Investigation of non-local symmetry energy by isospin tracing in HIC

苏军，中山大学，中法核学院
Jun SU
Sino-French Institute of Nuclear Engineering & Technology,
Sun Yat-sen University, Zhuhai 519082, China.
E-mail: sujun04@gmail.com

co-worker:
Feng-Shou Zhang, Beijing Normal University
Wen-Jie Xie, Institute of Modern Physics, Chinese Academy of Sciences
Ching-Yuan Huang, Sun Yat-sen University
Long Zhu, Sun Yat-sen University

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Overview

- ➢ Introduction:
  - Equation of state (EOS) of nuclear matter

    Density dependent, isospin dependent, momentum dependent
  - Investigation of np effective mass splitting by HICs

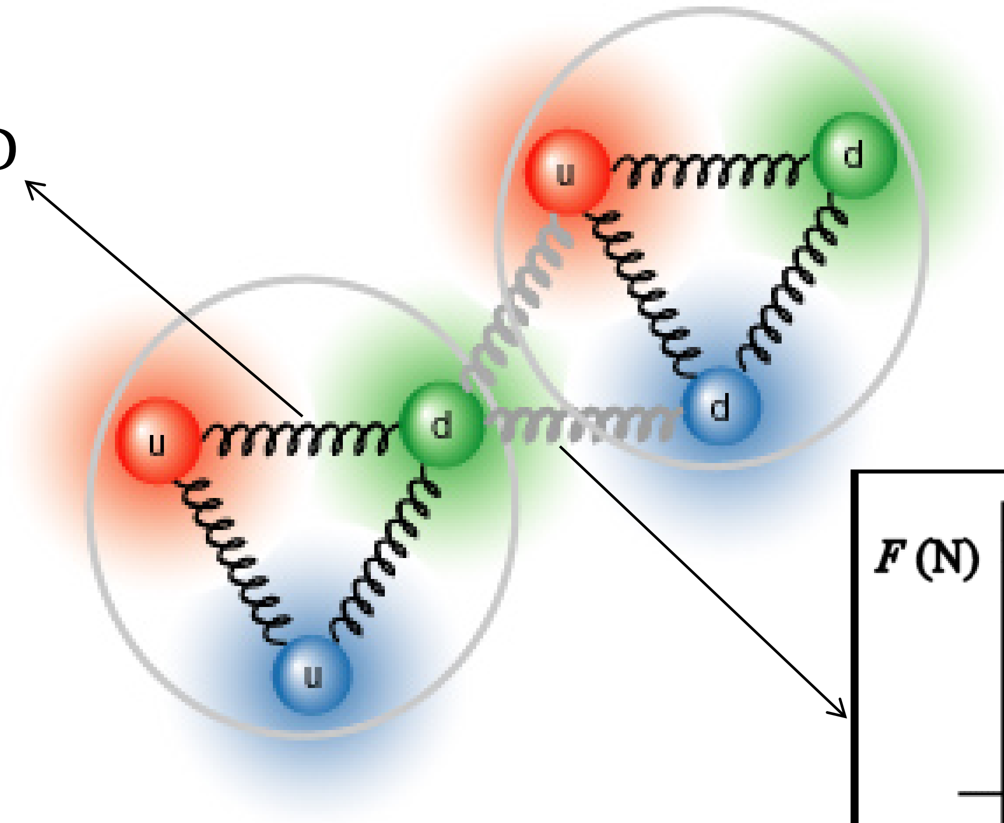
- ➢ Model:
  - Isospin-dependent quantum molecular dynamics
  - Parameters of EOS in this work

- ➢ Results
  - n/p double ratio
  - Isospin tracer

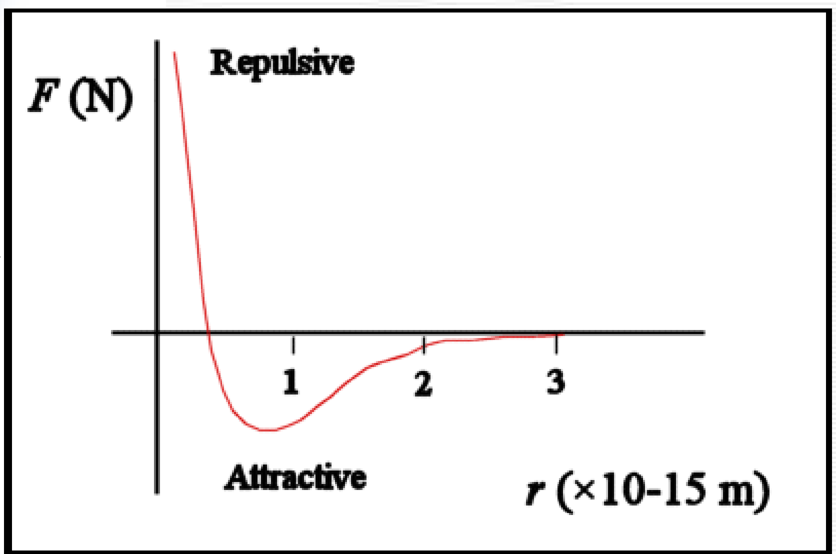
Introduction

- Strong interactions are described by Quantum ChromoDynamics.
- The residual strong interactions between nucleons is of fundamental importance in the understanding of natures asymmetric nuclear objects including supernovae, neutron stars as well as nuclei.

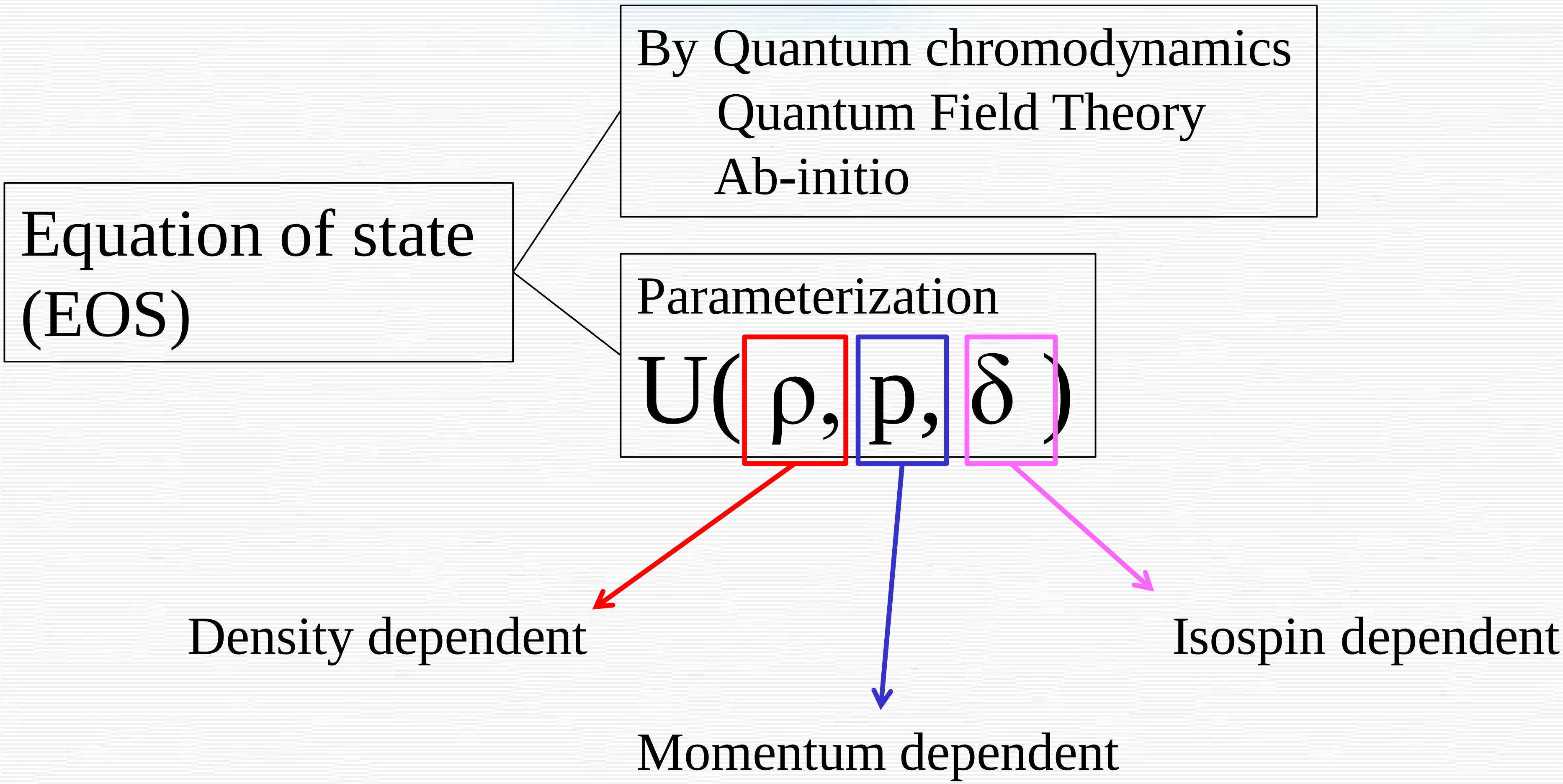
QCD



Parameterization



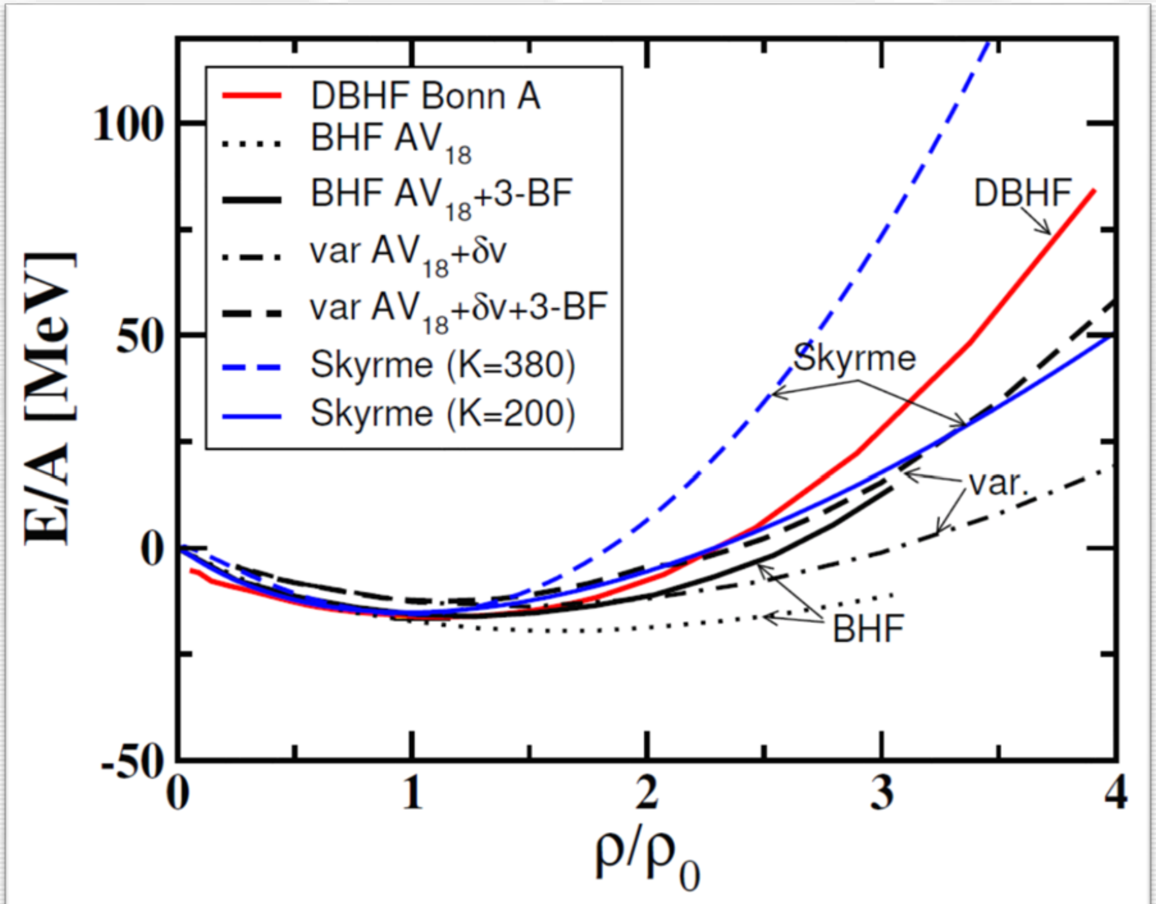
Introduction



Generally speaking EOS can be described by the nuclear potential $U(\rho,p,\delta)$ for nucleon with momentum p in asymmetric nuclear matter with density ρ and isospin asymmetry $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$.

Density dependent EOS

- Knowledge of EOS of symmetric nuclear matter is predicted from microscopic ab initio calculations, i.e. relativistic DBHF, non-relativistic BHF and variational calculations.
- EOS also calculated by modelization, such as Skyrme forces.



