Isospin properties of quark matter from a 3-flavor NJL model

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Outline:
1. Introduction and theoretical model
2. Isospin dependence of phase diagram
3. Quark matter symmetry energy
4. Applications to hybrid stars
5. Conclusion
1. Introduction and theoretical model

I. Introduction

- a nuclues
- the neutron-rich heavy-ion beams
- a neutron star

- QCD phase diagram in 3D
- the symmetry energy
- a hybrid star

L.W. Chen et al., PRL (2005)
J.Xu et al., PRC (2010)
II. Theoretical model

The Lagrangian of NJL model

\[ \mathcal{L}_{NJL} = \bar{q}(i\partial - m)q + \frac{G_S}{2} \sum_{a=0}^{8} \left[ (\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2 \right] + \mathcal{L}_V + \mathcal{L}_{KMT} + \mathcal{L}_{IS} + \mathcal{L}_{IV} \]

Global Symmetric group (if \( K=G_{IV}=G_{IS}=0 \))

\[ G = U_L(3) \times U_R(3) = SU_V(3) \otimes SU_A(3) \otimes U_V(1) \otimes U_A(1) \]

where

\[ \mathcal{L}_{KMT} = -K \{ \det [ \bar{q}(1 + \gamma_5)q] + \det [ \bar{q}(1 - \gamma_5)q] \} \quad \text{fit to QCD } U_A(1) \text{ anomaly} \]

\[ \mathcal{L}_V = \frac{G_V}{2} \sum_{a=0}^{8} \left[ (\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_\mu\gamma_5\lambda_a q)^2 \right] \]

\[ \mathcal{L}_{IS} = G_{IS} \sum_{a=1}^{3} \left[ (\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2 \right] \quad \text{the scalar-isovector coupling term} \]

\[ \mathcal{L}_{IV} = G_{IV} \sum_{a=1}^{3} \left[ (\bar{q}\gamma_\mu\lambda_a q)^2 + (\bar{q}\gamma_\mu\gamma_5\lambda_a q)^2 \right] \quad \text{the vector-isovector coupling term} \]

QCD has a global symmetric group \( G = SU_V(3) \otimes SU_A(3) \otimes U_V(1) \) and \( U_A(1) \) anom. If \( G_{IV}=G_{IS}=0 \), NJL model can fit to the symmetric group of QCD.
1. Introduction and theoretical model

Mean-Field Approximation

\[(\bar{q}f q_f)^2 \approx \bar{q}f q_f \langle \bar{q}f q_f \rangle - \langle \bar{q}f q_f \rangle^2 = \sigma_f \bar{q}f q_f - \sigma_f^2 \quad \sigma_f = \langle \bar{q}f q_f \rangle\]

if \(\sigma_f \neq 0\), the chiral symmetry will be broken.

- The NJL model can successfully interpret the dynamics of spontaneous breaking of chiral symmetry.
1. Introduction and theoretical model

Thermodynamic Potential

\[ \Omega(T, \mu) = - \frac{T}{V} \ln Z = - \frac{T}{V} \ln Tr e^{-\beta(H - \mu N)} \]

\[
\frac{\partial \Omega_{NJL}}{\partial \sigma_u} = \frac{\partial \Omega_{NJL}}{\partial \sigma_d} = \frac{\partial \Omega_{NJL}}{\partial \sigma_s} = 0
\]

\[ M_i = m_i - 2G_S \sigma_i + 2K \sigma_j \sigma_k - 2G_{IS} \tau_{3i} (\sigma_u - \sigma_d) \]

\[ \tilde{\mu}_i = \mu_i + 2G_V \rho_i + 2G_{IV} \tau_{3i} (\rho_u - \rho_d) \]

\[ \sigma_i = -2N_c \int_0^\Lambda \frac{d^3 p}{(2\pi)^3} \frac{M_i}{E_i} (1 - f_i - \bar{f}_i) \]

where

\[ f_i = 1 / (1 + e^{\beta(E_i - \bar{\mu}_i)}) \]

\[ \bar{f}_i = 1 / (1 + e^{\beta(E_i + \bar{\mu}_i)}) \]
2. Isospin dependence of phase diagram

In relativistic heavy-ion collision, the ratio of electric/baryon charge should be fixed and the $\rho_s = 0$ should also be satisfied.

$$\frac{2 \rho_u - \rho_d}{\rho_u + \rho_d} = Z / A,$$

$\rho_s = 0.$

For the Au+Au collision experiment,

- The chiral phase transitions of $u$ quark and $d$ quark just become to slightly separate and have only one critical point.
2. Isospin dependence of phase diagram

