Isospin properties of quark matter from a 3-flavor NJL model

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Outline:

1. Introduction and theoretical model
2. Isospin dependence of phase diagram
3. Quark matter symmtry energy
4. Applications to hybrid stars
5. Conclusion

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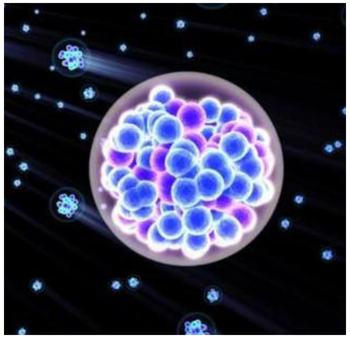
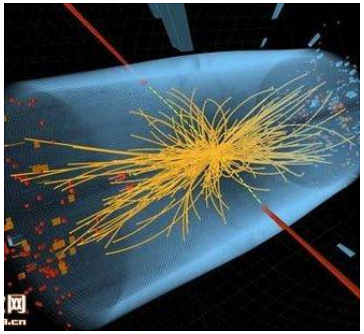
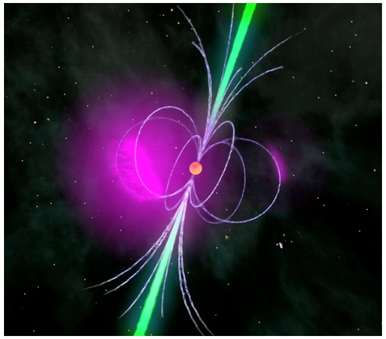
I.Introduction



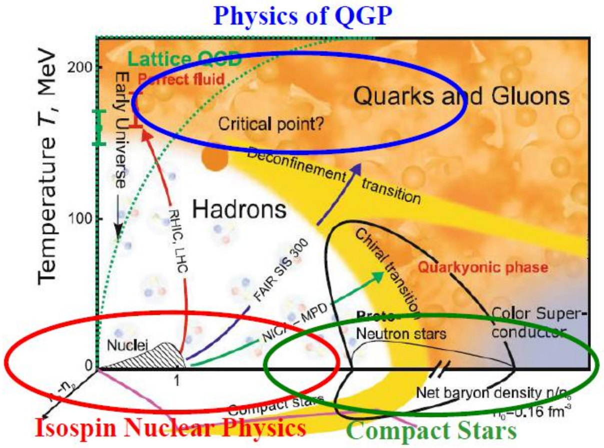
a nuclues



the neutron-rich heavy-ion beams

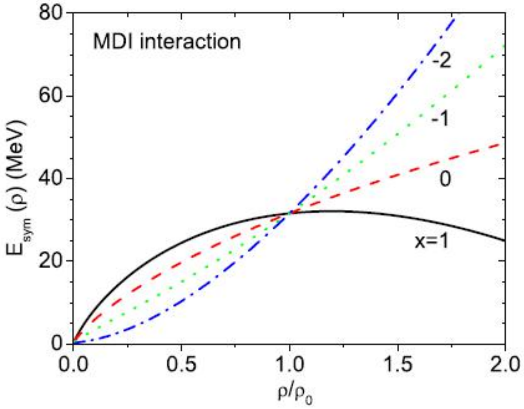


a neutron star



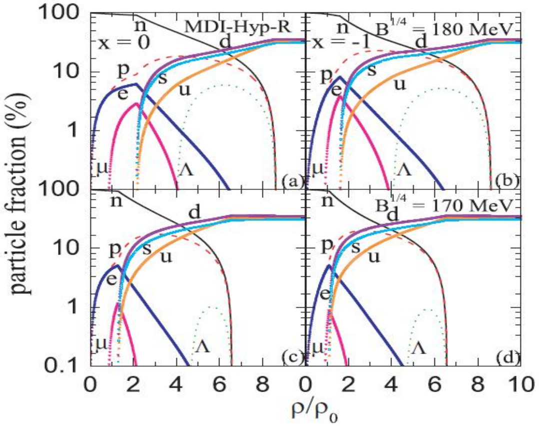
QCD phase diagram in 3D

L.W.Chen et al., PRL (2005)



the symmetry energy

J.Xu et al., PRC (2010)



a hybrid star

II.Theoretical model

The Lagrangian of NJL model

$$\mathscr{L}_{NJL} = \bar{q}(i\partial\!\!\!/ - m)q + \frac{G_S}{2}\sum_{a=0}^{8}[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2] + \mathscr{L}_V + \mathscr{L}_{KMT} + \mathscr{L}_{IS} + \mathscr{L}_{IV}$$

Global Symmetric group(if K=G_{IV}=G_{IS}=0)

$$G = \mathrm{U}_L(3)\times \mathrm{U}_R(3) = \mathrm{SU}_\mathrm{V}(3)\otimes \mathrm{SU}_\mathrm{A}(3)\otimes \mathrm{U}_\mathrm{V}(1)\otimes \mathrm{U}_\mathrm{A}(1)$$

where

$$\mathscr{L}_{KMT} = -K\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\}$$

fit to QCD $U_A(1)$ anomaly

$$\mathscr{L}_V = \frac{G_V}{2}\sum_{a=0}^{8}[(\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_\mu\gamma_5\lambda_a q)^2]$$

$$\mathscr{L}_{IS} = G_{IS}\sum_{a=1}^{3}[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2]$$

$$\mathscr{L}_{IV} = G_{IV}\sum_{a=1}^{3}[(\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_\mu\gamma_5\lambda_a q)^2]$$

the scalar-isovector coupling term

the vector-isovector coupling term

QCD has a global symmetric group $G = \mathrm{SU}_\mathrm{V}(3)\otimes\mathrm{SU}_\mathrm{A}(3)\otimes\mathrm{U}_\mathrm{V}(1)$ and $U_A(1)$ anomal.

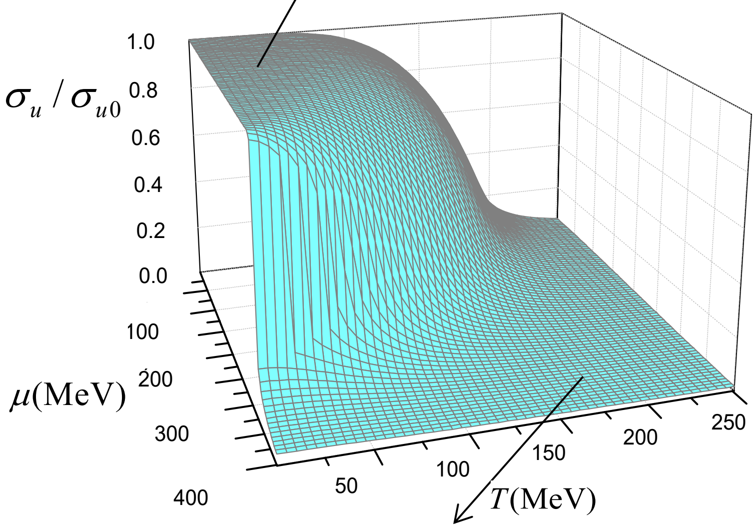
If G_{IV}=G_{IS}=0,NJL model can fit to the symmetric group of QCD.