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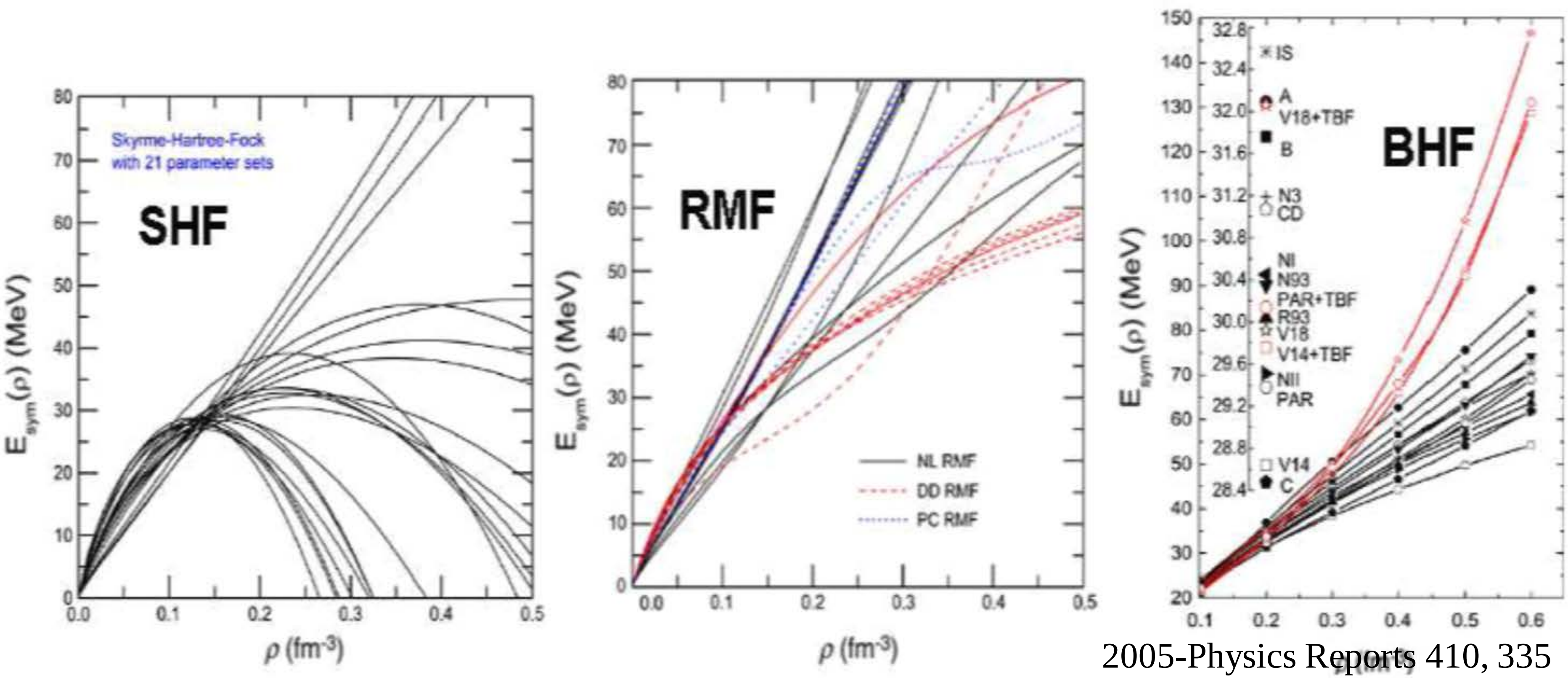
A. Akmal, V.R. Pandharipande, D.G. Ravenhall, Phys. Rev. C 58, 1804 (1998).

Isospin dependent EOS

$$E(\rho,\delta) - E(\rho,\delta=0) = E_{sym}(\rho)\delta^2 + \mathcal{O}\delta^4$$

- A consistently acknowledged picture at normal and subnormal densities ($\rho \leq$ 0.16 fm^{-3}) has been obtained.
- But at supernormal densities, the symmetry energy is not fully understood.



2005-Physics Reports 410, 335
2008-Physics Reports 464, 113

Momentum dependent EOS

- Momentum dependence of EOS has been confirmed by the cross section data of elastic proton-nucleus scattering in 80s.
- The momentum-dependent potential U(p) is parametrized from the measured energy dependence of the proton-nucleus optical potential.

Parameterization in 1987

$U(p) = 1.57\,[\ln^2(1+5\times10^{-4}p^2)]$

1987-PhysRevC.36.2170

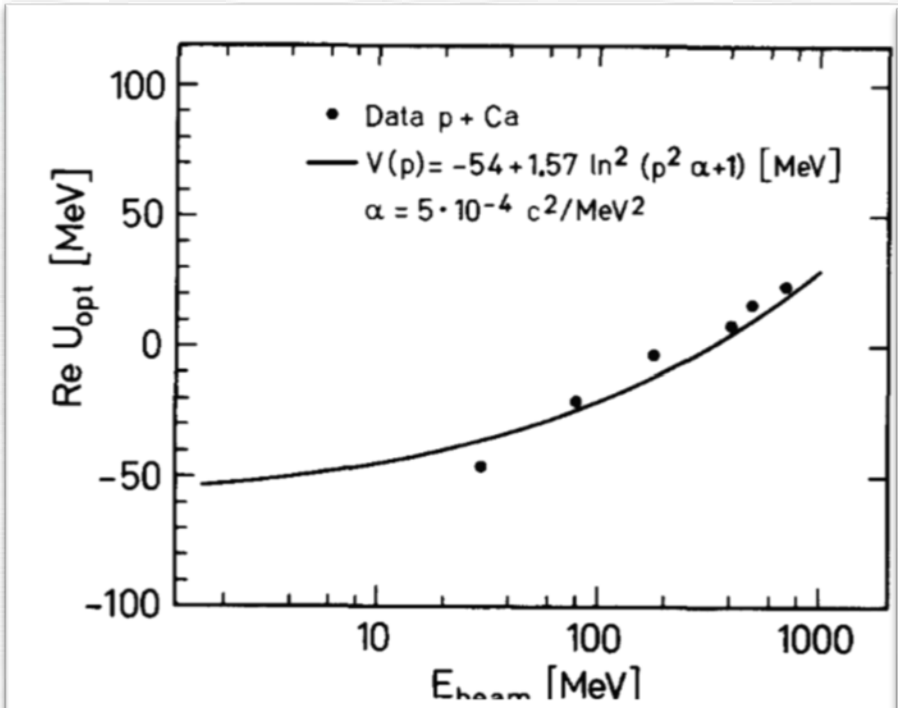
1991-QMD-PR.202.233 1

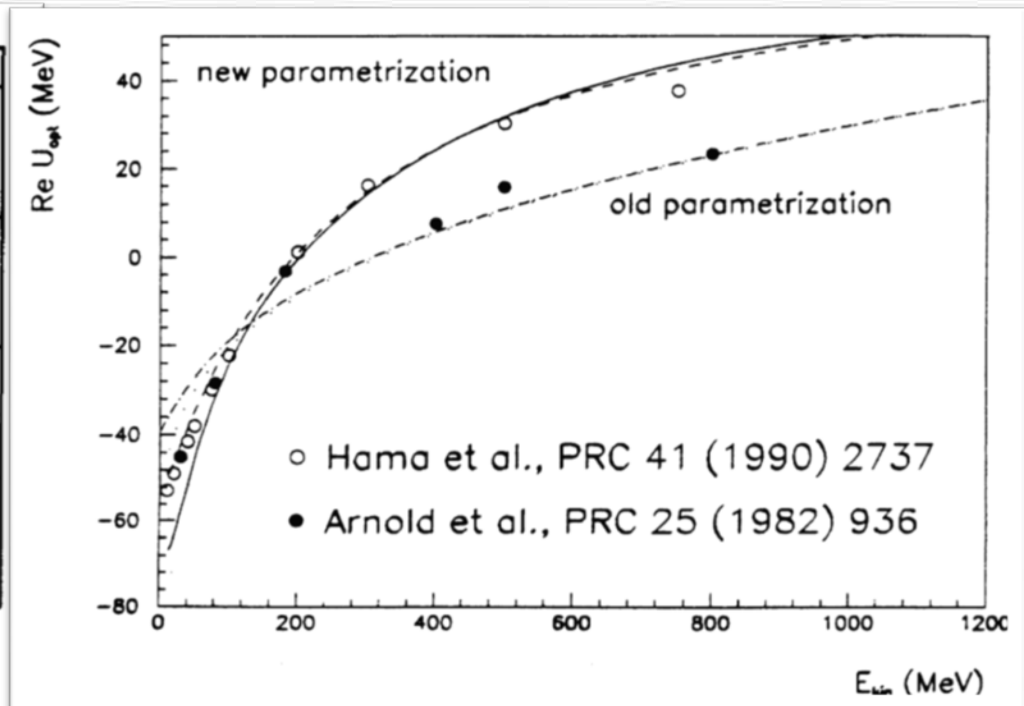
Parameterization in 1993

$U(p) = 0.0667-0.0589/(p^2+0.4837)$

1990-PhysRevC.41.2737

1994-PhysRevC.49.2801

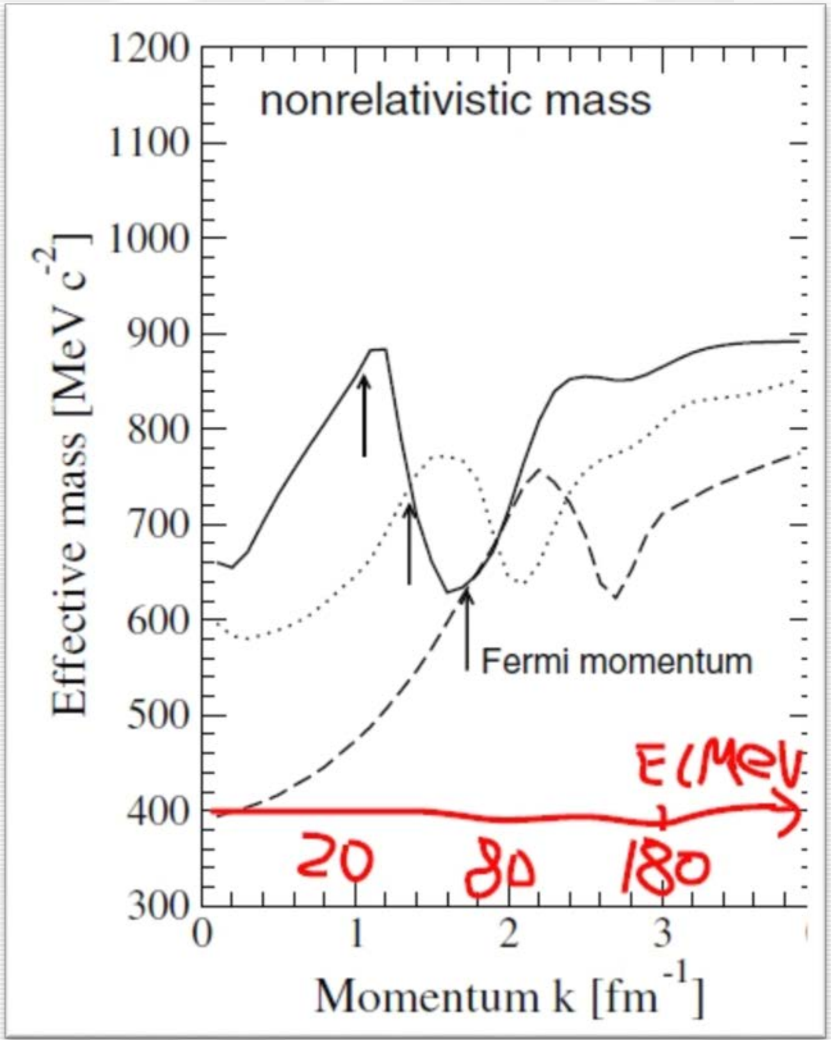




MDI → effective mass

- The gradient of the nuclear potential energy of momentum, i.e. dU/dp, is usually described by a common concept of non-relativistic effective mass m∗.
- Generally speaking the nucleon effective mass in symmetry nuclear medium is well determined, m∗/m = 0.7 ± 0.05, at normal density and Fermi momentum.

$$m_\tau^* = p\left(\frac{dE_\tau}{dp}\right)^{-1} = \left[\frac{1}{m_\tau} + \frac{\partial U_\tau(\rho,\delta,\mathbf{p})}{p\partial p}\right]^{-1}$$



2005-PRL95,022302

Investigation of np effective mass splitting by HICs

- Nuclear collective flows in HICs have been applied to constrain the np effective mass splitting.
- For instance, directed flow protons and nuetrons in semicentral Au+Au collisions at 400 MeV/u have been investigated by the IQMD model. It is found that the neutron–proton differential collective flows are proposed to be a good probe to the neutron–proton effective mass splitting.

