Nusym2016, Beijing,
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Understanding transport simulations of heavy-ion collisions at intermediate energies

-Comparison of heavy-ion transport codes under controlled condition

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J. Xu, L.W. Chen, B. Tsang, H. Wolter, Y.X. Zhang *et al.* (31 authors), Phys. Rev. C 93, 044609 (2016)
arXiv : 1603.08149 [nucl-th]

18 transport codes: 9 BUU + 9 QMD

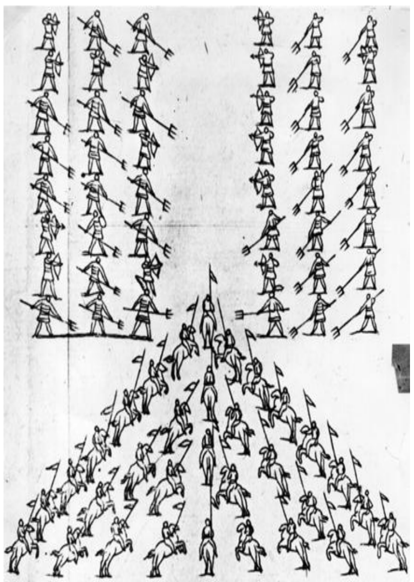
Nuclear EOS, E_{sym}

How reliable?

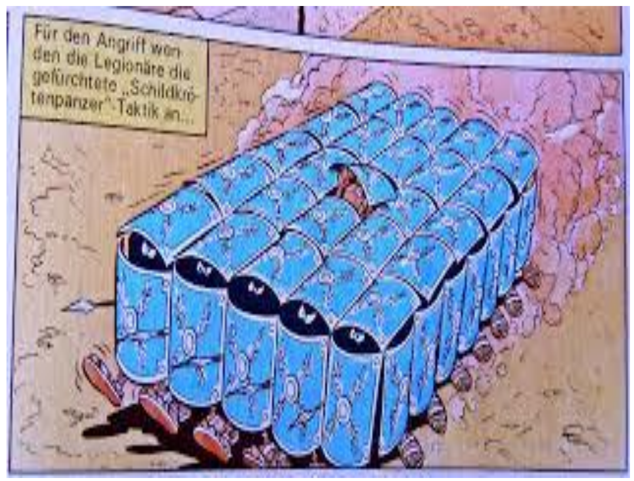
Heavy-ion experiments

Transport simulations

Mean-field potential



Initialization



Mean Field

NN scatterings

theoretical uncertainties?

Compare results from various models

Images from
H. Wolter

History of Transport Model Comparison Project

- **Trento I (2003):** energy 1-2 GeV/A, emphasis on particle production π,K

mean field and Pauli blocking not quite so important

Summary Published in **J.Phys. G31 (2005) 741**

- **Trento II (2009):** energy 100, 400 MeV/A

Uncertainties not well understood

results not published

Transport2014 in Shanghai

Mainly 100 AMeV, also 400 AMeV

observables: stopping, flow

Nonobservables: stability, scattering and Pauli blocking rate

Participating Codes

Boltzmann-Uehling-Uhlenbeck(BUU)-type models (9)

BUU-type	code correspondents	energy range	reference
BLOB	P.Napolitani,M.Colonna	$0.01 \sim 0.5$	[19]
GIBUU-RMF	J.Weil	$0.05 \sim 40$	[20]
GIBUU-Skyrme	J.Weil	$0.05 \sim 40$	[20]
IBL	W.J.Xie,F.S.Zhang	$0.05 \sim 2$	[21]
IBUU	J.Xu,L.W.Chen,B.A.Li	$0.05 \sim 2$	[11, 22]
pBUU	P.Danielewicz	$0.01 \sim 12$	[23]
RBUU	K. Kim,Y.Kim,T.Gaitanos	$0.05 \sim 2$	[24]
RVUU	T.Song,G.Q.Li,C.M.Ko	$0.05 \sim 2$	[25]
SMF	M.Colonna,P.Napolitani	$0.01 \sim 0.5$	[26]