Mean-Field Approximation

$$(\bar{q}_f q_f)^2 \approx \bar{q}_f q_f \langle \bar{q}_f q_f \rangle - \langle \bar{q}_f q_f \rangle^2 = \sigma_f \bar{q}_f q_f - \sigma_f^{\ 2} \qquad \sigma_f = < \bar{q}_f q_f >$$

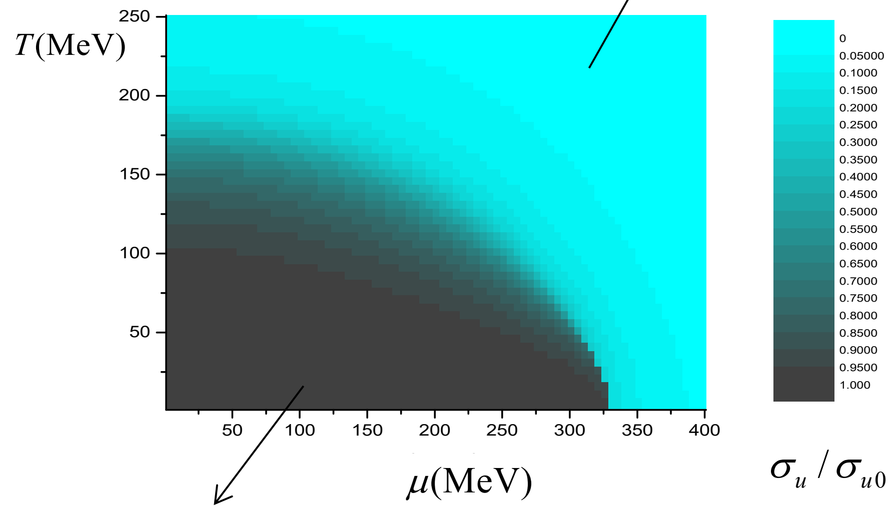
if $\sigma_f \neq 0$,the chiral symmetry will be broken.

chiral symmetry breaking



chiral symmetry restoration

chiral symmetry restoration



chiral symmetry breaking

- **The NJL model can successfully interpret the dynamics of spontaneous breaking of chiral symmetry.**

1.Introduction and theoretical model

Thermodynamic Potential

$$\Omega(T,\mu) = -\frac{T}{V}\ln Z = -\frac{T}{V}\ln Tr e^{-\beta(H-\mu N)}$$

$$\frac{\partial \Omega_{NJL}}{\partial \sigma_u} = \frac{\partial \Omega_{NJL}}{\partial \sigma_d} = \frac{\partial \Omega_{NJL}}{\partial \sigma_s} = 0$$

$$M_i = m_i - 2G_S\sigma_i + 2K\sigma_j\sigma_k - 2G_{IS}\tau_{3i}(\sigma_u - \sigma_d)$$

$$\widetilde{\mu}_i = \mu_i + 2G_V\rho_i + 2G_{IV}\tau_{3i}(\rho_u - \rho_d)$$

$$\sigma_i = -2N_c\int_0^{\Lambda}\frac{d^3p}{(2\pi)^3}\frac{M_i}{E_i}(1 - f_i - \bar{f}_i)$$

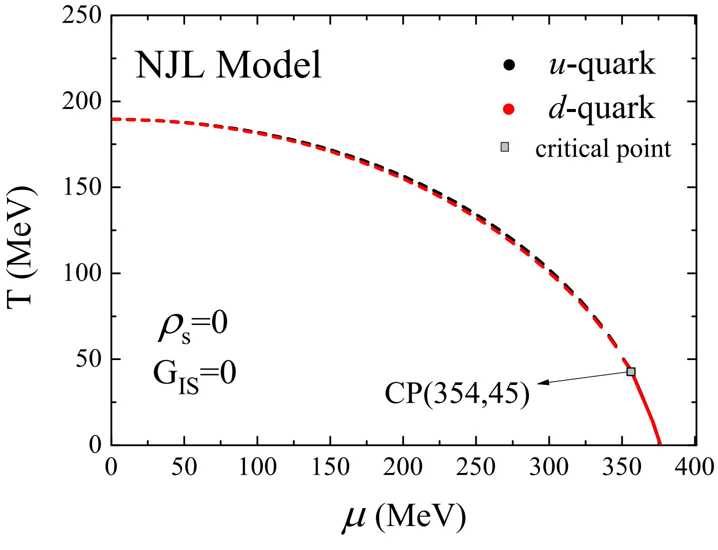
where

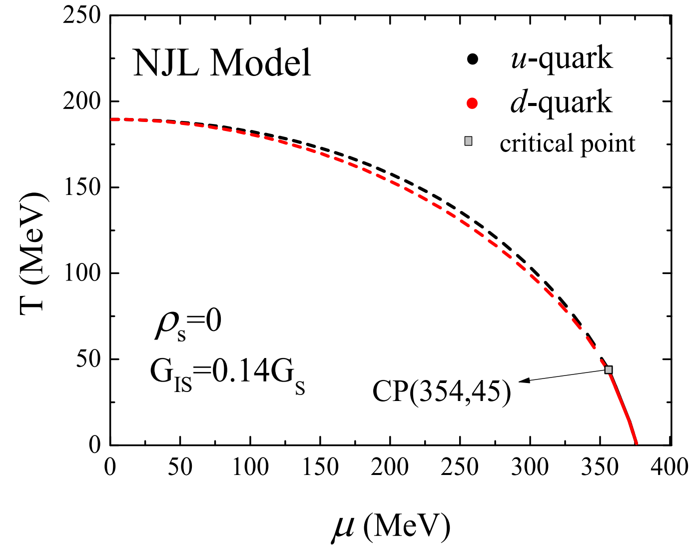
$$f_i = 1/(1 + e^{\beta(E_i - \widetilde{\mu}_i)}) \qquad \bar{f}_i = 1/(1 + e^{\beta(E_i + \widetilde{\mu}_i)})$$

In relativistic heavy-ion collision, the ratio of electric/baryon charge should be fixed and the $\rho_s = 0$ should also be satisfied.

$$\frac{2\rho_u - \rho_d}{\rho_u + \rho_d} = Z / A, \qquad \rho_s = 0.$$

For the Au+Au collision experiment,



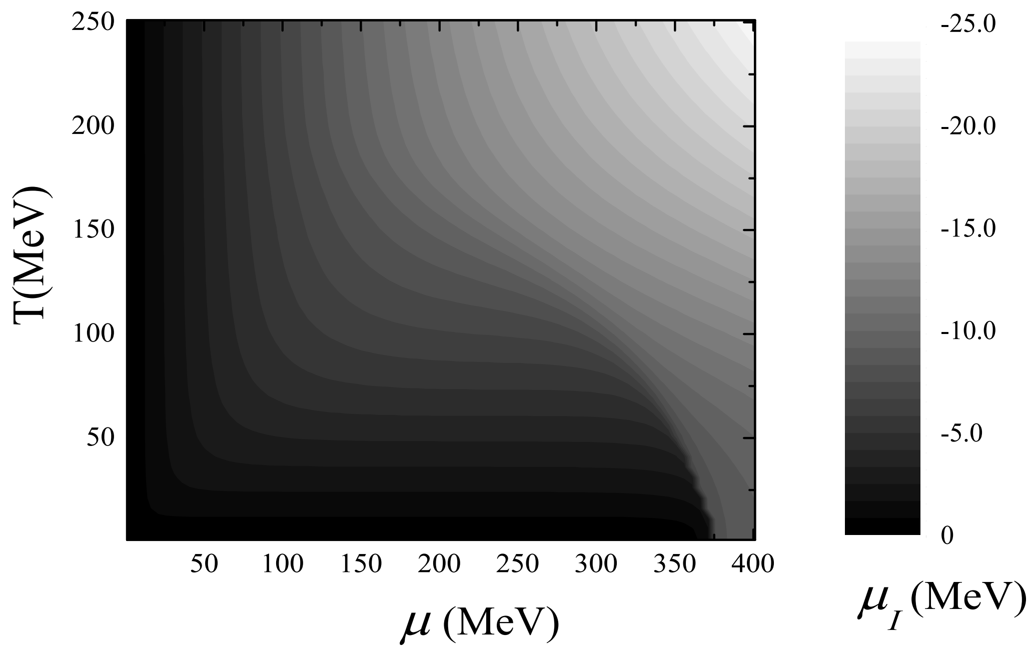


- The chiral phase transitions of u quark and d quark just become to slightly separate and have only one critical point.

