l = 2, m = 2$



$l = 3, m = 3$



$l = 4, m = 4$



slide by Kokkotas