Wen-Jie Xie, Feng-Shou Zhang, PLB 735 (2014) 250
P. Danielewicz, et. al, Science 298 (2002) 1592.
Q. Pan, P. Danielewicz, PRL 70 (1993) 2062;
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B.A. Li, et. al., Phys. Rev. Lett. 76 (1996) 4492.
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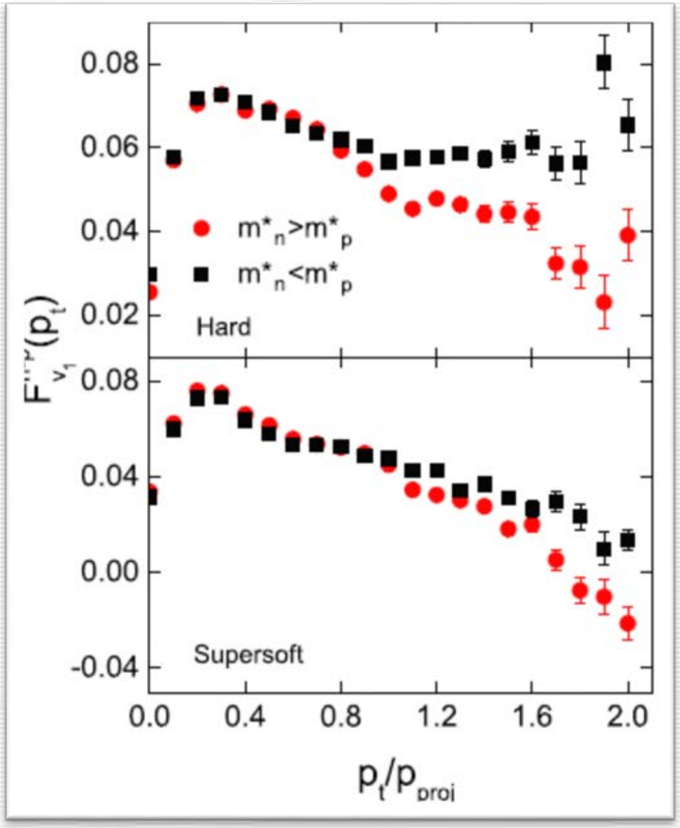
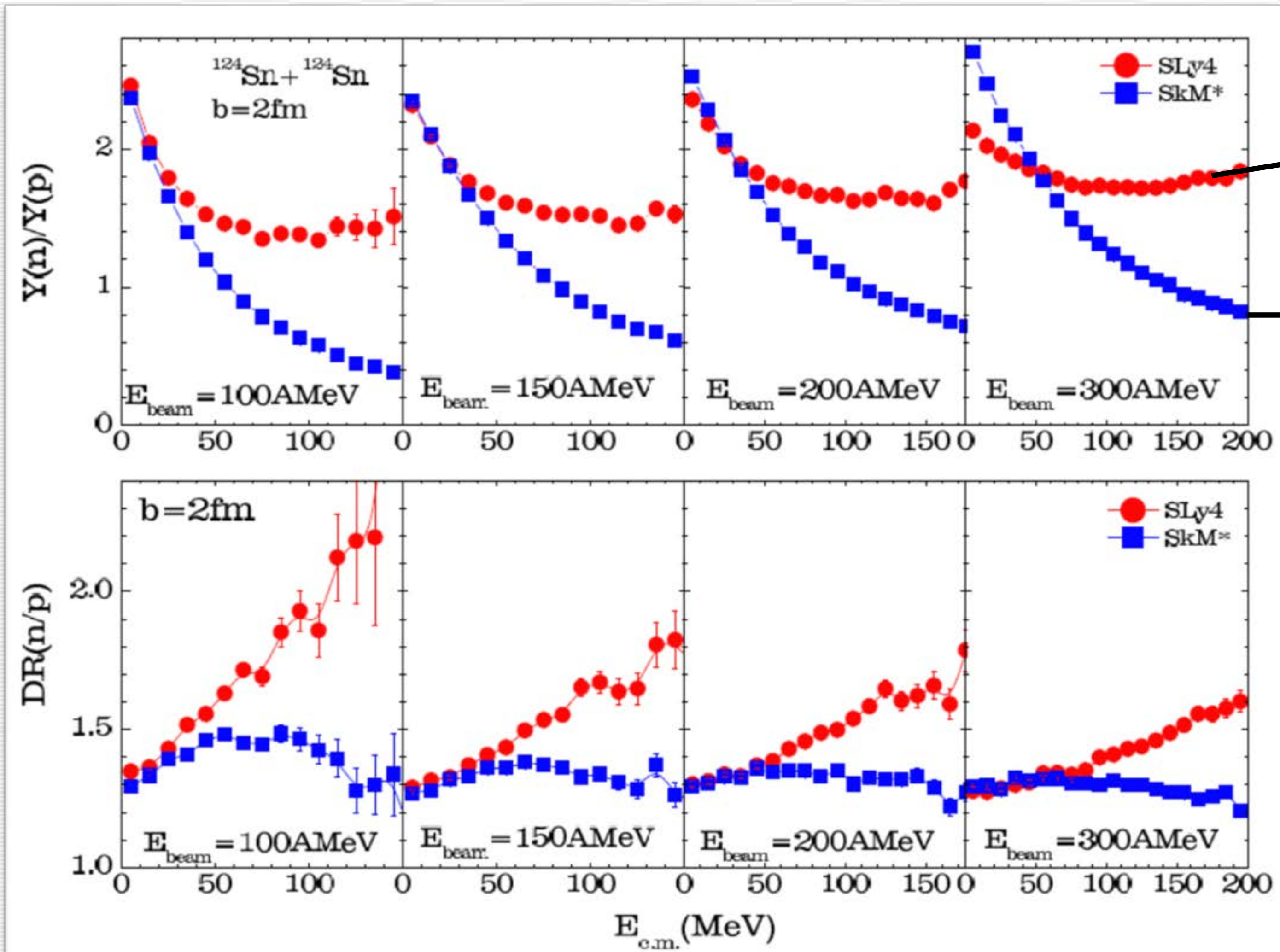
M. Di Toro, et. al., Eur. Phys. J. A 30 (2006) 153.
M.D. Cozma, Phys. Lett. B 700 (2011) 139.
Y. Wang, et. al, PRC 89 (2014) 044603.



Investigation of np effective mass splitting by HICs

- Zhang Yingxun et. al. showed that the high energy neutrons and protons and their ratios from HICs at 100–200 MeV/nucleon, provide a good observable to study the effective mass splitting.



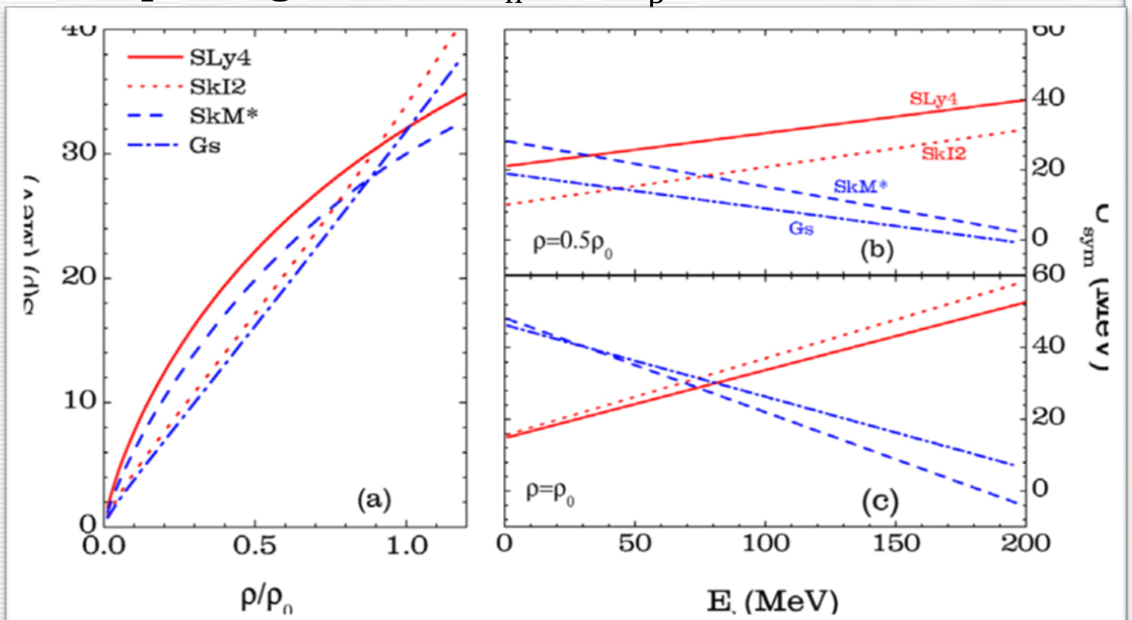
$m^*_n < m^*_p$

$m^*_n > m^*_p$

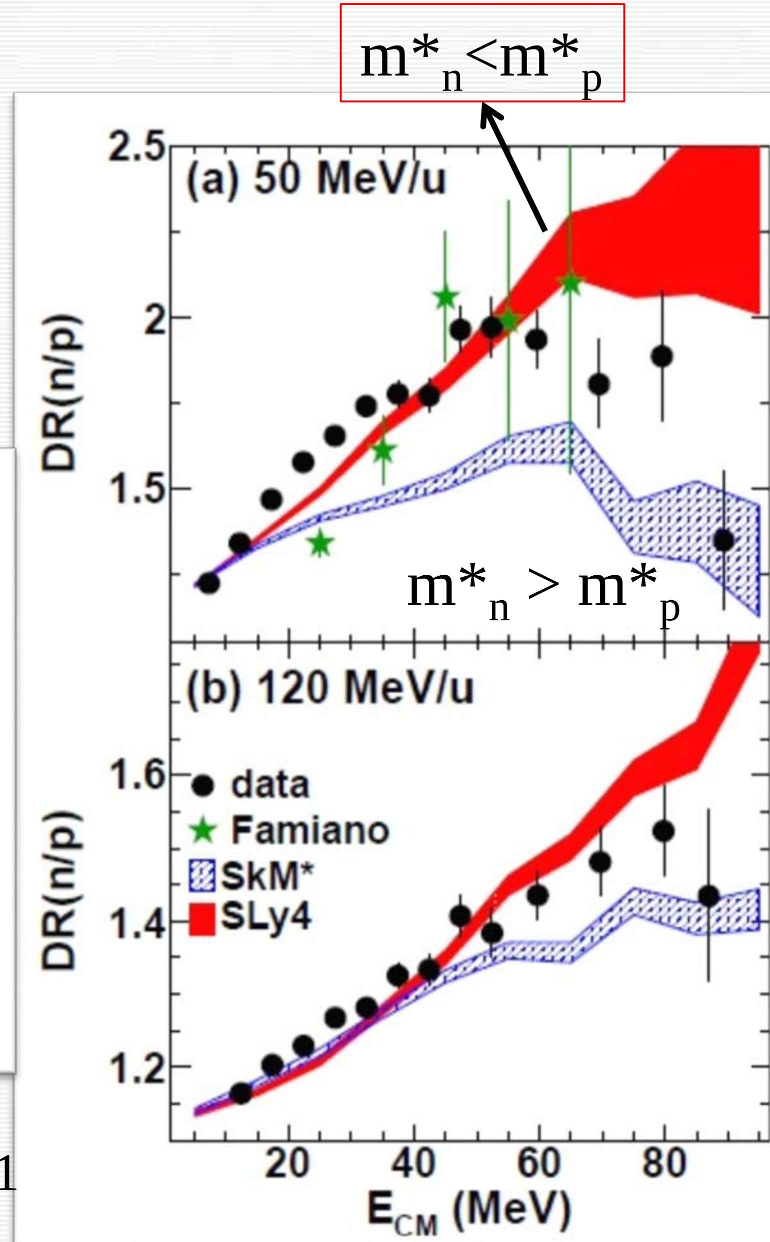
2014-PLB 732. 186

MSU data and ImQMD calculations propose $m^*_n<m^*_p$

- MSU have presented data of neutron/proton spectral double ratio from central $^{124}Sn+^{124}Sn$ and $^{112}Sn+^{112}Sn$ collisions 50 and 120 MeV/u.
- Together with the calculations by ImQMD, the data constrain the np effective mass splitting to be $m^*_n < m^*_p$.



Yingxun Zhang, et. al., PLB 732 (2014)186
D.D.S. Coupland et. al., 2014-arXiv:1406.4546v1



Our work

Energy-dependent global optical potentials

Parameters of EOS

Relation between symmetry energy
and effective mass splitting

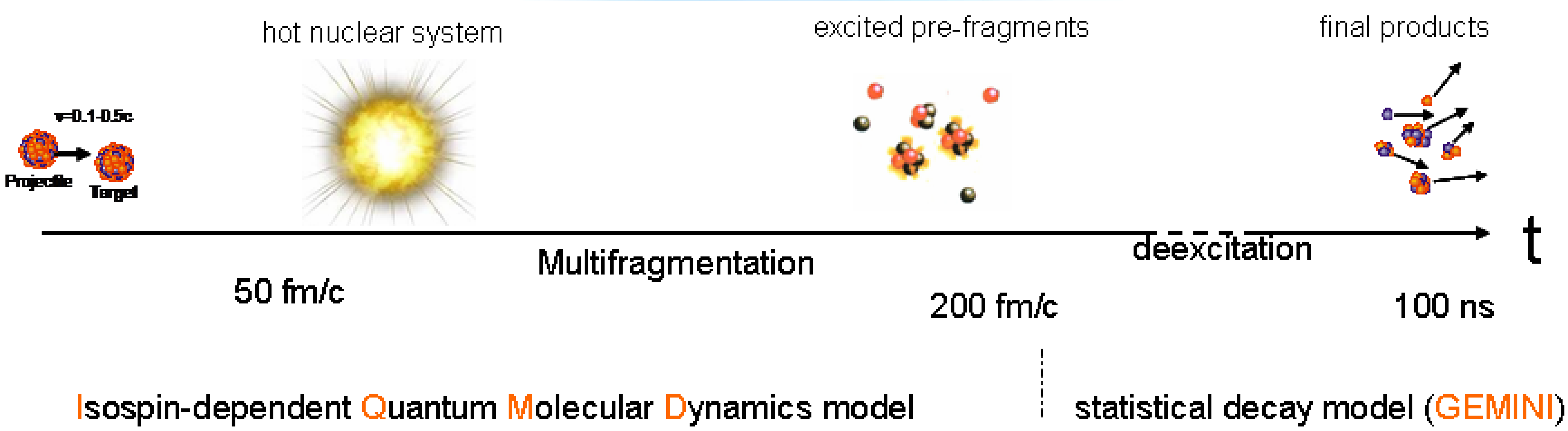
Transport model:
IQMD

HICs

n/p double ratio
at 50 and 120 MeV/u

Isospin tracer
at 400 MeV/u

Theoretical framework



- Density in phase space $$f(\mathbf{r},\mathbf{p},t)=\sum_{i=1}^{n}\frac{1}{(\pi\hbar)^3}e^{-\frac{[\mathbf{r}-\mathbf{r}_{i0}(t)]^2}{2L}}e^{-\frac{[\mathbf{p}-\mathbf{p}_{i0}(t)]^2\cdot 2L}{\hbar^2}}$$
- time evolution $$\dot{\mathbf{r}}_{i0}=\nabla_{p_{i0}}H_{eff}$$ $$\dot{\mathbf{p}}_{i0}=-\nabla_{r_{i0}}H_{eff}$$
- Hamiltonian $$H_{eff}=T+U_{Sky2}+U_{Sky3}+U_{sym}+U_{sur}+U_{mdi}+U_{Coul}$$
- Nucleon-nucleon collisions $b_{ij}<\sqrt{\sigma_t/\pi}$
- Pauli blocking $P_{block}=1-[1-min(P_1,1)][1-min(P_2,1)]$