We introduce the chemical potential matrix $\hat{\mu} = \text{diag}(\mu_u, \mu_d, \mu_s)$, and

$$\begin{align*}
\mu_u &= \mu_B / 3 + \mu_I, \\
\mu_d &= \mu_B / 3 - \mu_I, \\
\mu_s &= \mu_B / 3 - \mu_S
\end{align*}$$

$$\mu_I = \frac{\mu_u - \mu_d}{2}$$

The small effect of isospin is due to the small isospin chemical potential near phase transition district at low temperatures.
2. Isospin dependence of phase diagram

- **pNJL results**

\[ \mathcal{L}_{pNJL} = \mathcal{L}_{NJL} + \mathcal{U}(\Phi, \bar{\Phi}, T) \]

The potential of Polyakov loop

\[ \mathcal{U}(\Phi, \bar{\Phi}, T) = -b \cdot T \{54 e^{-a/T} \Phi \bar{\Phi} + \ln[1 - 6 \Phi \bar{\Phi} - 3(\Phi \bar{\Phi})^2 + 4(\Phi^3 + \bar{\Phi}^3)]\} \]

K. Fukushima, PRD (2008)

- \( \Phi = 1 \) \quad \text{deconfined phase}
- \( \Phi = 0 \) \quad \text{confined phase}

- The Polyakov loop doesn't strongly affect the isospin dependence of the phase diagram but moves the critical point to higher temperatures.

- The phase boundary of deconfinement transition is mostly independent of the isovector coupling constant.
In the relativistic heavy ion collisions, the isospin chemical potential is small near the phase transition region at low temperature, but in the calculation of hybrid stars, we found that there is a large isospin chemical potential in the mixing region.
2. Isospin dependence of phase diagram

With the increasing scalar-isovector coupling constant, the phase boundaries as well as the critical points of u and d quarks become to separate and their difference reaches the maximum around $G_{IS} = 0.14G_S$.

The isospin splitting of the chiral phase transition boundary also can be observed for positive $G_{IV}$.

- **NJL results (if $\mu_I = -30$ MeV)**

- **pNJL results (if $\mu_I = -30$ MeV)**

There is a similar result from the pNJL model.
2. Isospin dependence of phase diagram

\[ M_u = m_u - 2G_S \sigma_u + 2K \sigma_d \sigma_s - 2G_{IS} (\sigma_u - \sigma_d) \]
\[ M_d = m_d - 2G_S \sigma_d + 2K \sigma_u \sigma_s - 2G_{IS} (\sigma_d - \sigma_u) \]
Isospin asymmetric quark matter:

Similar to the case of nuclear matter, the binding energy of quark matter consisting of u, d, and s quarks can be expanded in isospin asymmetry as

$$E(\rho_B, \delta, \rho_s) = E_0(\rho_B, \rho_s) + E_{\text{sym}}(\rho_B, \rho_s)\delta^2 + \mathcal{O}(\delta^4)$$

where

$$\delta = -\frac{\rho_3}{\rho_B} \approx \frac{3\rho_d - \rho_u}{\rho_u + \rho_d}$$

can be extracted approximately:

$$E_{\text{sym}}(\rho_B, \rho_s) \approx \frac{E(\rho_B, \delta, \rho_s) - E(\rho_B, \delta = 0, \rho_s)}{\delta^2}$$

The energy density of the system

$$\varepsilon(\rho_B, \delta, \rho_s) = -\frac{\partial}{\partial \beta} \ln Z + \mu_f \rho_f = \Omega + \beta \frac{\partial \Omega}{\partial \beta} + \mu_f \rho_f$$

$$\varepsilon(\rho_B, \delta, \rho_s) = -2N_C \sum_{i=u,d,s} \int_0^\Lambda \frac{dp}{(2\pi)^3} \frac{E_i(1 - f_i - \bar{f}_i)}{E_i} + G_S (\sigma_u^2 + \sigma_d^2 + \sigma_s^2) + \sum_i (\bar{\mu}_i - \mu_i) \rho_i$$

$$+ G_V (\rho_u^2 + \rho_d^2 + \rho_s^2) - 4K \sigma_u \sigma_d \sigma_s + G_{IS} (\sigma_u - \sigma_d)^2 + G_{IV} (\rho_u - \rho_d)^2 - \varepsilon_0$$

where $\varepsilon_0$ is introduced to ensure $\varepsilon = 0$ in the vacuum.
3. Quark matter symmetry energy

- $E_{\text{sym}}$ decreases with increasing constant of both the scalar-isovector and vector-isovector couplings constants.
A similar result can be obtained from the pNJL model. A larger isovector coupling constant leads to a smaller symmetry energy.
3. Quark matter symmetry energy

- The larger quark matter symmetry energy in the pNJL model than in the NJL model is mainly due to their different kinetic energy contributions.