In GeV

Find representative references for each code in arXiv: 1603:08149 [nucl-th]

Participating Codes

Quantum-Molecular-Dynamics(QMD)-type models(9)

QMD-type	code correspondents	energy range	reference
AMD	A.Ono	$0.01 \sim 0.3$	[27]
IQMD-BNU	J.Su,F.S.Zhang	$0.05 \sim 2$	[28]
IQMD	C.Hartnack,J.Aichelin	$0.05 \sim 2$	[29, 30]
CoMD	M.Papa	$0.01 \sim 0.3$	[31]
ImQMD-CIAE	Y.X.Zhang,Z.X.Li	$0.02 \sim 0.4$	[32]
IQMD-IMP	Z.Q.Feng	$0.01 \sim 10$	[33]
IQMD-SINAP	G.Q.Zhang	$0.05 \sim 2$	[34]
TuQMD	D.Cozma	$0.1 \sim 2$	[35]
UrQMD	Y.J.Wang,Q.F.Li	$0.05 \sim 200$	[36, 37]

In GeV

ImQMD-GXNU: low-energy fusion reaction

Find representative references for each code in arXiv: 1603:08149 [nucl-th]

A taste of BUU-type models

BUU equation:

$$\left(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\cdot\nabla_r-\nabla_r U\cdot\nabla_p\right)f(\vec{r},\vec{p};t)=I_{coll}[f;\sigma_{12}]$$

$$I_{coll} = \frac{1}{(2\pi)^6}\int dp_2 dp_3 d\Omega |v-v_2| \frac{d\sigma_{12}^{med}}{d\Omega}(2\pi)^3\delta(p+p_2-p_3-p_4)$$
$$\times\ [f_3 f_4(1-f)(1-f_2)-f f_2(1-f_3)(1-f_4)]$$

test-particle (TP) method: parallel events

C.Y. Wong, PRC 25, 1460 (1982); G.F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988).

Point particle or finite size (triangular, Gaussian)

$$f(\vec{r},\vec{p};t)=\frac{1}{N_{TP}}\sum_{i=1}^{N_{TP}A} g(\vec{r}-\vec{r}_i(t))\tilde{g}(\vec{p}-\vec{p}_i(t))$$

Equation of motion from pseudoparticle method:

$$d\vec{r}_i/dt=\nabla_{\vec{p}_i}H; \qquad d\vec{p}_i/dt=-\nabla_{\vec{r}_i}H.$$

A taste of QMD-type models

Total wave function as products of single-particle wave function:

$$\Psi(\vec{r}_1, ..., \vec{r}_N; t) = \Pi \phi_i(\vec{r}_i; t),$$

$$\phi_i(\vec{r}_i; t) = \frac{1}{(2\pi)^{3/4}(\Delta x)^{3/2}} \exp\left[-\frac{(\vec{r}_i - \vec{R}_i(t))^2}{(2\Delta x)^2} + i\vec{r}_i \cdot \vec{P}_i(t)\right]$$

Wigner function (phase-space distribution):

$$f_i(\vec{r}, \vec{p}) = \frac{1}{(\pi\hbar)^3} \exp\left[-\frac{\left(\vec{r} - \vec{R}_i(t)\right)^2}{2(\Delta x)^2}\right] \exp\left[-\frac{\left(\vec{p} - \vec{P}_i(t)\right)^2 \cdot 2(\Delta x)^2}{\hbar^2}\right]$$

Canonical equation of motion:

$$d\vec{R}_i / dt = \nabla_{\vec{p}_i} H, \qquad d\vec{P}_i / dt = -\nabla_{\vec{r}_i} H$$

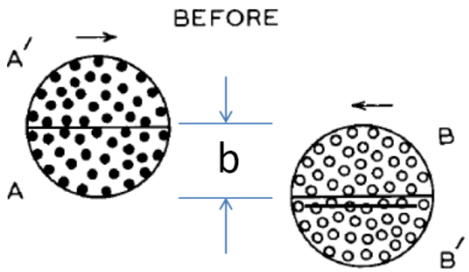
Ch. Hartnack et al., PRC 495, 303 (1989); J. Aichelin, Phys. Rep. 202, 233 (1988).