EOS: Potential energy density

- The potential energy density of the asymmetric nuclear matter with density ρ and asymmetry δ is given by

$$V(\rho,\delta) = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\gamma+1}\frac{\rho^{\gamma+1}}{\rho_0^{\gamma}}$$

$$+ \frac{C_{sp}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}\rho\delta^2$$

$$+ \sum_{\tau}\frac{(1+x)}{\rho_0}\iint v(\mathbf{p},\mathbf{p}')f_\tau(\mathbf{r},\mathbf{p})f_\tau(\mathbf{r},\mathbf{p}')d\mathbf{p}d\mathbf{p}'$$

$$+ \sum_{\tau}\frac{(1-x)}{\rho_0}\iint v(\mathbf{p},\mathbf{p}')f_\tau(\mathbf{r},\mathbf{p})f_{-\tau}(\mathbf{r},\mathbf{p}')d\mathbf{p}d\mathbf{p}'$$

Two body and three body interaction; Density depedent widely used in QMD model
1991-QMD-PR.202.233 1

Local symmetry potential; density and isospin dependent
2009-PhysRevLett.102.122701

Momentum dependent interaction; proposed by Welke et. al.
[1988-PhysRevC.38.2101].

$$v(\mathbf{p},\mathbf{p}') = \frac{C_m}{1+(\mathbf{p}-\mathbf{p}')^2/\Lambda^2}$$

EOS: Single particle potential

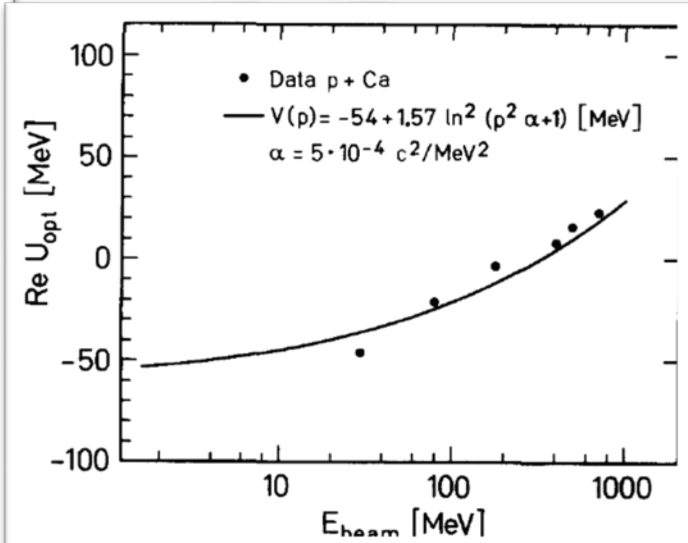
- The corresponding single particle potential of neutron and proton with momenta p in the asymmetric nuclear matter with density ρ and asymmetry δ can be calculated as,

$$
\begin{aligned}
U_n(\rho,\delta,\mathbf{p}) =& \frac{\partial V(\rho,\delta)}{\partial \rho_n} \\
=& \alpha\frac{\rho}{\rho_0} + \beta\frac{\rho^{\gamma}}{\rho_0^{\gamma}} + \frac{C_{sp}(\gamma_i+1)}{2}\frac{\rho^{\gamma_i}}{\rho_0^{\gamma_i}}\delta^2 + \frac{C_{sp}}{2}\frac{\rho^{\gamma_i+1}}{\rho_0^{\gamma_i}}2\delta \\
&+ (1+x)\int v(p-p')f_n(\mathbf{r},\mathbf{p}')d\mathbf{p}' \\
&+ (1-x)\int v(p-p')f_p(\mathbf{r},\mathbf{p}')d\mathbf{p}';
\end{aligned}
$$

$$
\begin{aligned}
U_p(\rho,\delta,\mathbf{p}) =& \frac{\partial V(\rho,\delta)}{\partial \rho_p} \\
=& \alpha\frac{\rho}{\rho_0} + \beta\frac{\rho^{\gamma}}{\rho_0^{\gamma}} + \frac{C_{sp}(\gamma_i+1)}{2}\frac{\rho^{\gamma_i}}{\rho_0^{\gamma_i}}\delta^2 - \frac{C_{sp}}{2}\frac{\rho^{\gamma_i+1}}{\rho_0^{\gamma_i}}2\delta \\
&+ (1+x)\int v(p-p')f_p(\mathbf{r},\mathbf{p}')d\mathbf{p}' \\
&+ (1-x)\int v(p-p')f_n(\mathbf{r},\mathbf{p}')d\mathbf{p}'
\end{aligned}
$$

- Single particle potential is energy dependent;
- Can be fixed by optical potential

[1987-PhysRevLett.58.1926]



EOS: Energy per nucleon

- The energy per nucleon of the asymmetric nuclear matter at zero temperature is the summation of kinetic energy and potential energy,

$$\frac{E}{A}(\rho,\delta) = \frac{3}{5}\frac{p_{Fn}^2}{2m}\frac{\rho_n}{\rho} + \frac{3}{5}\frac{p_{Fp}^2}{2m}\frac{\rho_p}{\rho} + \frac{V(\rho,\delta)}{\rho}$$

Here, p_{Fn} and p_{Fp} are the Fermi momenta of neutrons and protons

$$p_{Fn} = \hbar c\left(\frac{3\pi^2\rho}{2}\right)^{1/3}(1+\delta)^{1/3}$$

$$p_{Fp} = \hbar c\left(\frac{3\pi^2\rho}{2}\right)^{1/3}(1-\delta)^{1/3}$$

For the symmetric nuclear matter, there is a general consensus that

$$\frac{E}{A}(\rho_0,0) = -15MeV;$$

$$P(\rho_0,0) = \rho^2\frac{\partial\frac{E}{A}}{\partial\rho}\bigg|_{\rho=\rho_0} = 0;$$

$$K(\rho_0,0) = 9\frac{\partial P}{\partial\rho}\bigg|_{\rho=\rho_0} = 250 \pm 50MeV$$

2014-Progress in Particle and Nuclear Physics 76 116

EOS: Symmetry energy and effective mass

- The parameter C_{sp}, γ_i and x are introduced to fix the theoretical predictions for the density dependence of the nuclear symmetry energy.
- The parabolic approximation to the nucleon energy in the asymmetric nuclear matter has been widely used.

$$E(\rho,\delta) - E(\rho,\delta=0) = E_{sym}(\rho)\delta^2 + \mathcal{O}\delta^4$$

Kinetic part

$$\frac{E_k}{A}(\rho,\delta) - \frac{E_k}{A}(\rho,\delta=0) = \frac{3}{5}\frac{p_{Fn}^2}{2m}\frac{\rho_n}{\rho} + \frac{3}{5}\frac{p_{Fp}^2}{2m}\frac{\rho_p}{\rho} - \frac{3}{5}\frac{p_F^2}{2m} \approx 12.57\delta^2;$$

$$\frac{E_p}{A}(\rho,\delta) - \frac{E_p}{A}(\rho,\delta=0) = \frac{C_{sp}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}\delta^2;$$

Local part

$$\begin{aligned}\frac{E_m}{A}(\rho,\delta) - \frac{E_m}{A}(\rho,\delta=0) = &\sum_\tau (1+x)\frac{1}{\rho}\iint v(p-p')f_\tau(\mathbf{r},\mathbf{p})f_\tau(\mathbf{r},\mathbf{p}')d\mathbf{p}d\mathbf{p}' \\ &+ \sum_\tau (1-x)\frac{1}{\rho}\iint v(\mathbf{p},\mathbf{p}')f_\tau(\mathbf{r},\mathbf{p})f_{-\tau}(\mathbf{r},\mathbf{p}')d\mathbf{p}d\mathbf{p}' \\ &- \iint v(\mathbf{p},\mathbf{p}')f(\mathbf{r},\mathbf{p})f(\mathbf{r},\mathbf{p}')d\mathbf{p}d\mathbf{p}'.\end{aligned}$$

Momentum dependent part

Parameters of EOS in this work

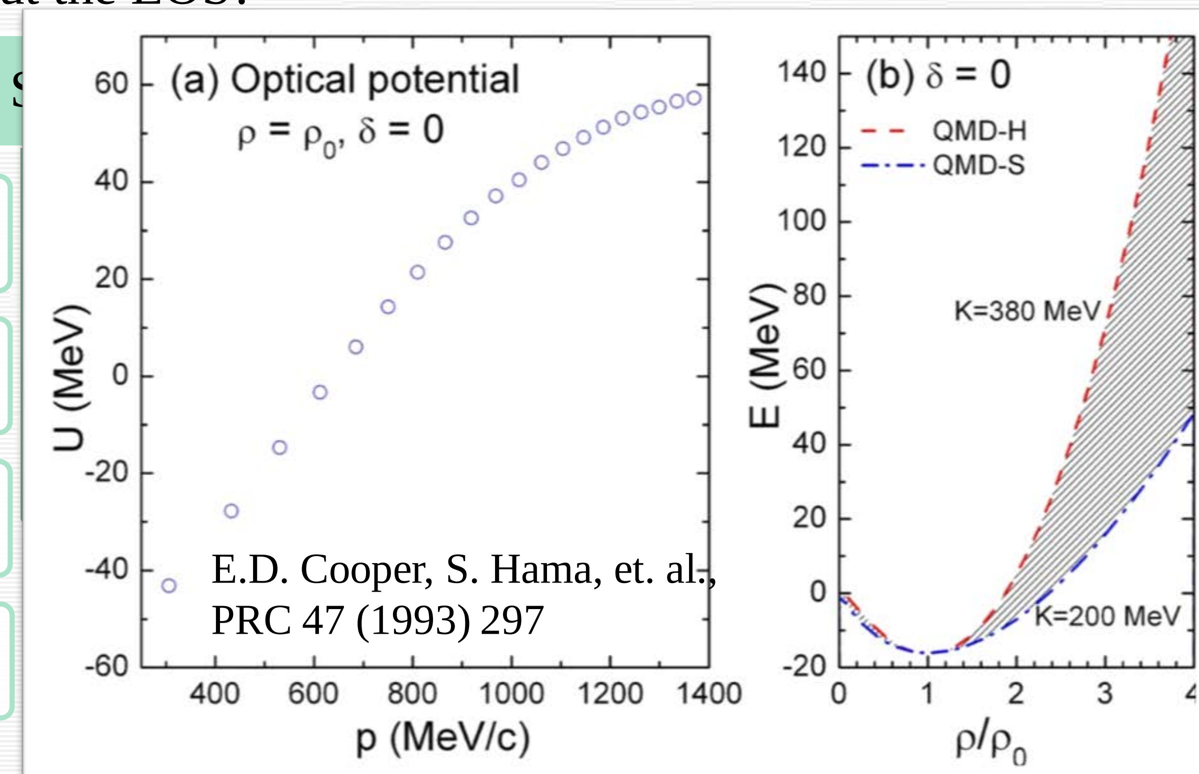
EOS of symmetric nuclear matter					Symmetry energy		
Two body	three body		MDI		MDI part	Local part	
α	β	γ	C_m	Λ	x	C_{sp}	γ_i

- What have we known about the EOS?