**NJL**

\[
f_i = \frac{1}{1 + e^{\beta(E_i - \bar{\mu}_i)}}
\]

\[
\bar{f}_i = \frac{1}{1 + e^{\beta(E_i + \bar{\mu}_i)}}
\]

**pNJL**

\[
F_i = \frac{1 + 2\Phi \xi_i + \Phi \xi_i^2}{1 + 3\Phi \xi_i + 3\Phi \xi_i^2 + \xi_i^3}
\]

\[
\bar{F}_i = \frac{1 + 2\Phi \xi'_i + \Phi \xi_i'^2}{1 + 3\Phi \xi'_i + 3\Phi \xi_i'^2 + \xi_i'^3}
\]

\[
\xi_i = e^{(E_i - \bar{\mu}_i)/T}
\]

\[
\xi'_i = e^{(E_i + \bar{\mu}_i)/T}
\]
4. Applications to hybrid stars

A schematic presentation of the different structures of neutron stars

- Baryon number conservation
- Charge neutrality condition
- The \( \beta \) equilibrium condition

Gibbs condition (two phase equilibrium condition)

\[ T^H = T^Q = 0 \]

\[ P^H = P^Q \]
4. Applications to hybrid stars

For positive isovector coupling constants, quarks of different flavors appear at different densities and their fractions are quite different.

But for the negative vector-isovector coupling constants, the quarks appear at a larger density.
A negative $G_{IV}$ leads to the later appearance of quarks and thus a stiffer EOS at intermediate densities, although at higher densities a positive $G_{IV}$ somehow leads to a larger pressure. As a result, a negative vector-isovector coupling gives the largest maximum mass of hybrid stars.
5. Conclusion

Conclusion

1. In the relativistic heavy ion collisions, the isospin chemical potential is small near the phase transition region. However, if the isospin chemical potential can reach as large as about 30 MeV, we find that the phase boundaries as well as the critical points of u quark and d quark become to separate with the increasing isovector coupling constant.

2. The isospin splittings of quark condensate, constituent quark mass, and chiral phase transition as well as the critical point are more sensitive to the scalar-isovector coupling, while the quark matter symmetry energy is more sensitive to the vector-isovector coupling.

3. A positive scalar-isovector coupling constant can lead to an unstable isospin asymmetric quark matter and hybrid star matter. The particle fraction as well as the equation of state in hybrid stars depends on the isovector couplings as well.

Thanks for your attention!
Appendix A

a. the hadron phase

\[ \rho_B = \rho_n + \rho_p \rightarrow \text{the baryon number conservation} \]
\[ \rho_p = \rho_e + \rho_\mu \rightarrow \text{the charge neutrality condition} \]
\[ \mu_p = \mu_n - \mu_e \]
\[ \mu_\mu = \mu_e \]

b. the hadron-quark phase transition

\[ T^H = T^Q = 0 \]
\[ P^H = P^Q \]
\[ \rho_B = (1 - Y)(\rho_n + \rho_p) + \frac{Y}{3}(\rho_u + \rho_d + \rho_s) \]
\[ \frac{Y}{3} \rho_p + \frac{Y}{3}(2\rho_u - \rho_d - \rho_s) - \rho_e - \rho_\mu = 0 \]
\[ \mu_u = \frac{1}{3} \mu_n - \frac{2}{3} \mu_e \]
\[ \mu_d = \mu_s = \frac{1}{3} \mu_n + \frac{2}{3} \mu_e \]

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\[ \varepsilon(\rho_B, \delta, \rho_s) = -2N_C \sum_{i=u,d,s} \int_0^\Lambda \frac{dp}{(2\pi)^3} E_i \left( 1 - f_i - \tilde{f}_i \right) + G_S(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) + \sum_i (\tilde{\mu}_i - \mu_i) \rho_i \]

\[ + G_V(\rho_u^2 + \rho_d^2 + \rho_s^2) - 4K\sigma_u \sigma_d \sigma_s + G_{IS}(\sigma_u - \sigma_d)^2 + G_{IV}(\rho_u - \rho_d)^2 - \varepsilon_0 \]

**NJL**

\[ f_i = \frac{1}{1 + e^{\beta(E_i - \tilde{\mu}_i)}} \]

\[ \tilde{f}_i = \frac{1}{1 + e^{\beta(E_i + \tilde{\mu}_i)}} \]

**pNJL**

\[ F_i = \frac{1 + 2\Phi \tilde{\xi}_i + \Phi \tilde{\xi}_i^2}{1 + 3\Phi \tilde{\xi}_i + 3\Phi \tilde{\xi}_i^2 + \tilde{\xi}_i^3} \]

\[ \tilde{F}_i = \frac{1 + 2\Phi \xi_i' + \Phi \xi_i'^2}{1 + 3\Phi \xi_i' + 3\Phi \xi_i'^2 + \xi_i'^3} \]

\[ \tilde{\xi}_i = e^{(E_i - \tilde{\mu}_i)/T} \]

\[ \xi_i' = e^{(E_i + \tilde{\mu}_i)/T} \]