2.Isospin dependence of phase diagram

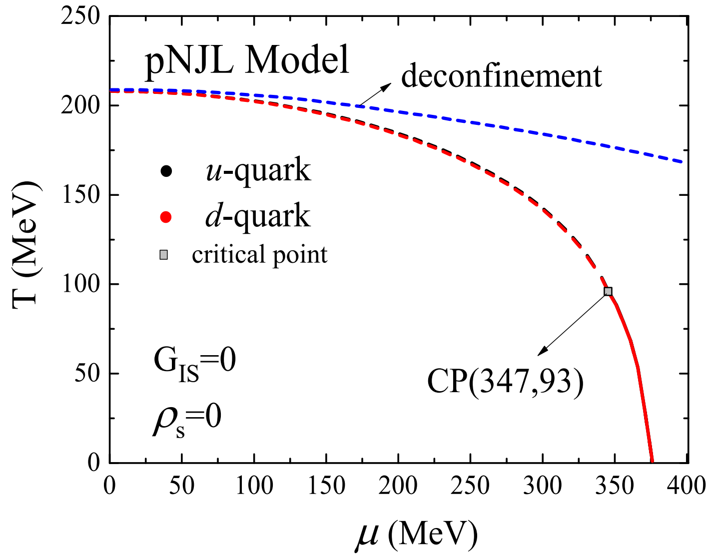
We introduce the chemical potential matrix $\hat{\mu} = diag(\mu_u, \mu_d, \mu_s)$, and

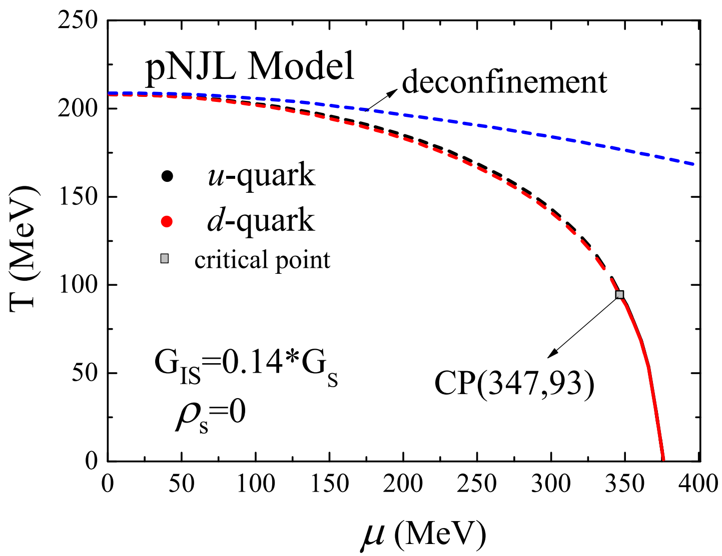
$$\mu_u = \mu_B / 3 + \mu_I,$$
$$\mu_d = \mu_B / 3 - \mu_I, \quad \longrightarrow \quad \mu_I = \frac{\mu_u - \mu_d}{2}$$
$$\mu_s = \mu_B / 3 - \mu_S$$



- The small effect of isopin is due to the small isospin chemical potential near phase transition district at low temperatures.

2.Isospin dependence of phase diagram





- **pNJL results**

$$\mathscr{L}_{pNJL} = \mathscr{L}_{NJL} + \mathcal{U}(\Phi, \bar{\Phi}, T)$$

The potential of Polyakov loop

$$\mathcal{U}(\Phi, \bar{\Phi}, T) = -b \cdot T\{54\, e^{-a/T}\Phi\bar{\Phi} + \ln[1 - 6\Phi\bar{\Phi} - 3(\Phi\bar{\Phi})^2 + 4(\Phi^3 + \bar{\Phi}^3)]\}$$

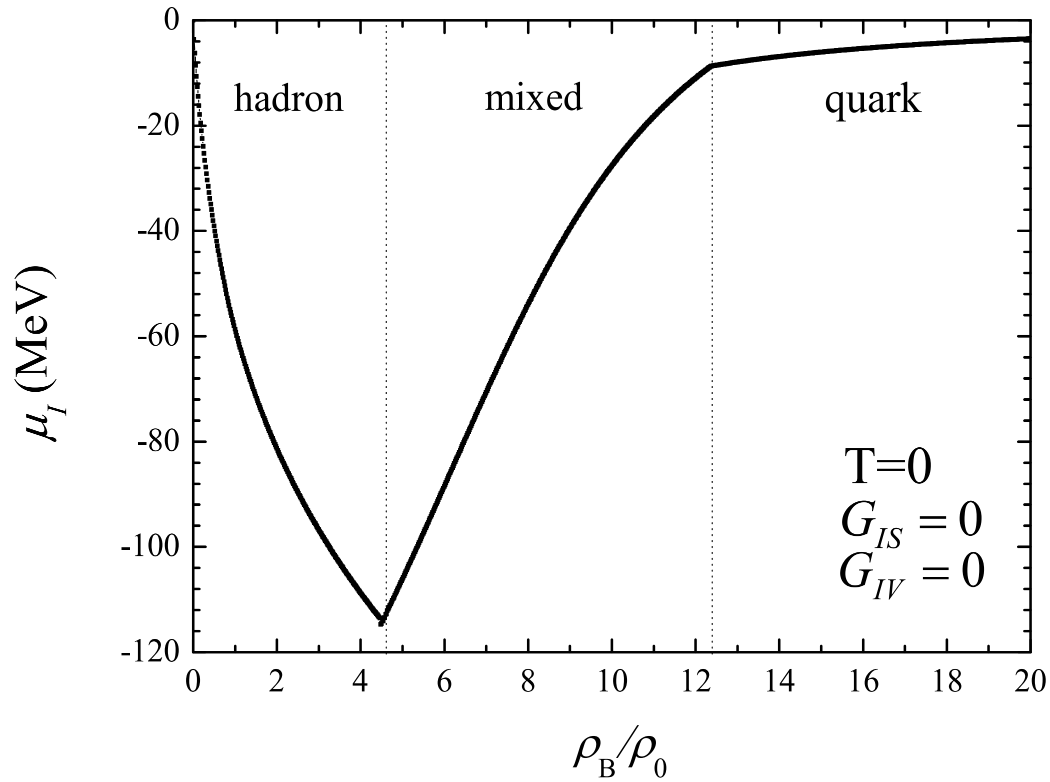
K.Fukushima, PRD (2008)