EOS at δ=0

- $E/A(\rho_0)$ = -16 ~ -15 MeV
- $P(\rho_0) = 0$
- $K(\rho_0)$ = 200 ~ 380 MeV
- Optical potential

2014-Prog. Part. & Nucl. Phys. 76, 116



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α	β	γ	C_m	Λ	x	C_{sp}	γ_i
-75.86 MeV	166.43 MeV	1.226	-88.21 MeV	664.86 MeV c			

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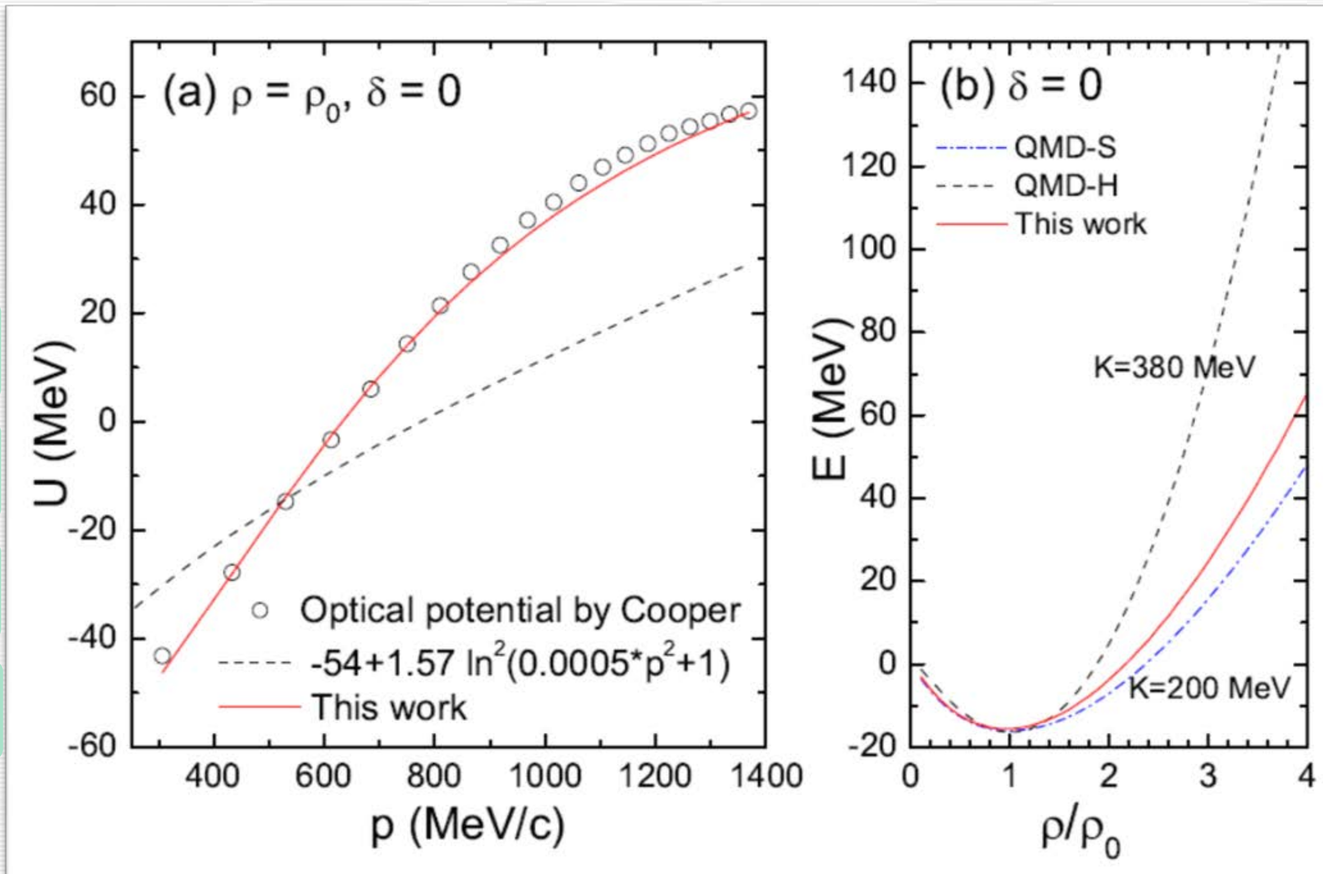
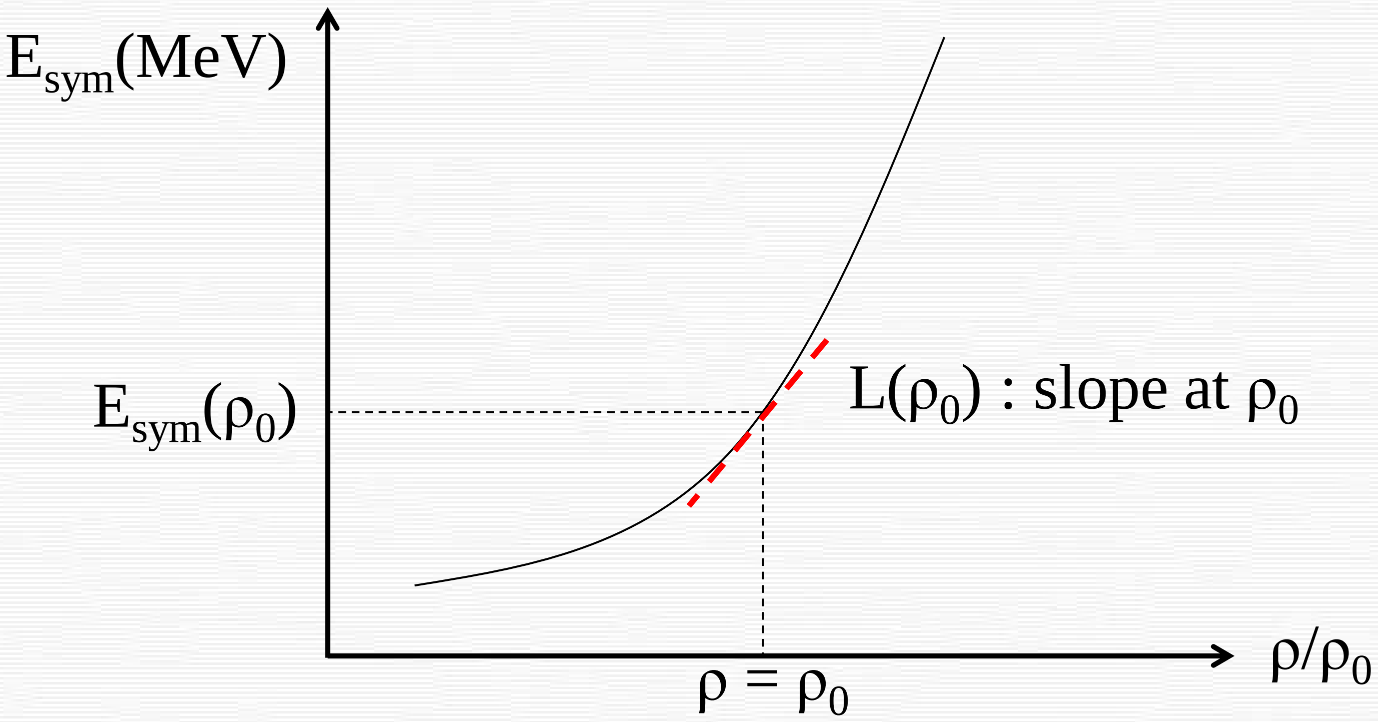


What have we known about the symmetry energy

$$E(\rho,\delta) - E(\rho,\delta=0) = E_{sym}(\rho)\delta^2 + \mathcal{O}\delta^4$$

$$E_{sym} = E_{sym}^{kin} + E_{sym}^{loc} + E_{sym}^{mdi}$$

$$L = 3\rho\frac{\partial E_{sym}}{\partial\rho} = 25.14 + \frac{3C_{sp}\gamma_i}{2} + 3\rho\frac{\partial E_{sym}^{mdi}}{\partial\rho}$$



What have we known about the symmetry energy

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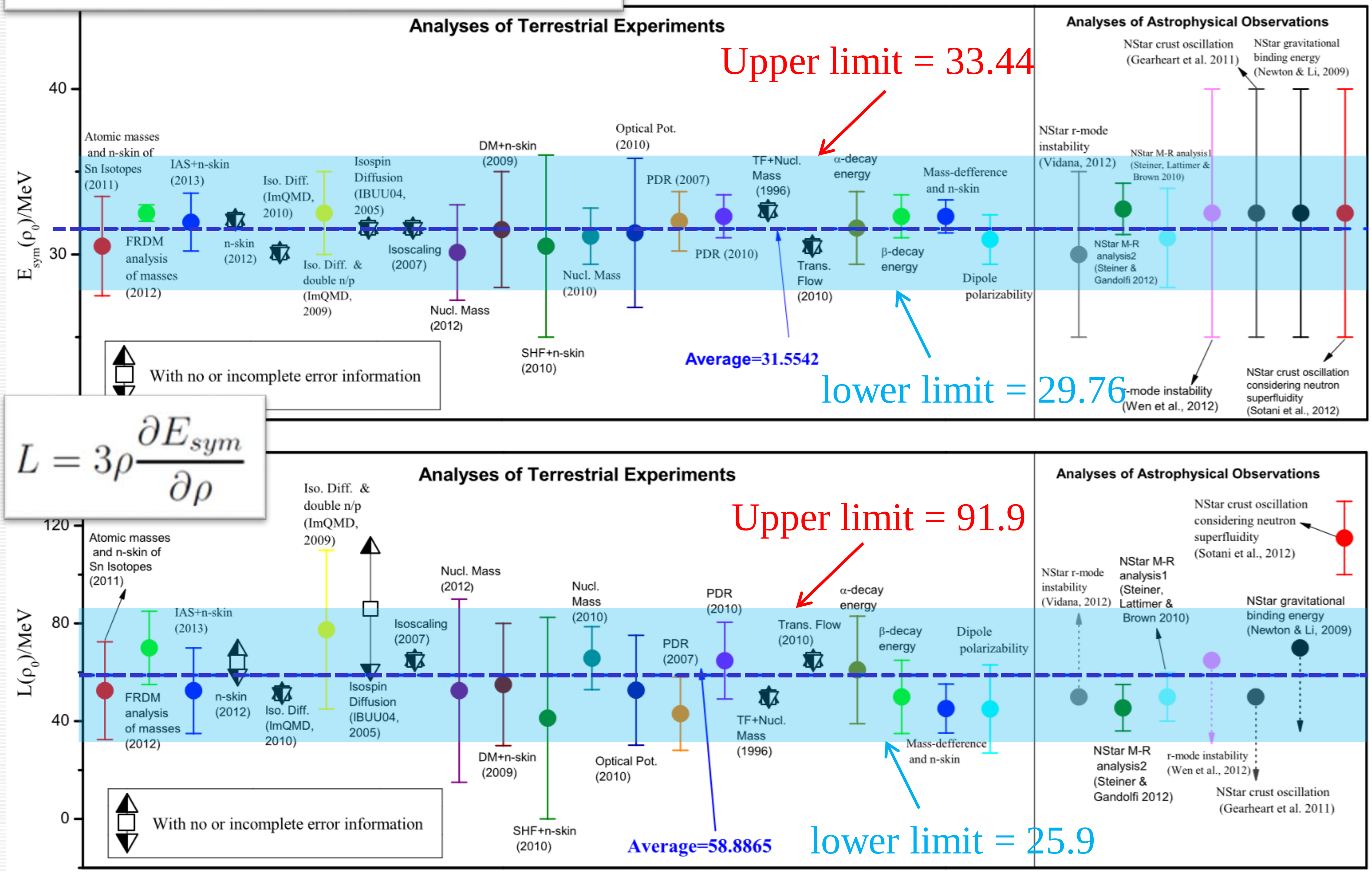
$$E_{sym} = E_{sym}^{kin} + E_{sym}^{loc} + E_{sym}^{mdi}$$

B.-A. Li, X. Han, Physics Letters B 727 (2013) 276

$$L = 3\rho \frac{\partial E_{sym}}{\partial \rho}$$



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α	β	γ	C_m	Λ	x	C_{sp}	γ_i
-75.86 MeV	166.43 MeV	1.226	-88.21 MeV	664.86 MeV c			

- We fit four groups of parameters to provide different density dependences of symmetry energy and effective mass splitting.

	x	$C_{s,p}$/MeV	γ_i	$E_{sym}(\rho_0)$/MeV	$L(\rho_0)$/MeV	m^*
Pos-S	-0.4	-30.87	2.002	29.76	25.9	$m_n^* > m_p^*$
Pos-H	-0.4	-23.51	0.757	33.44	91.9	$m_n^* > m_p^*$
Neg-S	0.4	78.21	0.408	29.76	25.9	$m_n^* < m_p^*$
Neg-H	0.4	85.57	0.888	33.44	91.9	$m_n^* < m_p^*$

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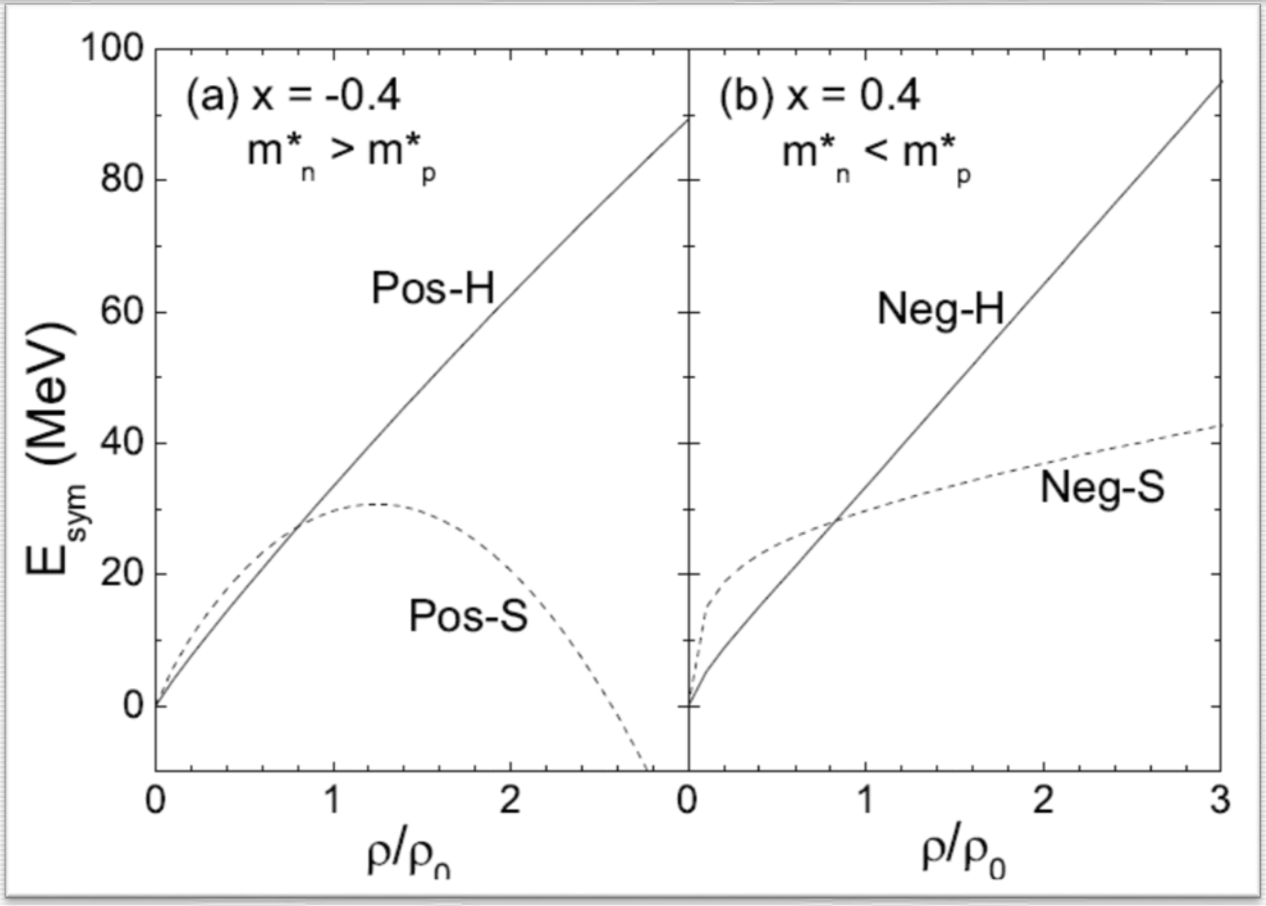