AMD and FMD: wave function antisymmetrized

Homework description: Initialization and reaction parameters

Initial coordinate space: Woods-Saxon distribution

2) Reaction:
a) **Au + Au, at 100 AMeV**
b) Initialisation: Initialize your (test)particles to obtain in coordinate space a Fermi function density profile with radius $R=xA^{1/3}$ (x=1.12 fm) and diffuseness a=0.6 fm. In case you are using finite size (test)particles, use a sphere of the above radius and try to adjust the width in such a way as to approach the diffuseness (if possible).
The momentum of the particles should then be chosen randomly in the local Fermi sphere.
c) impact parameter b= 7. and 20. fm, with initial distance (between the centers of project and target) in z-direction 16 fm
d) number of runs and (test)particles:
BUU: 10 runs, 100 TP/nucleon (for finite size TP), 500 TP/nucleon (for point TP)
MD: 1000 runs.
e) time evolution: run until t=140 fm/c. Use the time step you are usually using. For a second order integration method we suggest 0.5 fm/c.



b=7 fm: real collisions
b=20 fm: single nucleus evolution-stability

Homework description:
Mean-field potential and NN scattering

1) Physical input parameters:

a) Equation-of-state:

For non-relativistic transport codes: use a standard *soft Skyrme* parametrization (without momentum dependence) with the following parameters:
$a = -209.2$ MeV, $b = 156.4$ MeV, $\tau=1.35$, $M=938$ MeV
with a symmetry potential energy with linear density dependence $S_{pot}*(\rho/\rho_0)$, $S_{pot}=18$ MeV.
The total single-particle potential is then:
$U_{n/p}=a(\rho/\rho_0)+b(\rho/\rho_0)^{\tau}$ +/- $2S_{pot}*\rho/\rho_0*\delta$ $\quad (\delta=(\rho_n-\rho_p)/\rho)$
(Properties of this parameterization: Compression modulus $K_0=240$ MeV, saturation density $\rho_0=0.16$ fm^{-3}, binding energy at saturation density $E_0= -16$ MeV, symmetry energy $S(\rho_0)\sim 30.3$ MeV)

For relativistic transport codes: use a *nonlinear σ-ω-ρ RMF* parameterization *"NLρ"* (see parameter set I in PRC65, 045201, by Liu B et al.)
(properties of this parameterization: $K_0=240$ MeV, $\rho_0=0.16$ fm^{-3}, $E_0=-16$ MeV, $S(\rho_0)\sim 30.3$ MeV)

b) use a constant isotropic elastic cross section of 40 mb.

c) turn off all inelastic collisions.

Homework list for code contributors

Mode A). Homework1

Mode B). Au+Au@100 AMeV

B.1) No Surface Term mode: Turn off the surface term in the mean field (e.g., the Yukawa interaction in the QMD-like models, the gradient term in the BUU-like models). Allow collisions between all nucleons (or TP).

B-Full: both mean field and NN scattering

B.2) Vlasov mode: Turn off all collisions and use mean field as in B.1 (no surface terms)..