$\Phi = 1$ **deconfined phase**

$\Phi = 0$ **confined phase**

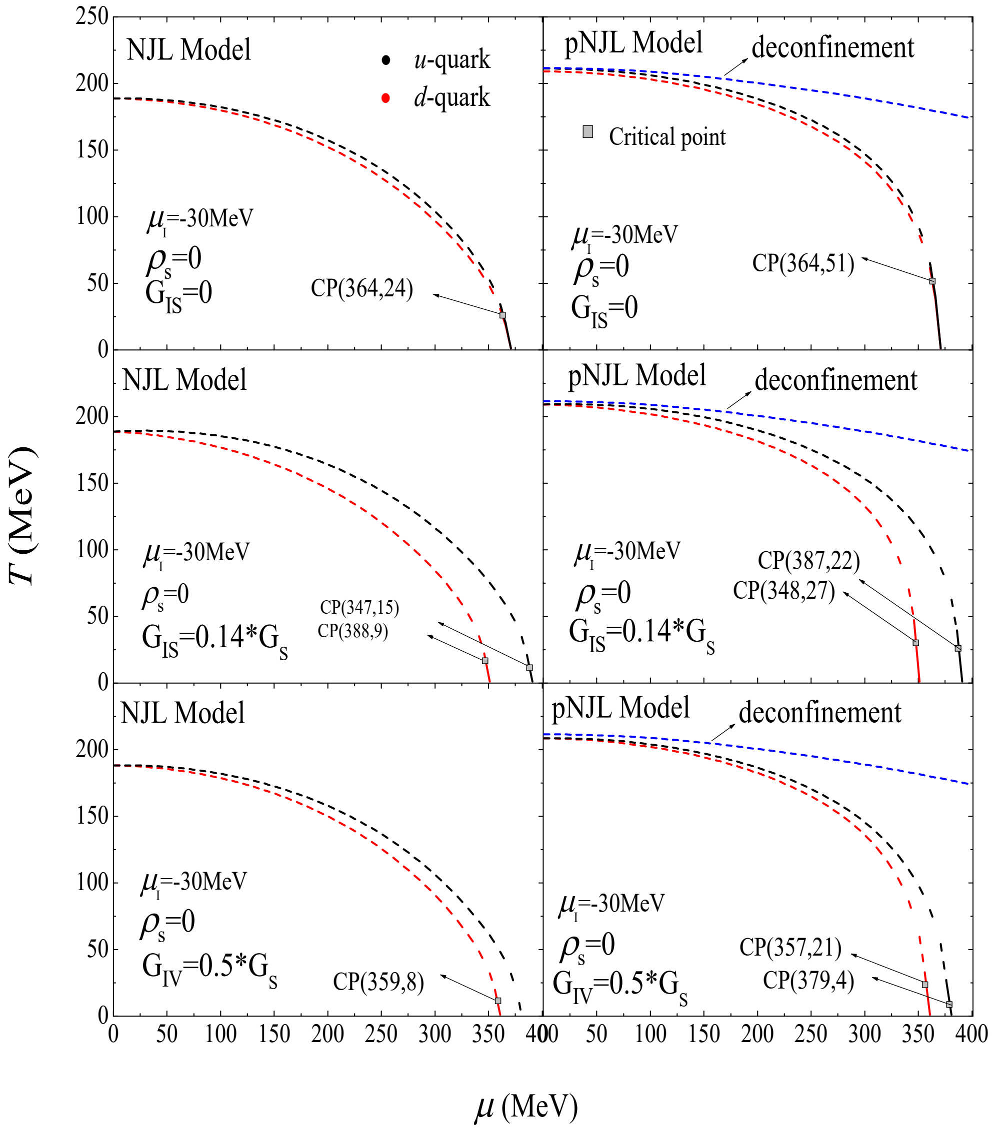
- **The Polyakov loop doesn't strongly affect the isospin dependence of the phase diagram but moves the critical point to higher temperatures.**
- **The phase boundary of deconfinement transition is mostly independent of the isovector coupling constant.**

2.Isospin dependence of phase diagram



- In the relativistic heavy ion collisions, the isospin chemical potential is small near the phase transition region at low temperature, but in the calculation of hybrid stars,we found that there is a large isopin chemical potential in the mixing region.

2.Isospin dependence of phase diagram



- **NJL results (if μ_I = -30 MeV)**

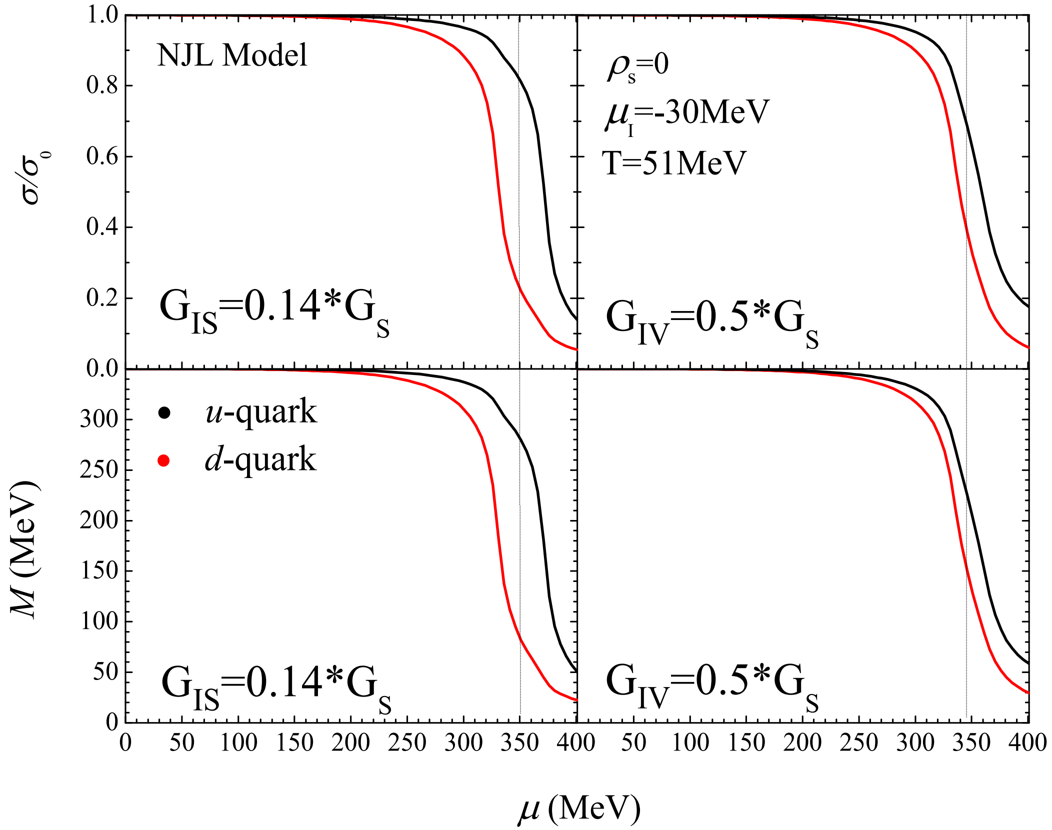
With the increasing scalar-isovector coupling constant, the phase boundaries as well as the critical points of u and d quarks become to separate and their difference reaches the maximum around G_{IS}=0.14G_S.

The isospin splitting of the chiral phase transition boundary also can be observed for positive G_{IV}.

- **pNJL results (if μ_I = -30 MeV)**

There is a similar result from the pNJL model

2.Isospin dependence of phase diagram



$$M_u = m_u - 2G_S\sigma_u + 2K\sigma_d\sigma_s - 2G_{IS}(\sigma_u - \sigma_d)$$
$$M_d = m_d - 2G_S\sigma_d + 2K\sigma_u\sigma_s - 2G_{IS}(\sigma_d - \sigma_u)$$

3.Quark matter symmtry energy

Isospin asymmetric quark matter:

Similar to the case of nuclear matter, the binding energy of quark matter consisting of u, d, and s quarks can be expanded in isospin asymmetry as

$$E(\rho_B,\delta,\rho_s) = E_0(\rho_B,\rho_s) + E_{sym}(\rho_B,\rho_s)\delta^2 + \vartheta(\delta^4) \qquad E(\rho_B,\delta,\rho_s) = \varepsilon/\rho_B$$

where

$$\delta = -\rho_3/\rho_B = 3\frac{\rho_d-\rho_u}{\rho_u+\rho_d} \qquad E_{sym}(\rho_B,\rho_s) = \frac{1}{2!}\frac{\partial^2 E(\rho_B,\delta,\rho_s)}{\partial\delta^2}\bigg|_{\delta=0}$$

can be extracted approximately:

$$E_{sym}(\rho_B,\rho_s) \approx \frac{E(\rho_B,\delta,\rho_s) - E(\rho_B,\delta=0,\rho_s)}{\delta^2}$$