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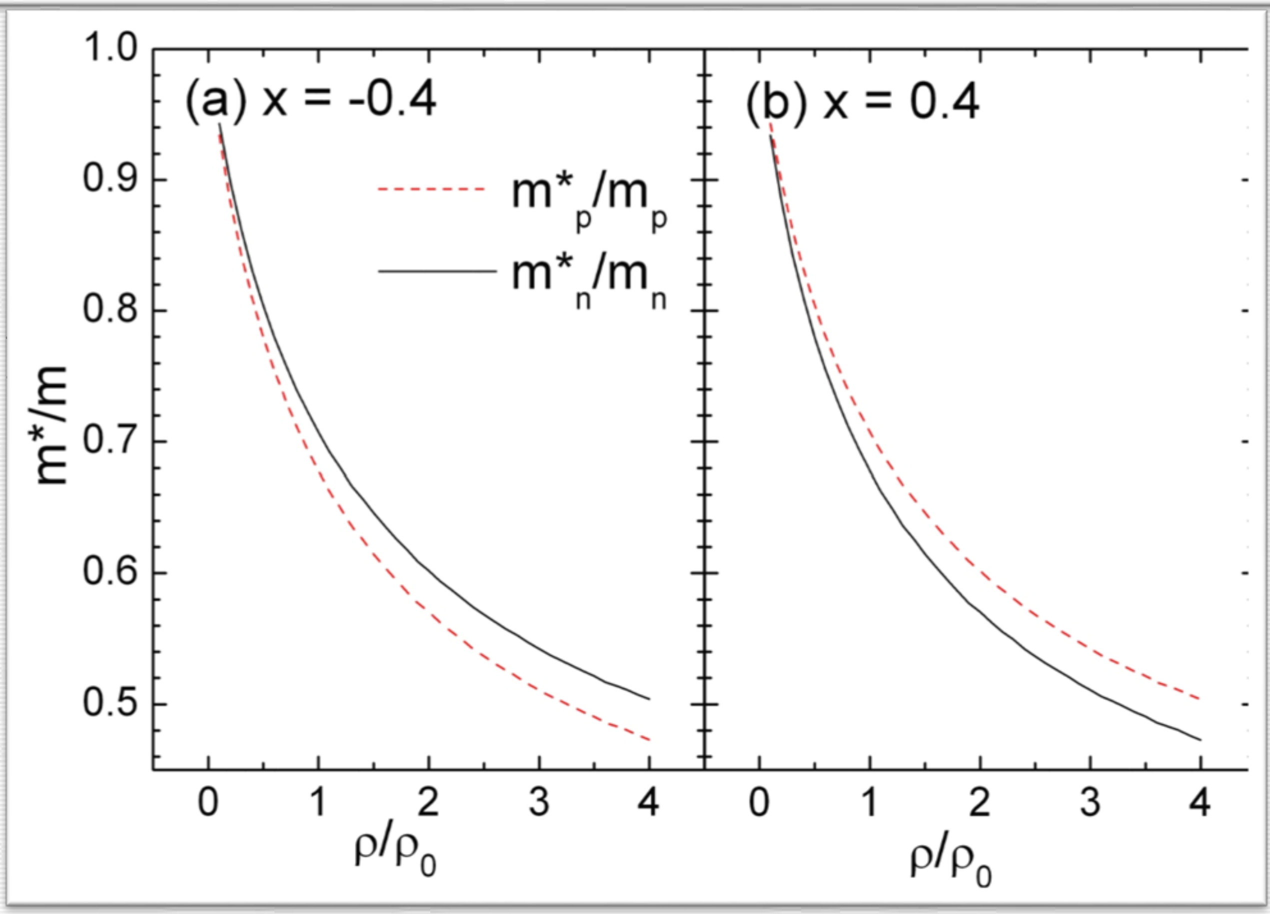
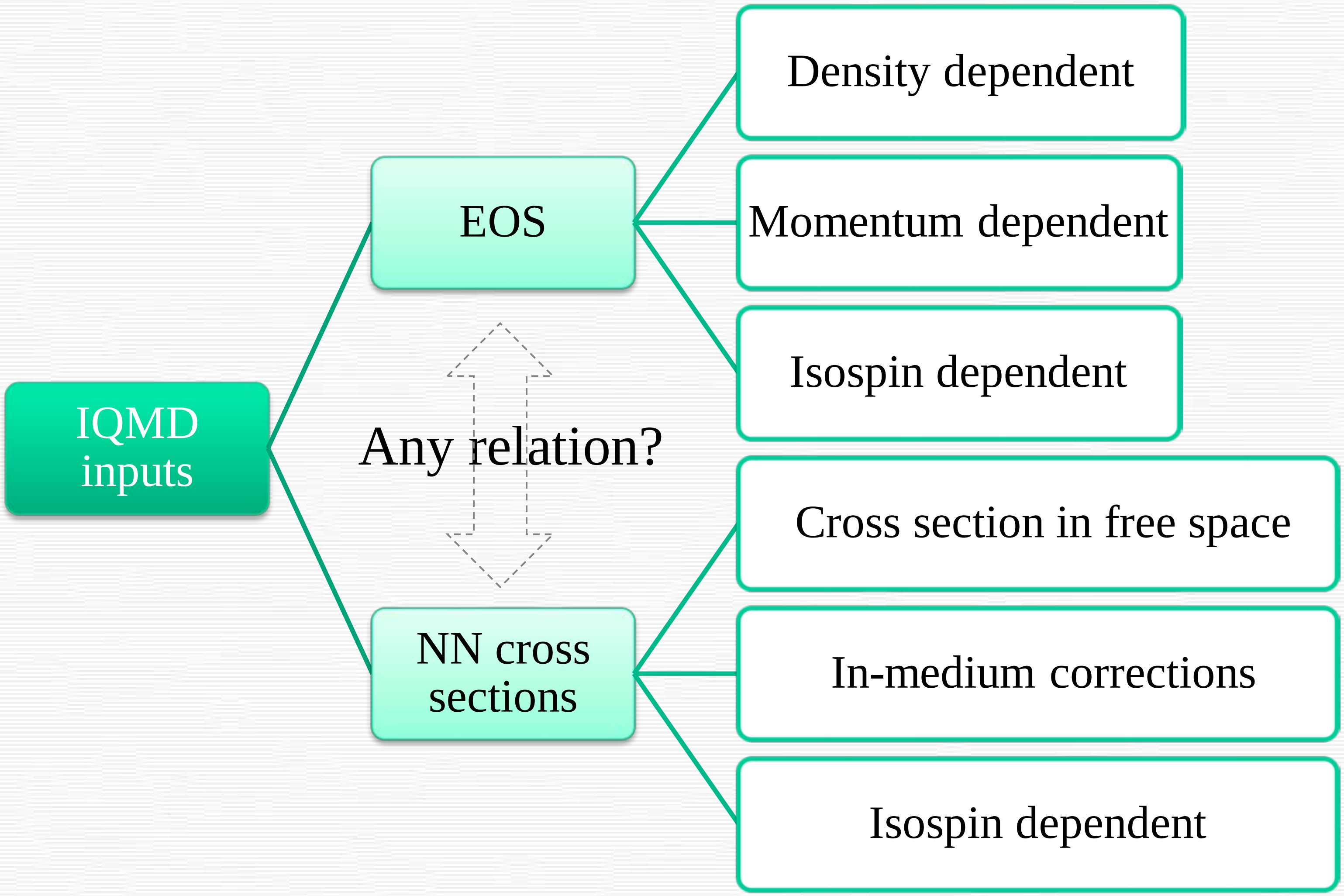


In-medium corrections to NN cross sections



In-medium corrections to NN cross sections

- The differential cross sections of NN collisions can be written as

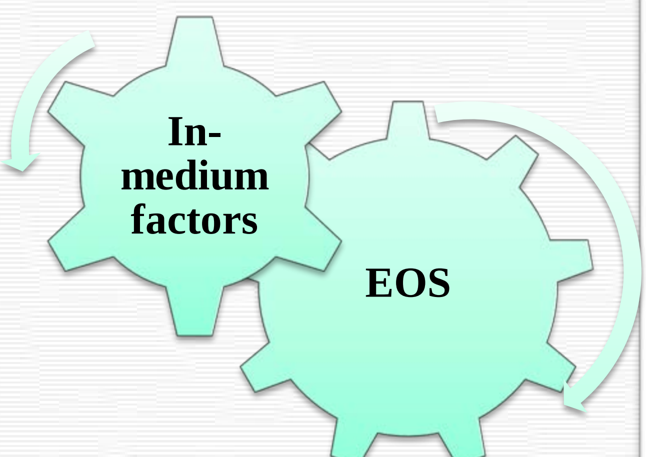
$$\left(\frac{d\sigma}{d\Omega}\right)_i = \sigma_i^{free} f_i^{angl} f_i^{med}$$

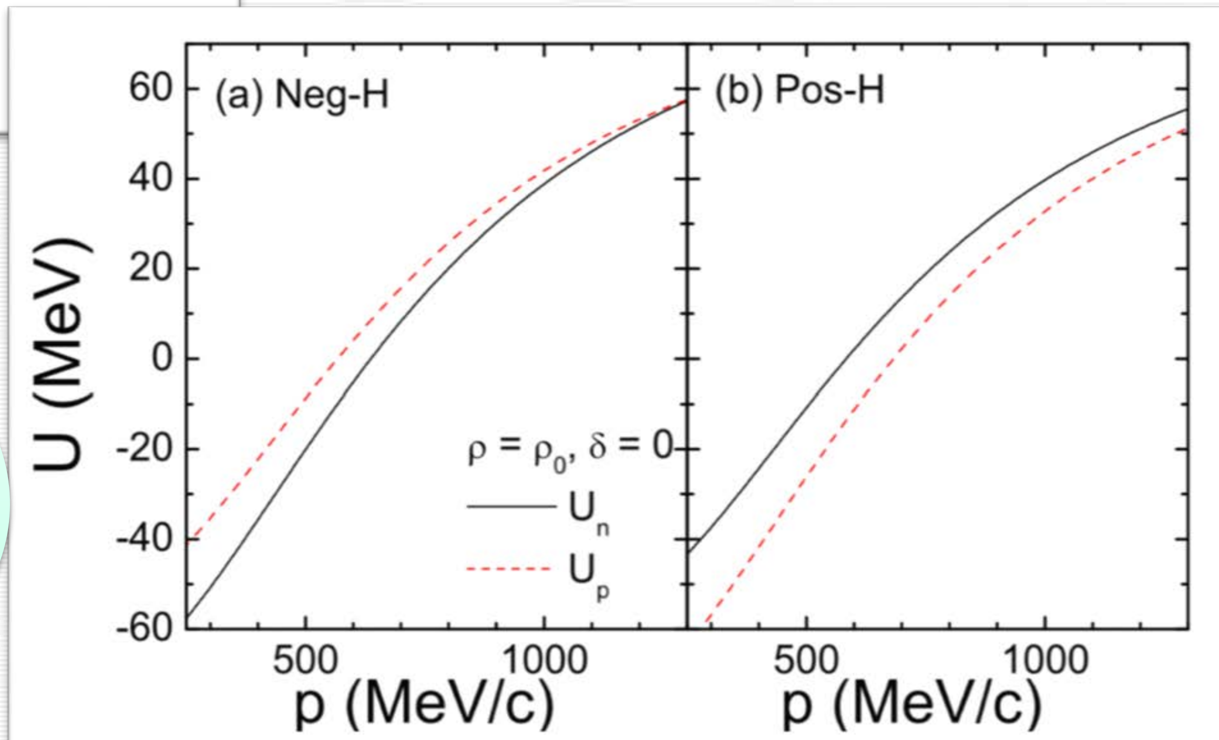
- σ^{free} is the cross section of NN collisions in free space,
- f^{ang} gives the angular dis- tribution,
- f^{med} gives the in-medium corrections.

$$f_{ij}^{med} = \frac{\sigma_{med}}{\sigma_{free}} = \left[\frac{m_i^* m_j^*/(m_i^* + m_j^*)}{m_i m_j/(m_i + m_j)}\right]^2,$$

$$m_i^* = \left[\frac{1}{m_i} + \frac{\partial U}{p_i \partial p_i}\right]^{-1}.$$

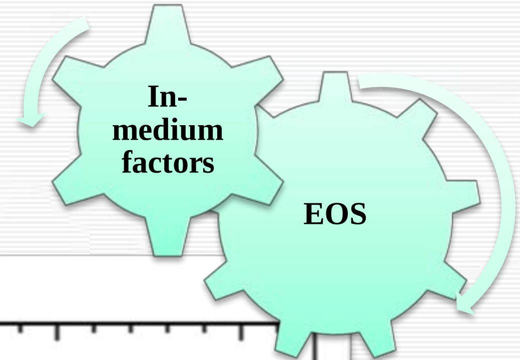
V. R. Pandharipande and S. C. Pieper, Phys. Rev. C 45, 791 (1991).
D. Persram and C. Gale, Phys. Rev. C 65, 064611 (2002).
B. A. Li and L. W. Chen, Phys. Rev. C 72, 064611 (2002).
Z. Q. Feng, Phys. Rev. C 85, 014604 (2012).