B-Vlasov: only mean field

B.3) Cascade mode: Turn off all interaction potentials in B.1 mode

B-Cascade: only NN scattering

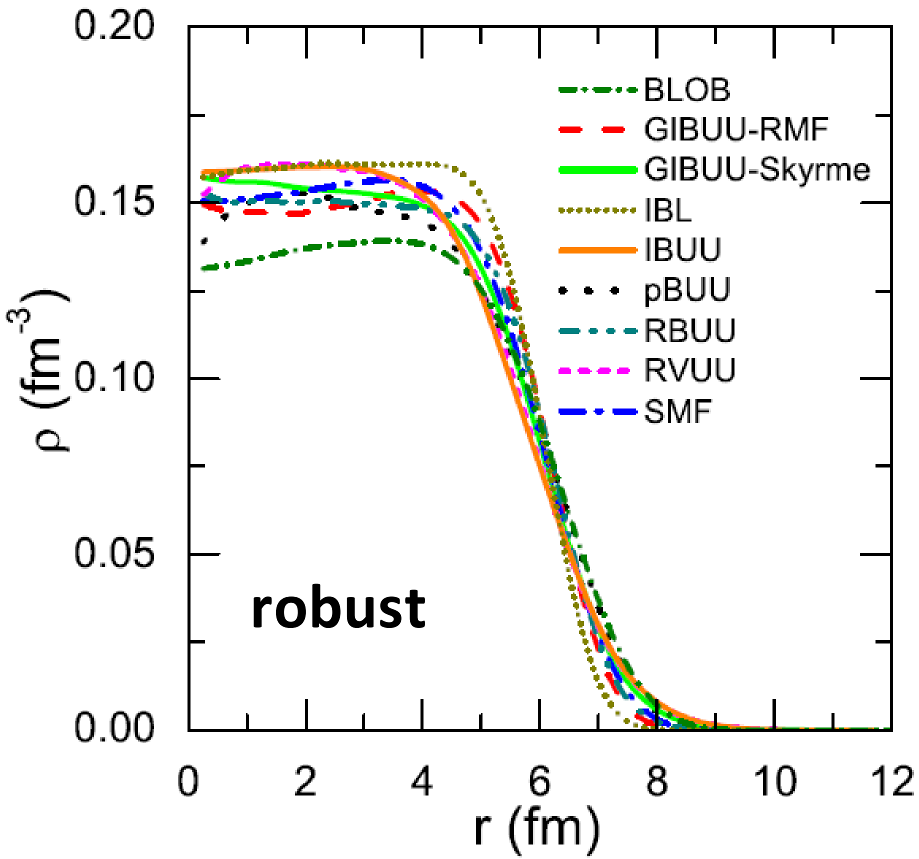
Mode C). Common initialization

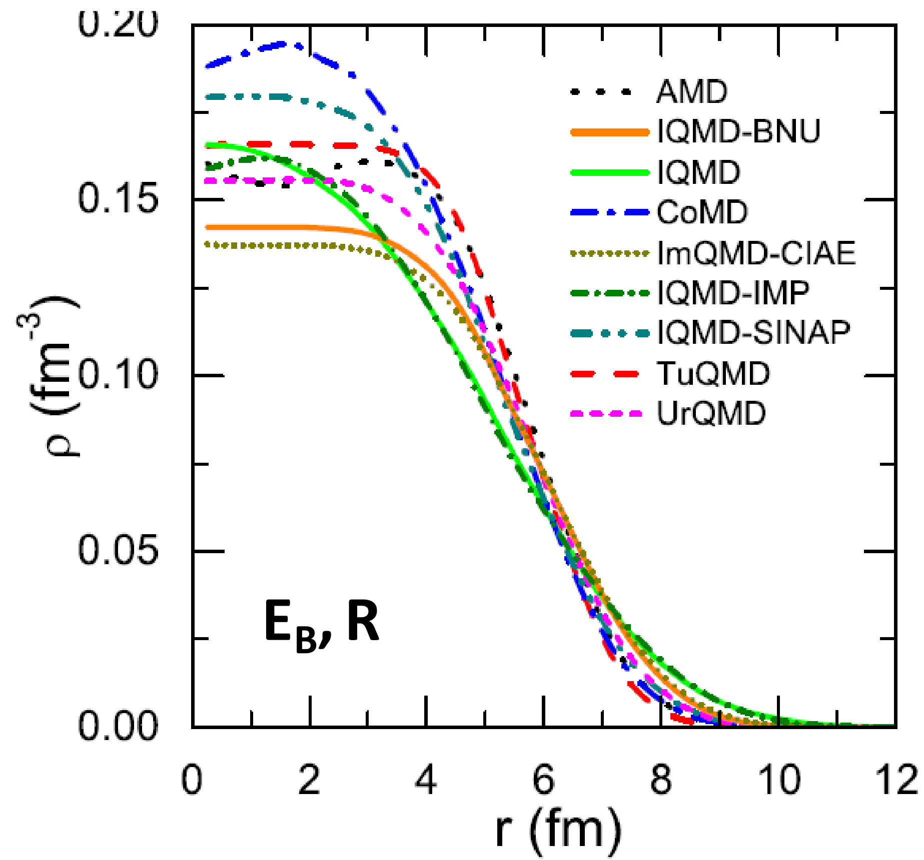
Mode D). Au+Au@400 AMeV