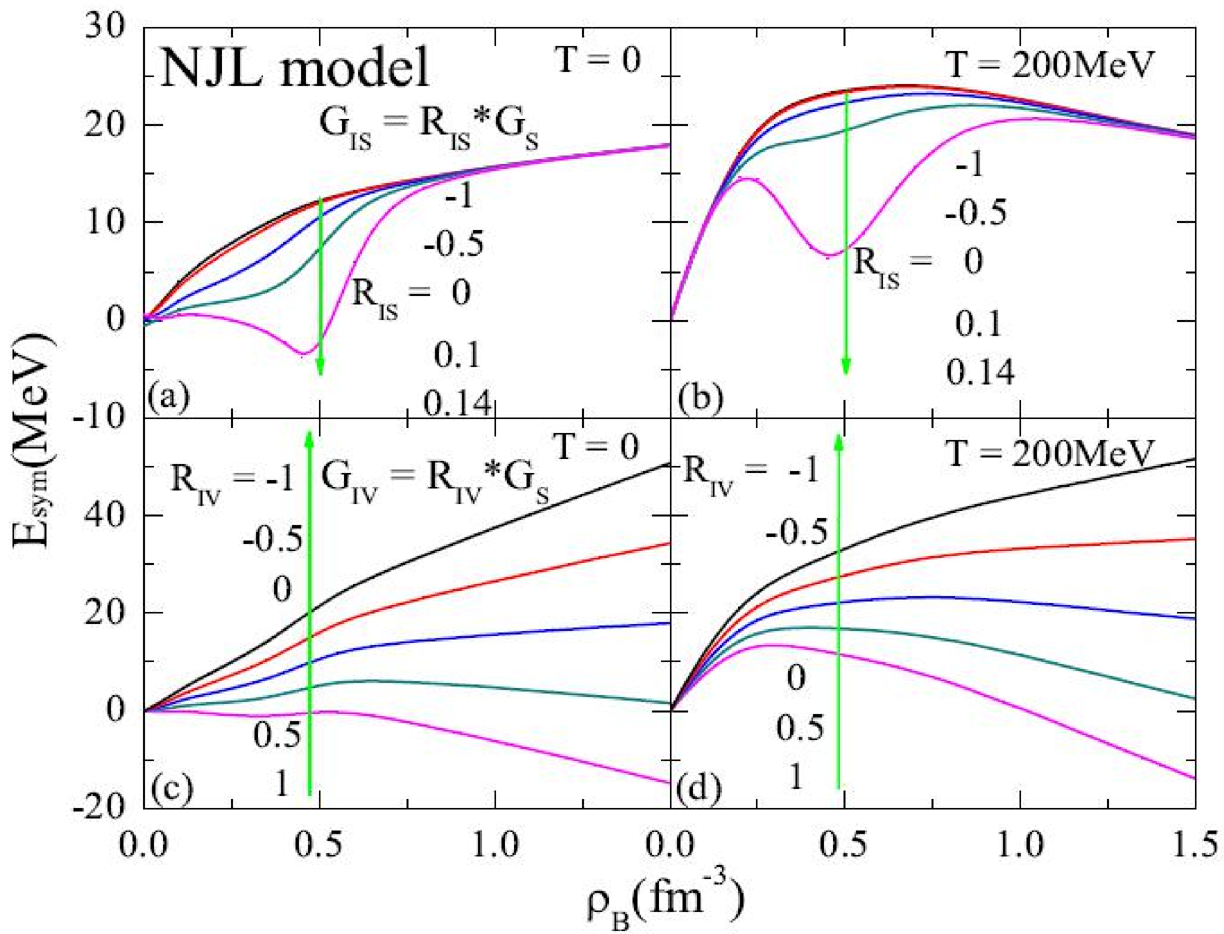
The energy density of the system

$$\varepsilon(\rho_B,\delta,\rho_s) = -\frac{\partial}{\partial\beta}\ln Z + \mu_f\rho_f = \Omega + \beta\frac{\partial}{\partial\beta}\Omega + \mu_f\rho_f$$

$$\varepsilon(\rho_B,\delta,\rho_s) = -2N_C\sum_{i=u,d,s}\int_0^{\Lambda}\frac{dp}{(2\pi)^3}E_i(1-f_i-\bar{f}_i) + G_S(\sigma_u^2+\sigma_d^2+\sigma_s^2) + \sum_i(\tilde{\mu}_i-\mu_i)\rho_i$$
$$+G_V(\rho_u{}^2+\rho_d{}^2+\rho_s{}^2) - 4K\sigma_u\sigma_d\sigma_s + G_{IS}(\sigma_u-\sigma_d)^2 + G_{IV}(\rho_u-\rho_d)^2 - \varepsilon_0$$

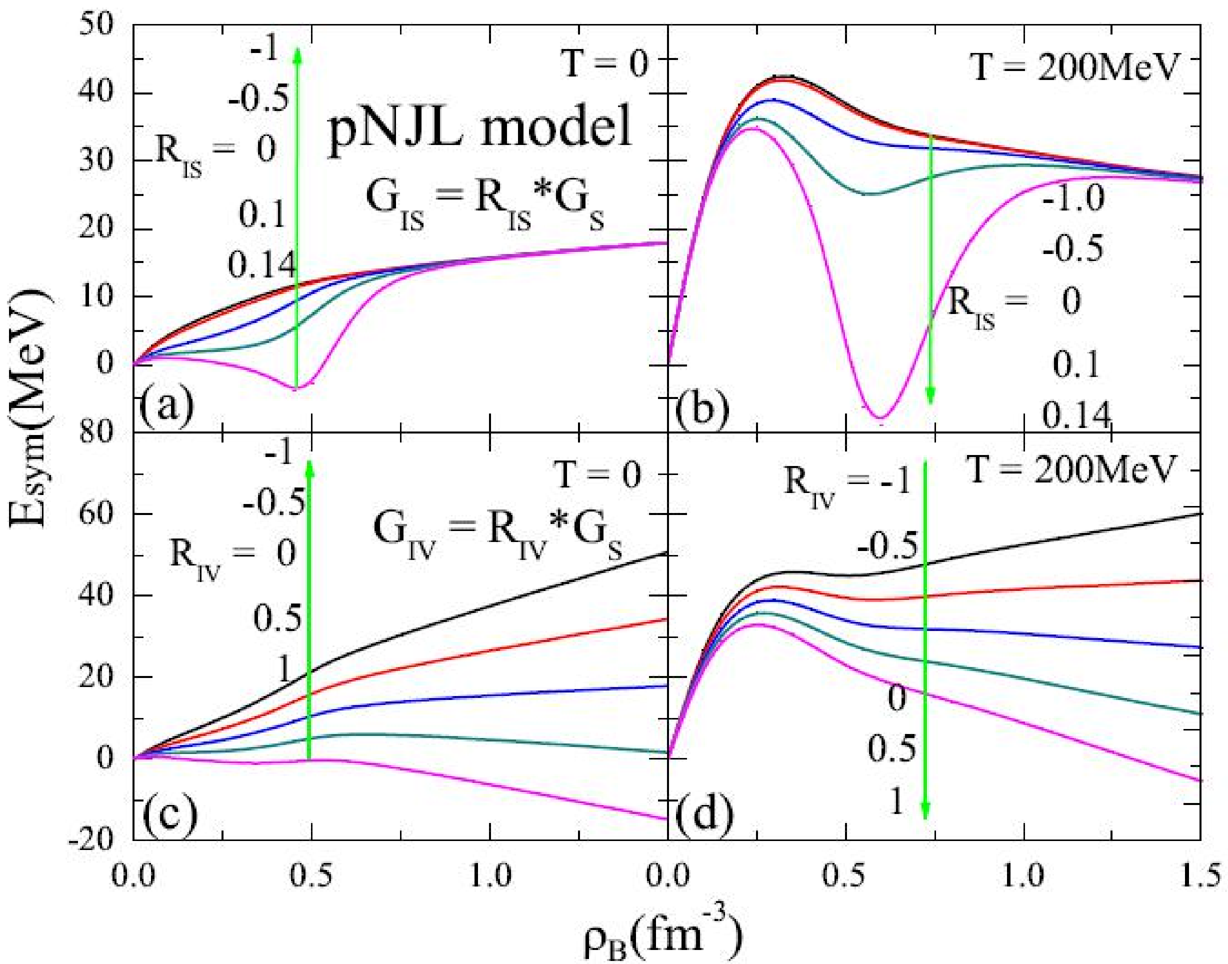
where ε_0 is introduced to ensure $\varepsilon=0$ in the vacuum.