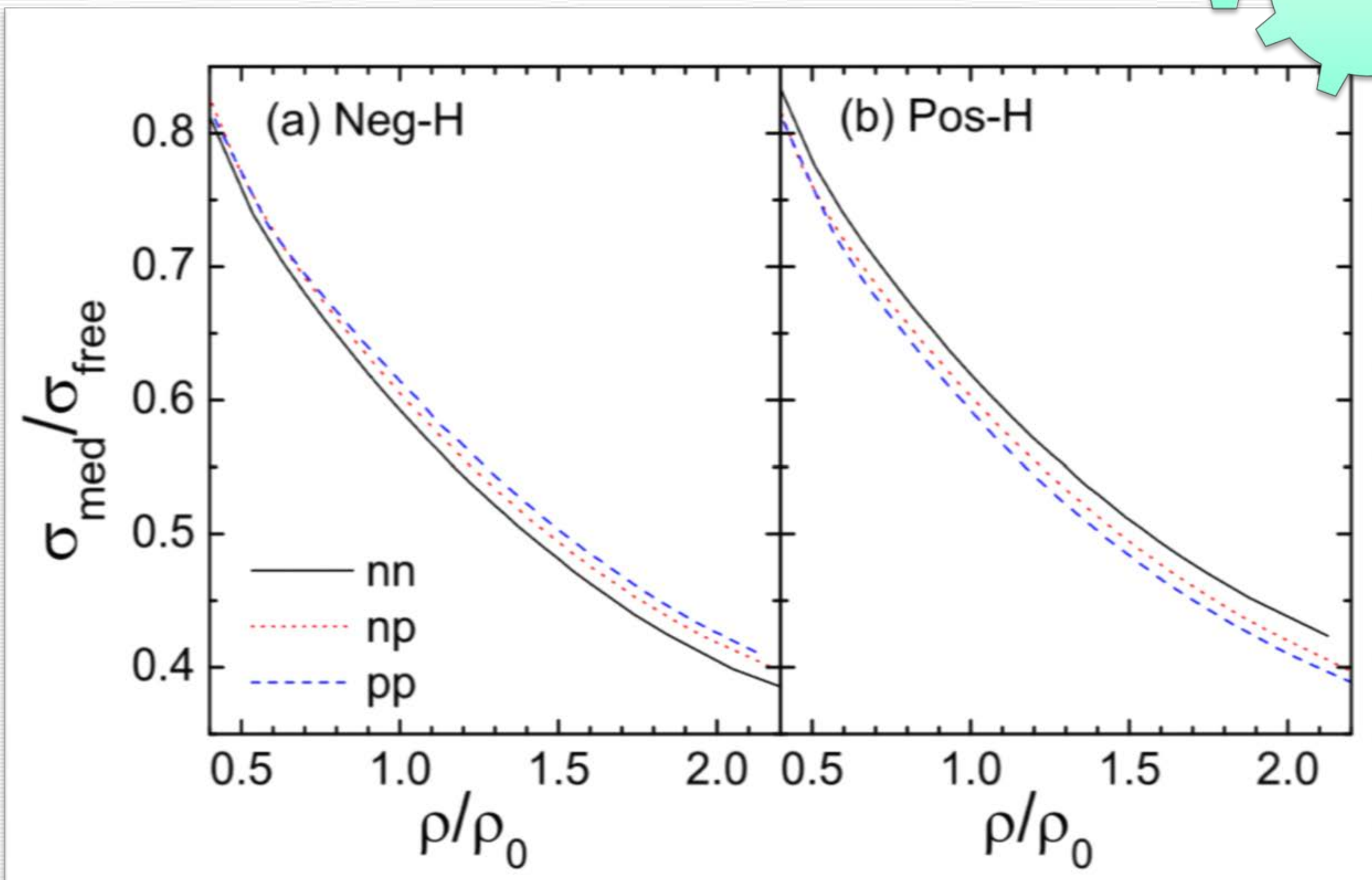




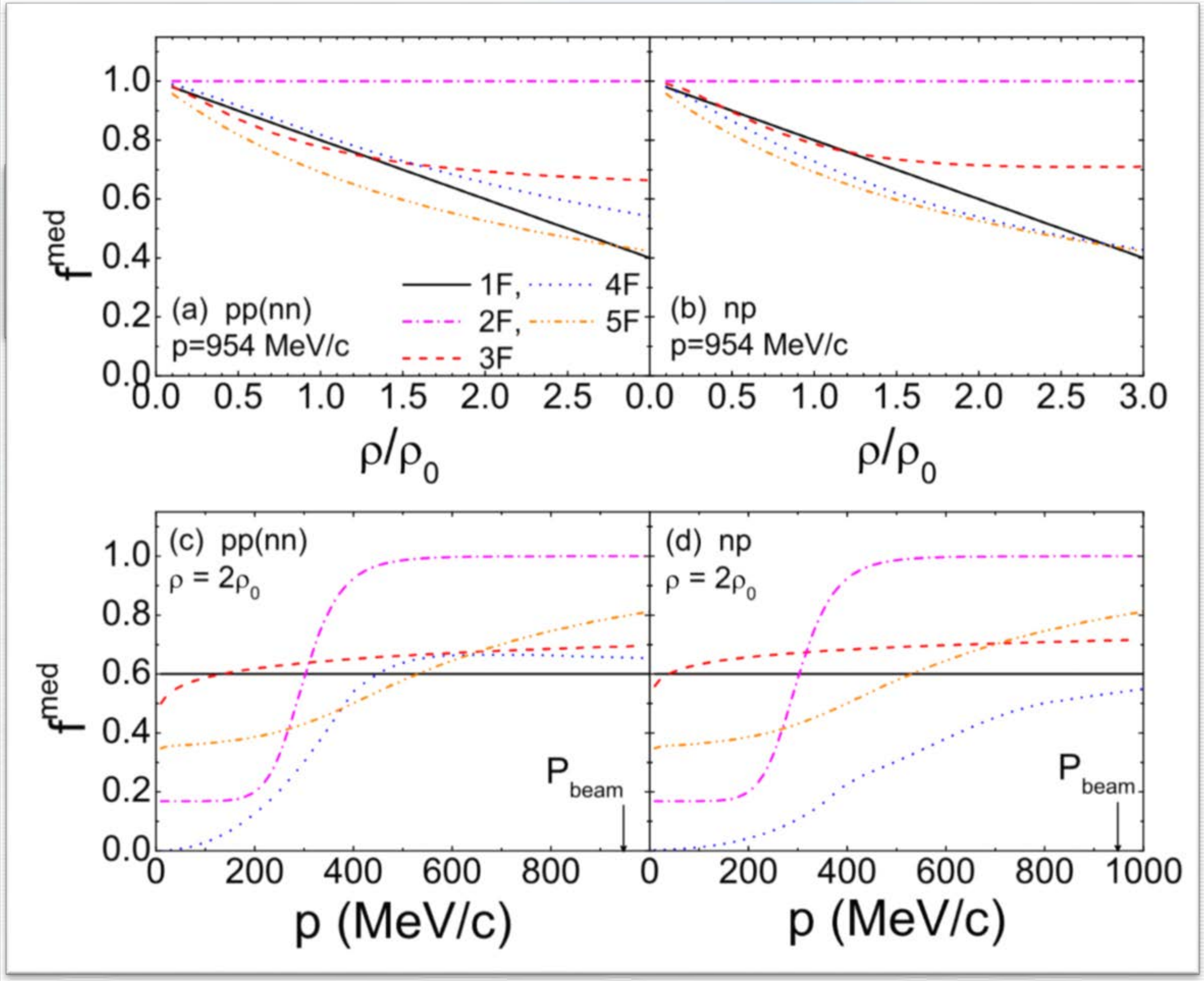
In-medium corrections to NN cross sections

- The in-medium factors depend on the EOS
- Due to the effective mass splitting, the in-medium factors are isospin dependent.





In-medium corrections to NN cross sections



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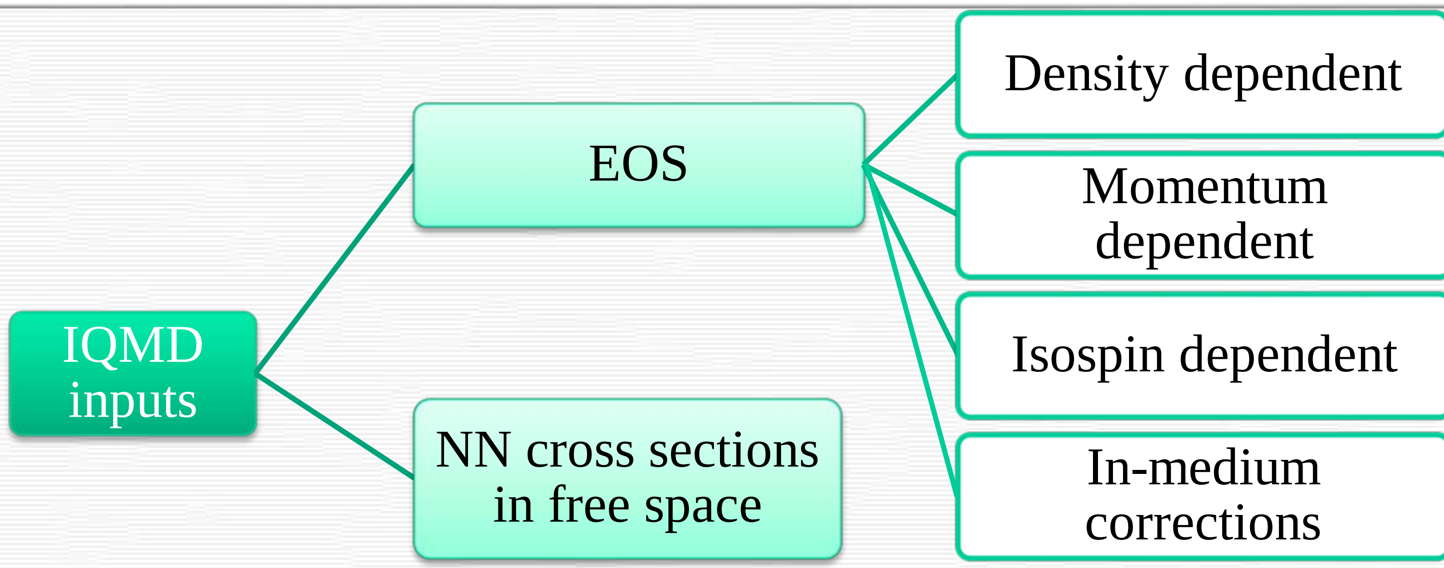
	x	$C_{s,p}$/MeV	γ_i	$E_{sym}(\rho_0)$/MeV	$L(\rho_0)$/MeV	m*
Pos-S	-0.4	-30.87	2.002	29.76	25.9	$m_n^* > m_p^*$
Pos-H	-0.4	-23.51	0.757	33.44	91.9	$m_n^* > m_p^*$
Neg-S	0.4	78.21	0.408	29.76	25.9	$m_n^* < m_p^*$
Neg-H	0.4	85.57	0.888	33.44	91.9	$m_n^* < m_p^*$



Results

Energy-dependent global optical potentials

Parameters of EOS

Relation between symmetry energy and effective mass splitting

IQMD

HICs

n/p double ratio at 50 and 120 MeV/u

Isospin tracer at 400 MeV/u

Results:
DR at 50 and 120 MeV/u

	$E_{sym}(\rho_0)$ MeV	$L(\rho_0)$ MeV	m^*
Pos-S	29.76	25.9	$m_n^* > m_p^*$
Pos-H	33.44	91.9	$m_n^* > m_p^*$
Neg-S	29.76	25.9	$m_n^* < m_p^*$
Neg-H	33.44	91.9	$m_n^* < m_p^*$