D-Full: 400AMeV, both mean field and NN scattering

Mode E). Box calculation

Initial density profile





average over 1000 events

BUU: mostly follow the suggested Woods-Saxon distribution, easily stable

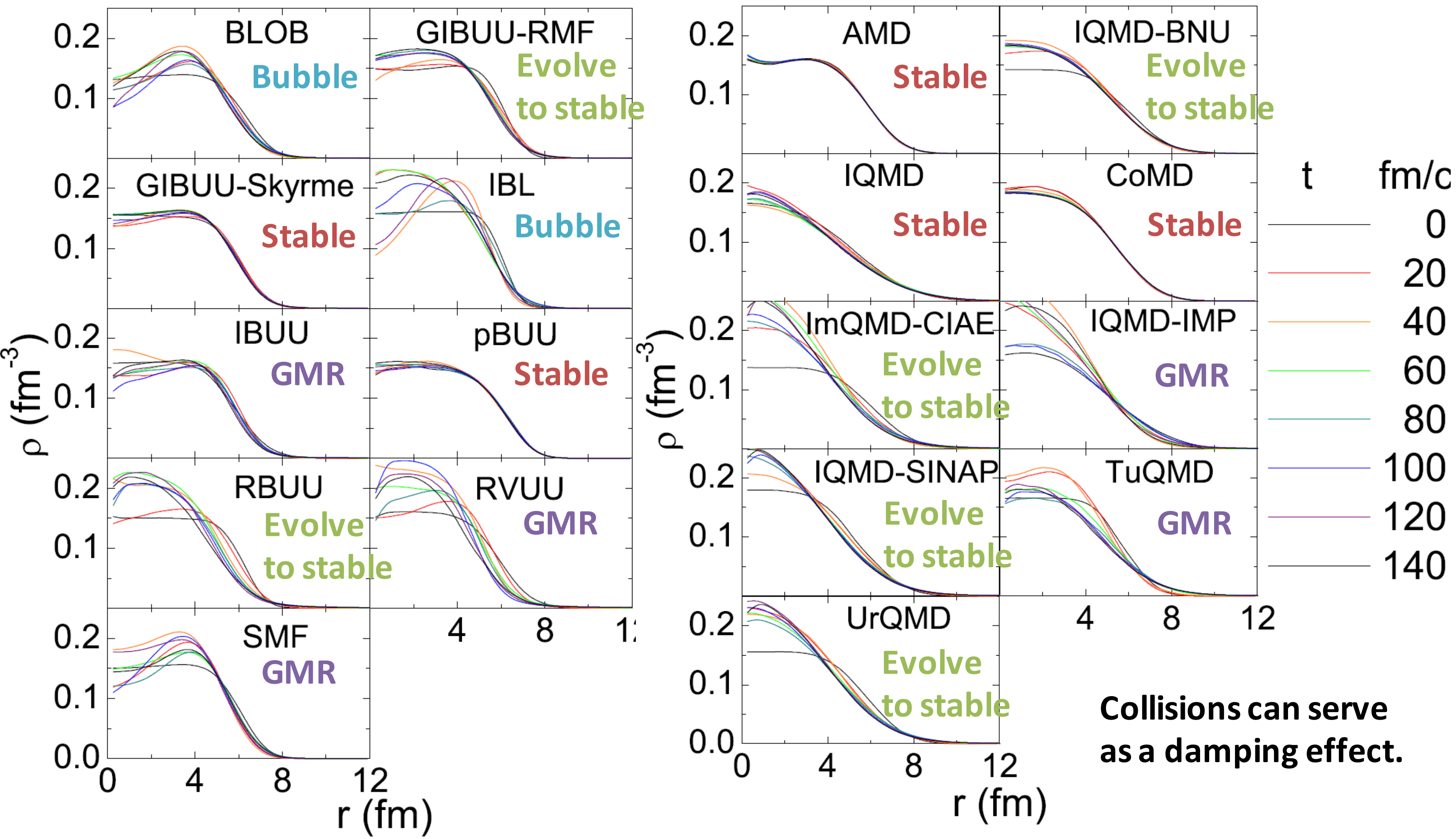
QMD: mostly deviate from the suggested Woods-Saxon distribution