3.Quark matter symmtry energy



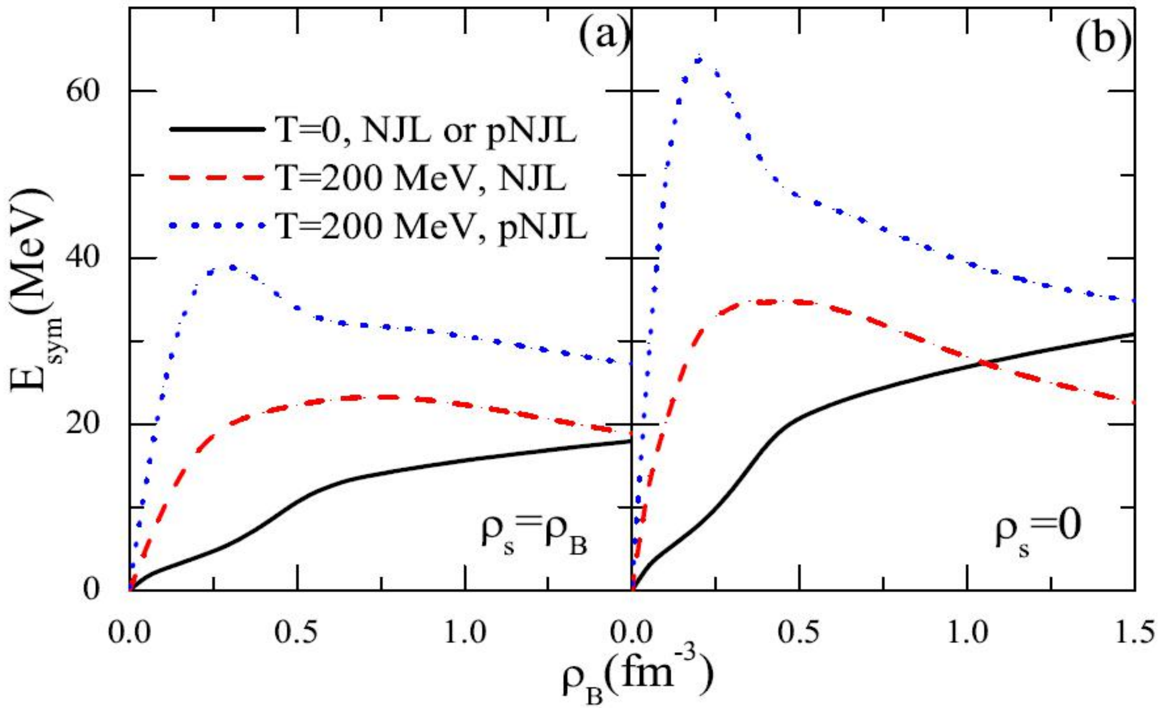
- **Esym decreases with increasing constant of both the scalar-isovector and vector-isovector couplings constants.**

3.Quark matter symmtry energy



- A similar result can be obtained from the pNJL model. A larger isovector coupling constant leads to a smaller symmetry energy.

3.Quark matter symmtry energy



- **The larger quark matter symmetry energy in the pNJL model than in the NJL model is mainly due to their different kinetic energy contributions.**

NJL

$$f_i = 1/(1+e^{\beta(E_i-\tilde{\mu}_i)})$$

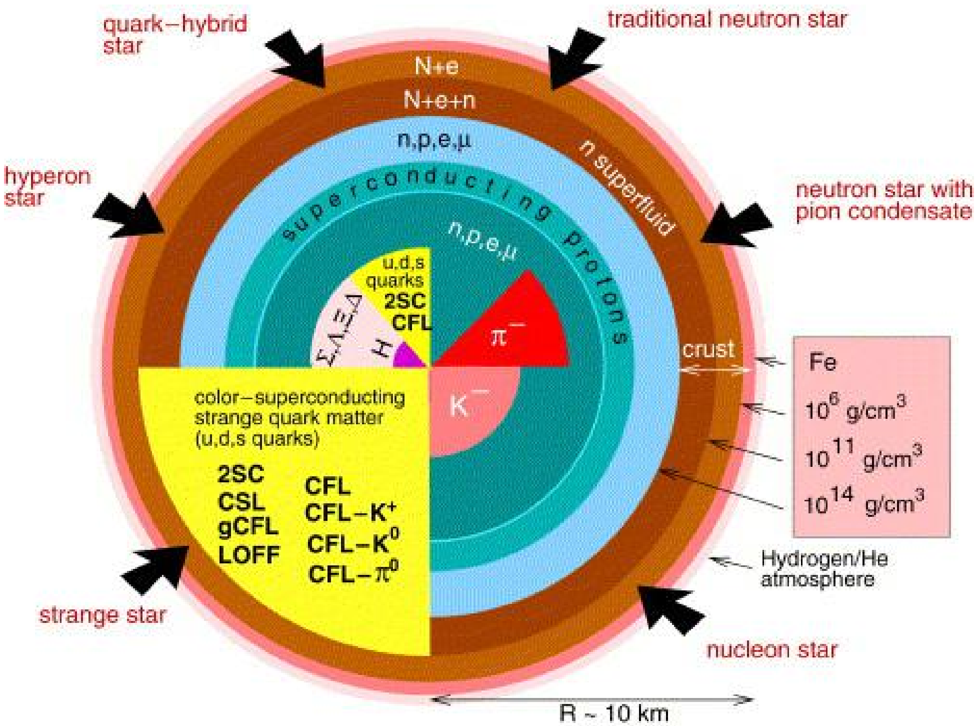
$$\bar{f}_i = 1/(1+e^{\beta(E_i+\tilde{\mu}_i)})$$

pNJL

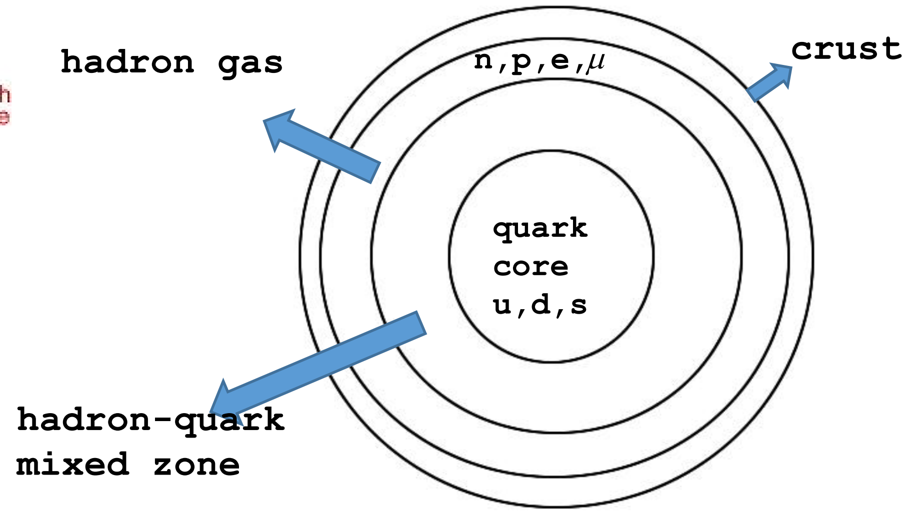
$$F_i = \frac{1+2\bar{\Phi}\xi_i+\Phi\xi_i^2}{1+3\bar{\Phi}\xi_i+3\Phi\xi_i^2+\xi_i^3} \qquad \xi_i = e^{(E_i-\tilde{\mu}_i)/T}$$

$$\bar{F}_i = \frac{1+2\Phi\xi_i'+\bar{\Phi}\xi_i'^2}{1+3\Phi\xi_i'+3\bar{\Phi}\xi_i'^2+\xi_i'^3} \qquad \xi_i' = e^{(E_i+\tilde{\mu}_i)/T}$$