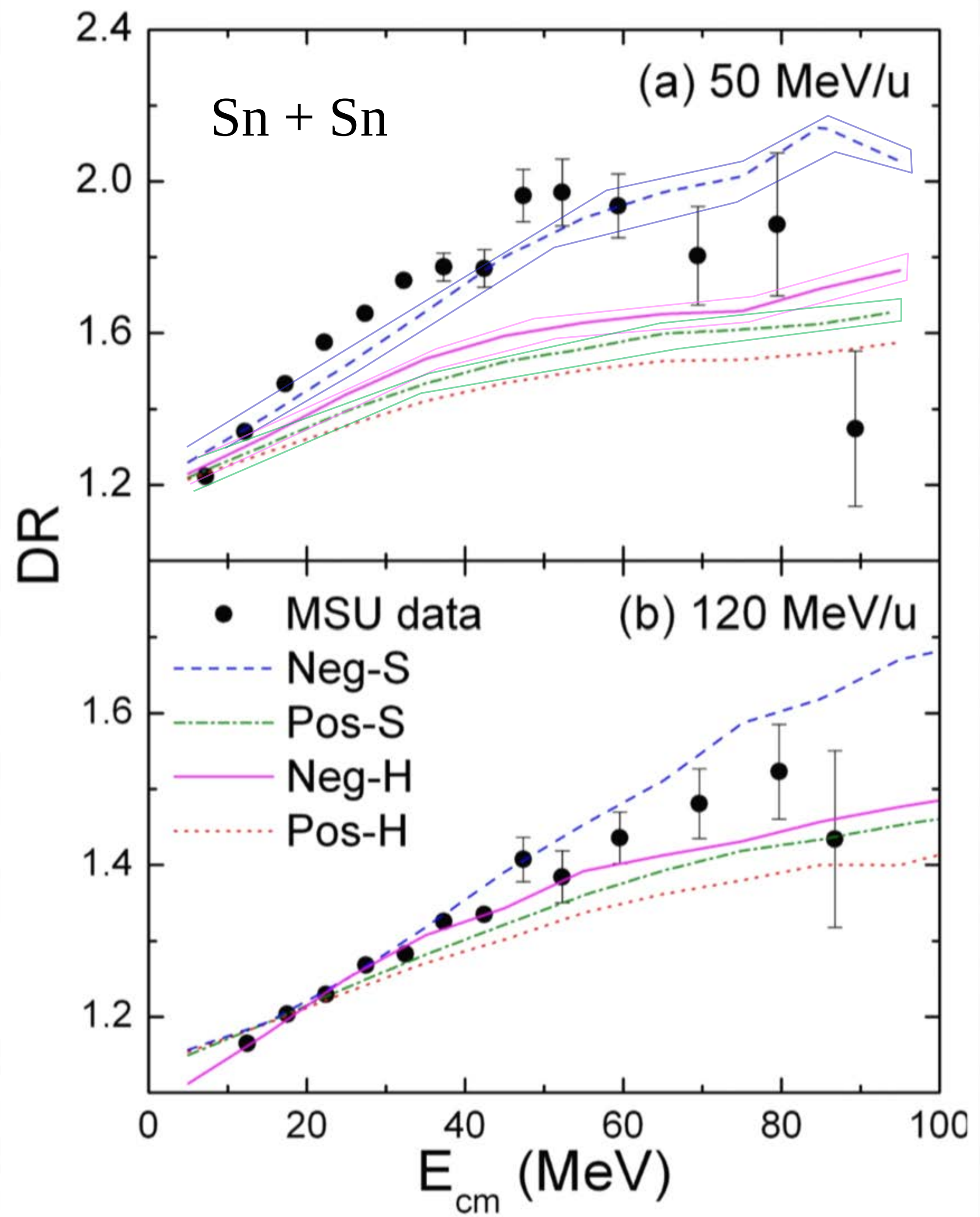
- Pos-S vs Pos-H
- Neg-S vs Neg-H

  effects of Esym

- Pos-S and Neg-S
- Pos-H and Neg-H

  effects of eff. mass splitting

$$\mathrm{DR}\,(Y(n)/Y(p)) = R_{n/p}(A)/R_{n/p}(B)$$
$$= \frac{dM_n(A)/dE_{\mathrm{c.m.}}}{dM_p(A)/dE_{\mathrm{c.m.}}} \cdot \frac{dM_p(B)/dE_{\mathrm{c.m.}}}{dM_n(B)/dE_{\mathrm{c.m.}}}$$

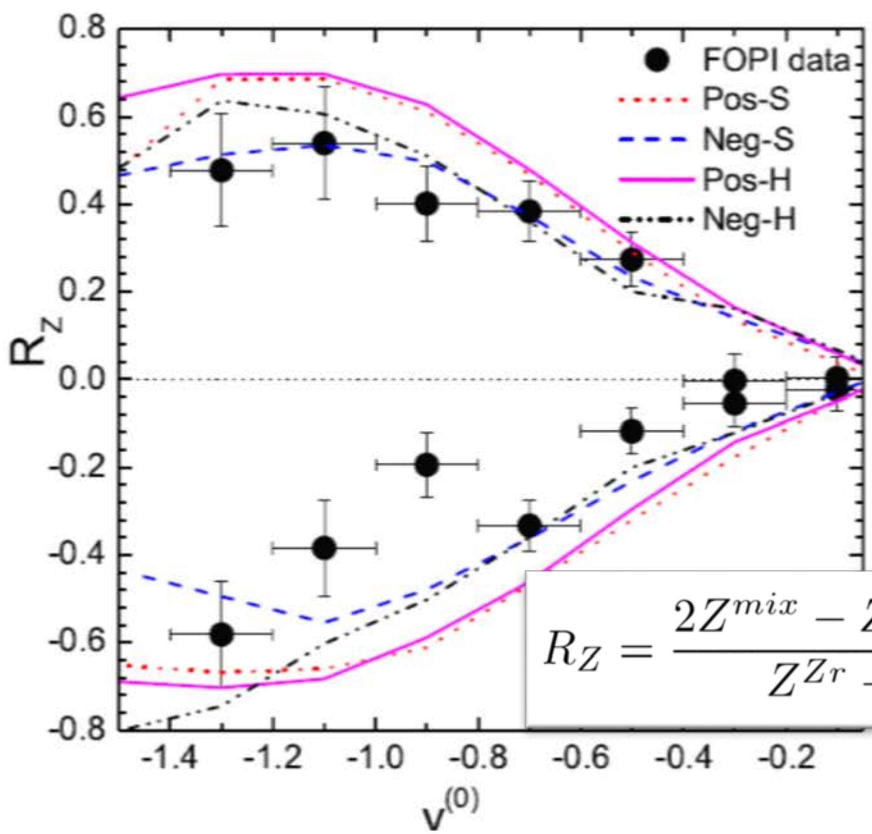


Results: Rz at 400 MeV/u

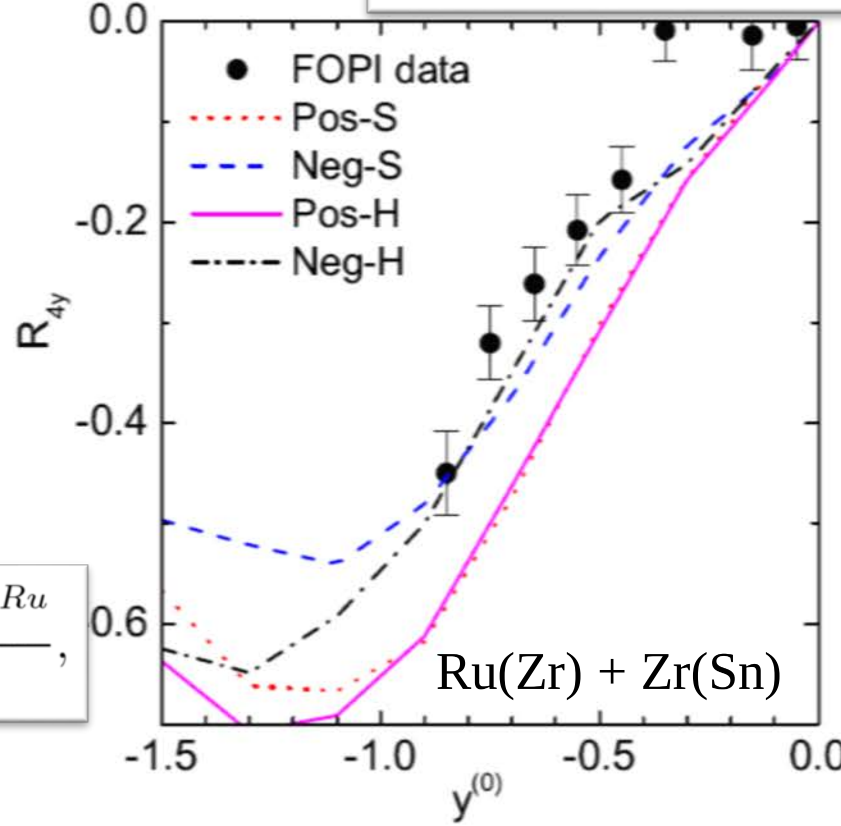
- Pos-S vs Pos-H: weak effects of Esym
- Pos-S vs Neg-S: effects of eff. mass splitting

	$E_{sym}(\rho_0)$ MeV	$L(\rho_0)$ MeV	m*
Pos-S	29.76	25.9	$m_n^* > m_p^*$
Pos-H	33.44	91.9	$m_n^* > m_p^*$
Neg-S	29.76	25.9	$m_n^* < m_p^*$
Neg-H	33.44	91.9	$m_n^* < m_p^*$

$$R_{4y}(p) = \frac{N_y^{\mathrm{RuZr}}(p) - N_y^{\mathrm{ZrRu}}(p)}{N_y^{\mathrm{RuRu}}(p) - N_y^{\mathrm{ZrZr}}(p)}$$



$$R_Z = \frac{2Z^{mix} - Z^{Zr} - Z^{Ru}}{Z^{Zr} - Z^{Ru}},$$



Discussion

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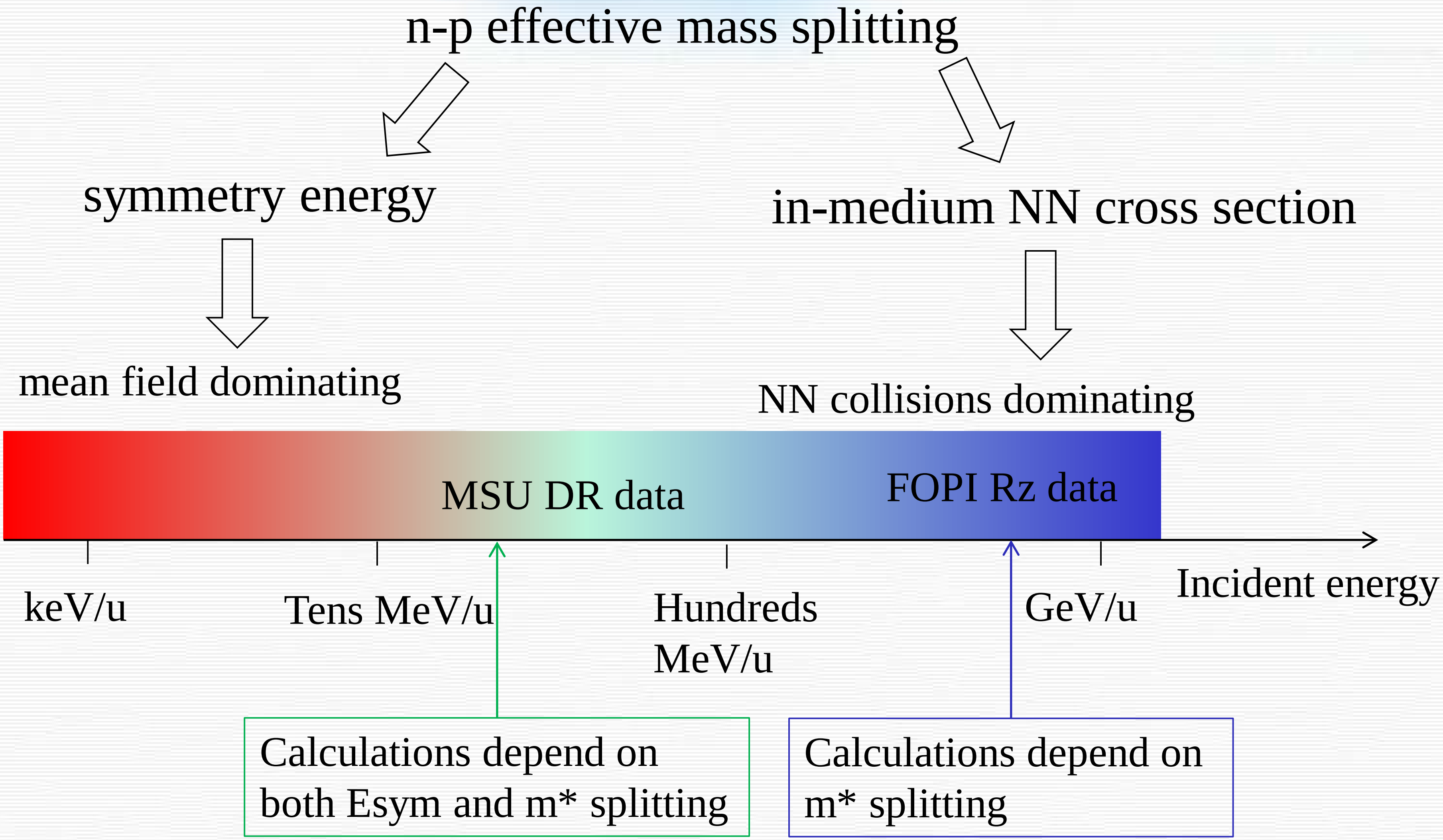

Constrain n-p effective mass splitting using data on HICs at hundreds MeV/u

Summary

- Theoretical framework
  1. Using the energy-dependent global optical potentials proposed by E. D. Cooper, we extract new parameters of EOS for IQMD.
  2. Estimate the in-medium factors of NN cross sections by EOS.
- Calculation
  1. n/p double ratio at 50 and 120 MeV/u.
  2. isospin tracer at 400 MeV/u.
- Found that
  1. The calculations of DR depend on local symmetry energy and effective mass splitting.
  2. The isospin tracer depend on m* splitting.
- Suggestion
  1. Constrain n-p effective mass splitting using data on HICs at hundreds MeV/u