ground state? Thomas-Fermi or Hartree-Fock, frictional cooling

Difficult to get a common initialization

Code name	Shape of particles	$(\Delta x)^2 (\mathrm{fm}^2)$[a] Size of particles	$\delta\langle r^2\rangle^{1/2}$ (fm)[b] $\langle r^2\rangle^{1/2} - \langle r^2\rangle_{\mathrm{WS}}^{1/2}$	$\delta\langle r^4\rangle^{1/4}$ (fm)[c] $\langle r^4\rangle^{1/4} - \langle r^4\rangle_{\mathrm{WS}}^{1/4}$
AMD	Gaussian	1.56	− 0.01	0.01
IQMD-BNU	Gaussian	1.97	0.32	0.39
IQMD	Gaussian	2.16	0.64	0.85
CoMD	Gaussian	1.32	− 0.11	− 0.04
ImQMD-CIAE	Gaussian	2.02	0.39	0.47
IQMD-IMP	Gaussian	1.92	0.61	0.80
IQMD-SINAP	Gaussian	2.16	0.03	0.12
TuQMD	Gaussian	2.16	− 0.17	− 0.17
UrQMD	Gaussian	2	0.12	0.18
	Shape of test particle	$(\Delta x)^2$ (fm^2) or l (fm)[f] Size of test particles		
BLOB	Triangle	2	0.10	0.07
GIBUU-RMF	Gaussian	1	− 0.18	− 0.26
GIBUU-Skyrme	Gaussian	1	− 0.03	− 0.03
IBL	Gaussian	2	− 0.32	− 0.42
IBUU	Triangle	1	0.01	0.04
pBUU	Point	0[g]	0.01	− 0.02
RBUU	Invar. Gauss	1.4	− 0.12	− 0.19
RVUU	Point	0	0.01	0.03
SMF	Triangle	2	− 0.13	− 0.18