4.Applications to hybrid stars



a schematic presentation of the different structures of neutron stars



quark-hybrid star

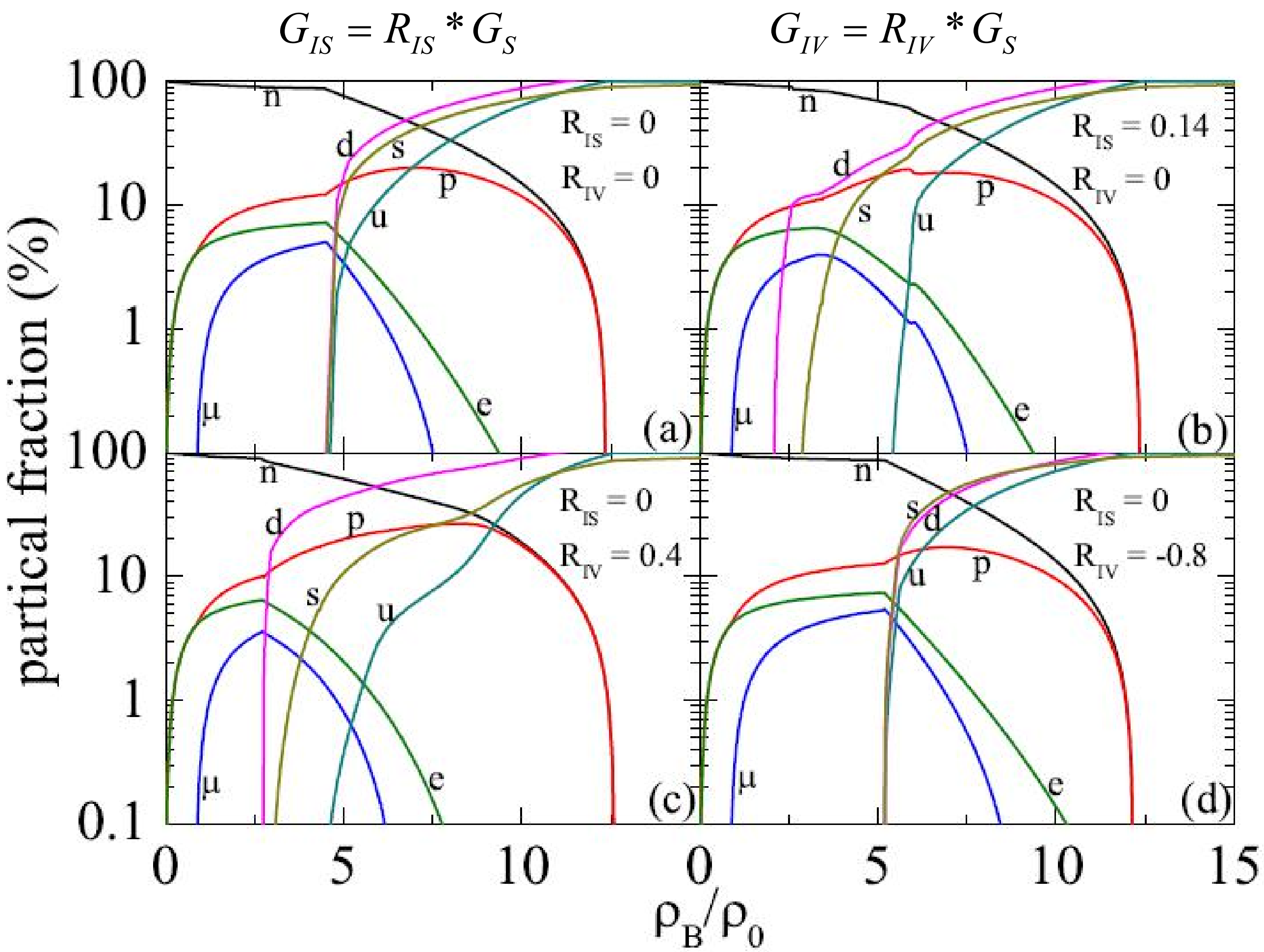
Gibbs condition (two phase equilibrium condition)

$$T^H = T^Q = 0$$

$$P^H = P^Q$$

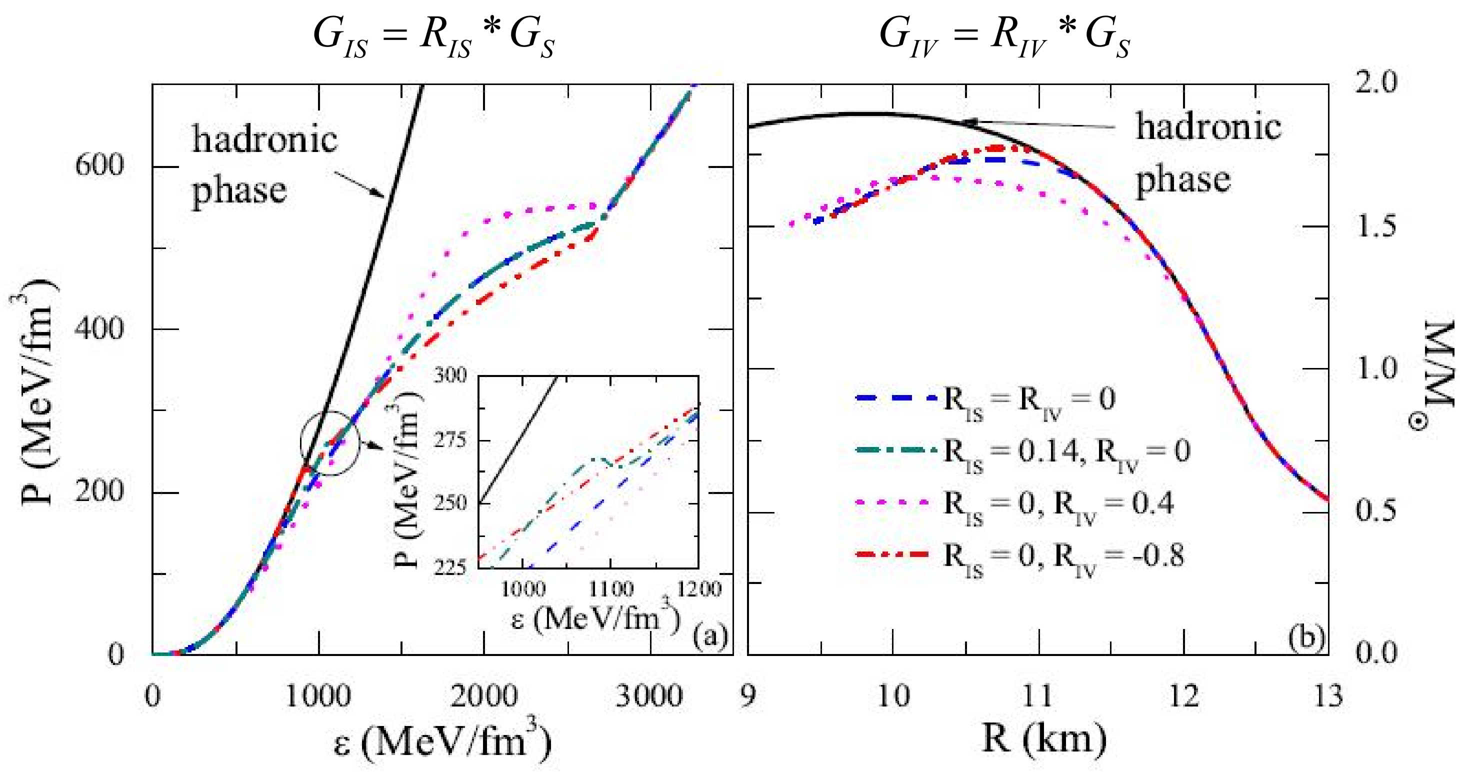
●Baryon number conservation ●Charge neutrality condition ●the β equilibrium condition