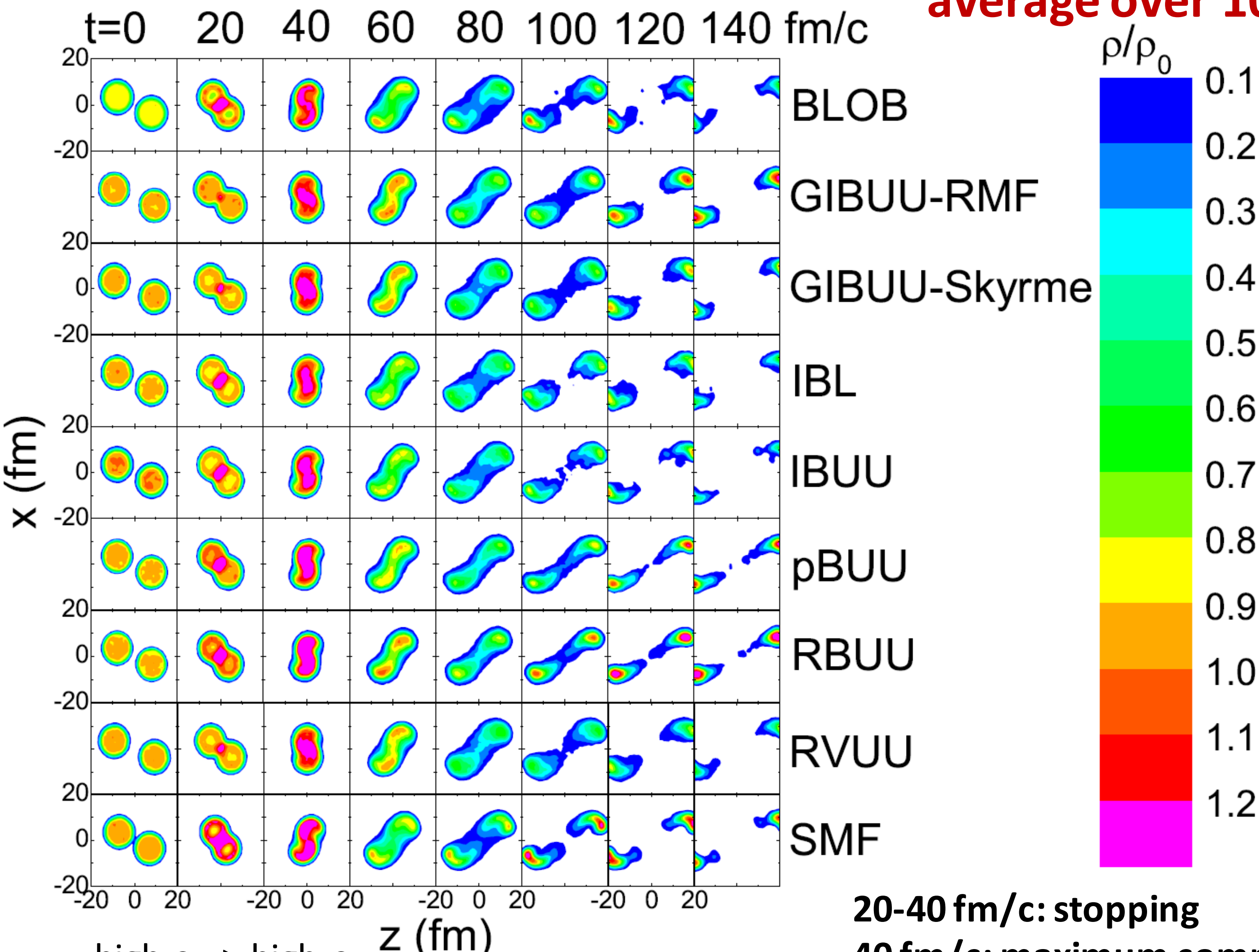
Stability (b=20 fm)



Collisions can serve as a damping effect.