4.Applications to hybrid stars



- **For positive isovector coupling constants, quarks of different flavors appear at different densities and their fractions are quite different.**
- **But for the negative vector-isovector coupling constants, the quarks appear at a larger density.**

4.Applications to hybrid stars



- A negative G_{IV} leads to the later appearance of quarks and thus a stiffer EOS at intermediate densities, although at higher densities a positive G_{IV} somehow leads to a larger pressure. As a result, a negative vector-isovector coupling gives the largest maximum mass of hybrid stars.

Conclusion

1.In the relativistic heavy ion collisions, the isospin chemical potential is small near the phase transition region.However,if the isospin chemical potential can reach as large as about 30 MeV, we find that the phase boundaries as well as the critical points of u quark and d quark become to separate with the increasing isovector coupling constant.

2.The isospin splittings of quark condensate, constituent quark mass, and chiral phase transition as well as the critical point are more sensitive to the scalar-isovector coupling, while the quark matter symmetry energy is more sensitive to the vector-isovector coupling.

3.A positive scalar-isovector coupling constant can lead to an unstable isospin asymmetric quark matter and hybrid star matter. The particle fraction as well as the equation of state in hybrid stars depends on the isovector couplings as well.

Thanks for your attention!

Appendix.A

a. the hadron phase

$$\rho_B = \rho_n + \rho_p \longrightarrow$$ **the baryon number conservation** $$\longleftarrow \rho_B = \frac{1}{3}(\rho_u + \rho_d + \rho_s)$$

$$\rho_p = \rho_e + \rho_\mu \rightarrow$$ **the charge neutrality condition** $$\longleftarrow \frac{2}{3}\rho_u - \frac{1}{3}(\rho_d + \rho_s) - \rho_e = 0$$

$$\left.\begin{array}{l}\mu_p = \mu_n - \mu_e \\ \mu_\mu = \mu_e\end{array}\right] \longrightarrow$$ **the β equilibrium condition** $$\longleftarrow \mu_s = \mu_d = \mu_u + \mu_e$$

c. the quark phase (right-hand column above)

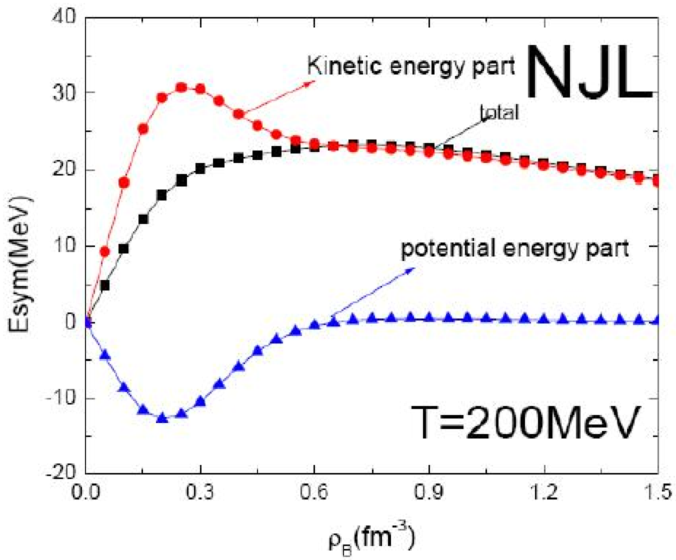
b. the hadron-quark phase transition

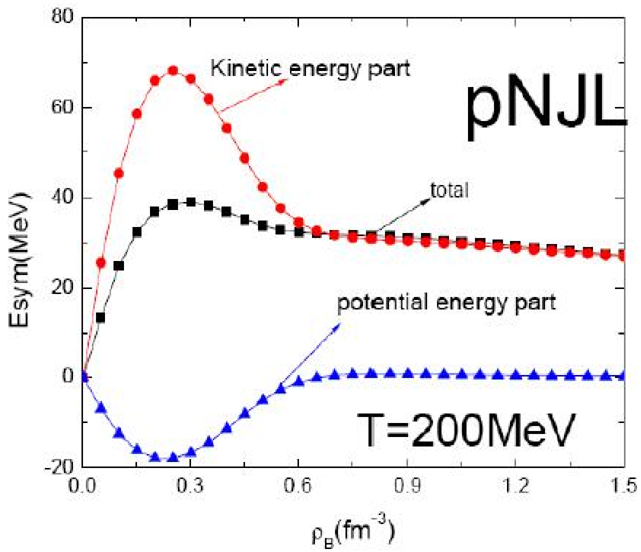
$$\left.\begin{array}{l}T^H = T^Q = 0 \\ P^H = P^Q\end{array}\right]$$ **Gibbs condition (two phase equilibrium condition)**

$$\rho_B = (1-Y)(\rho_n + \rho_p) + \frac{Y}{3}(\rho_u + \rho_d + \rho_s)$$ **the baryon number conservation**

$$\frac{Y}{3}\rho_p + \frac{Y}{3}(2\rho_u - \rho_d - \rho_s) - \rho_e - \rho_\mu = 0$$ **the charge neutrality condition**

$$\left.\begin{array}{l}\mu_u = \frac{1}{3}\mu_n - \frac{2}{3}\mu_e \\ \mu_d = \mu_s = \frac{1}{3}\mu_n + \frac{2}{3}\mu_e\end{array}\right]$$ **the β equilibrium condition**





Kinetic energy part

$$\varepsilon(\rho_B,\delta,\rho_s) = -2N_C \sum_{i=u,d,s} \int_0^{\Lambda} \frac{dp}{(2\pi)^3} E_i(1-f_i-\bar{f}_i) + G_S(\sigma_u^2+\sigma_d^2+\sigma_s^2) + \sum_i (\tilde{\mu}_i-\mu_i)\rho_i$$

$$+G_V(\rho_u{}^2+\rho_d{}^2+\rho_s{}^2) - 4K\sigma_u\sigma_d\sigma_s + G_{IS}(\sigma_u-\sigma_d)^2 + G_{IV}(\rho_u-\rho_d)^2 - \varepsilon_0$$

NJL $f_i = 1/(1+e^{\beta(E_i-\tilde{\mu}_i)})$

$\bar{f}_i = 1/(1+e^{\beta(E_i+\tilde{\mu}_i)})$

pNJL $F_i = \frac{1+2\bar{\Phi}\xi_i+\Phi\xi_i{}^2}{1+3\bar{\Phi}\xi_i+3\Phi\xi_i{}^2+\xi_i{}^3}$ $\xi_i = e^{(E_i-\tilde{\mu}_i)/T}$

$\bar{F}_i = \frac{1+2\Phi\xi_i'+\bar{\Phi}\xi_i'^2}{1+3\Phi\xi_i'+3\bar{\Phi}\xi_i'^2+\xi_i'^3}$ $\xi_i' = e^{(E_i+\tilde{\mu}_i)/T}$