Density evolution at b = 7 fm - BUU

average over 1000 events



high ρ => high ρ

Density => attempted collisions, mean field

Pauli blocking => successful collisions

20-40 fm/c: stopping

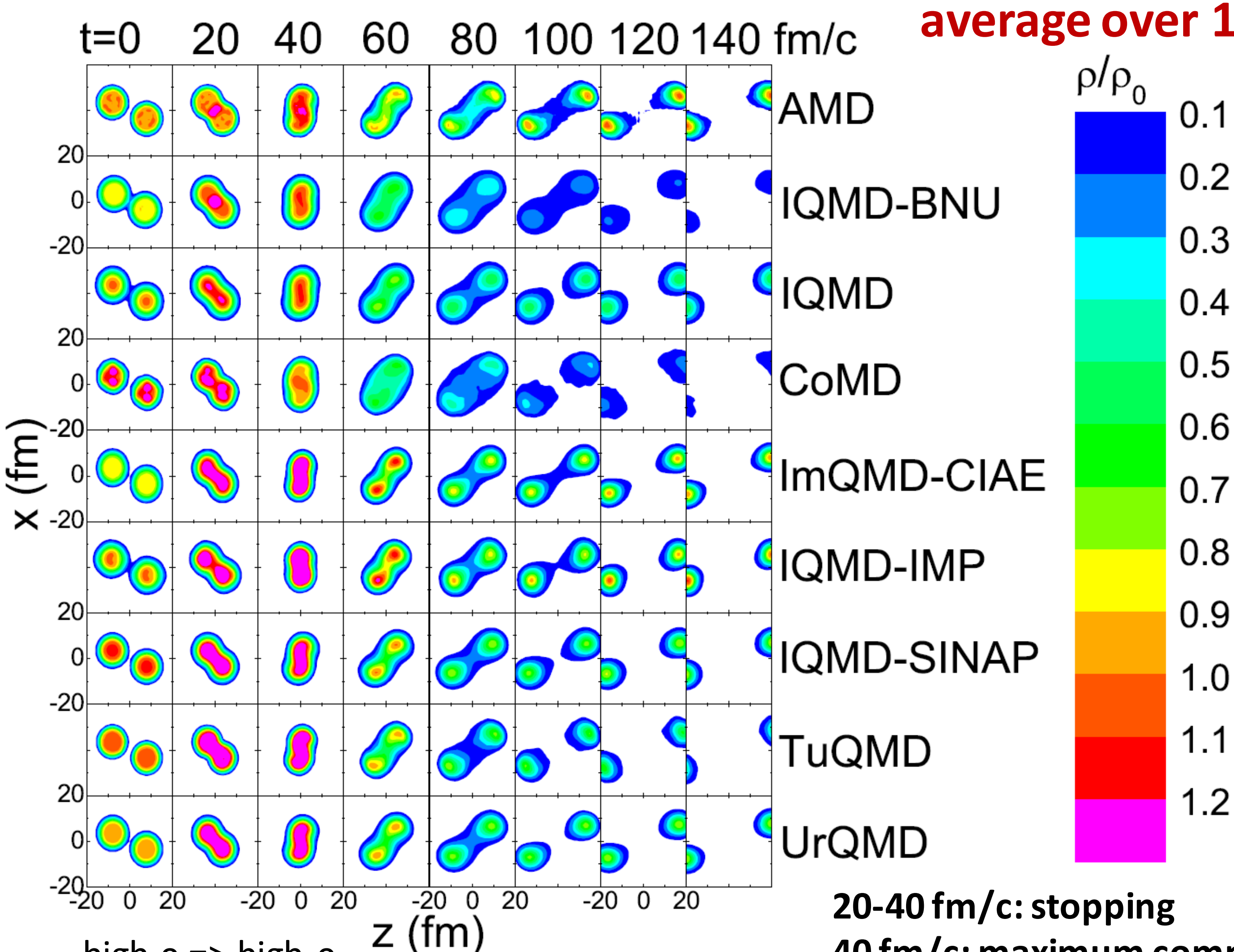
40 fm/c: maximum compression

60-80 fm/c: sideward flow developed

Around 100 fm/c: neck

Density evolution at b = 7 fm - QMD

average over 1000 events



high ρ => high ρ

Density => attempted collisions, mean field

Pauli blocking => successful collisions

20-40 fm/c: stopping

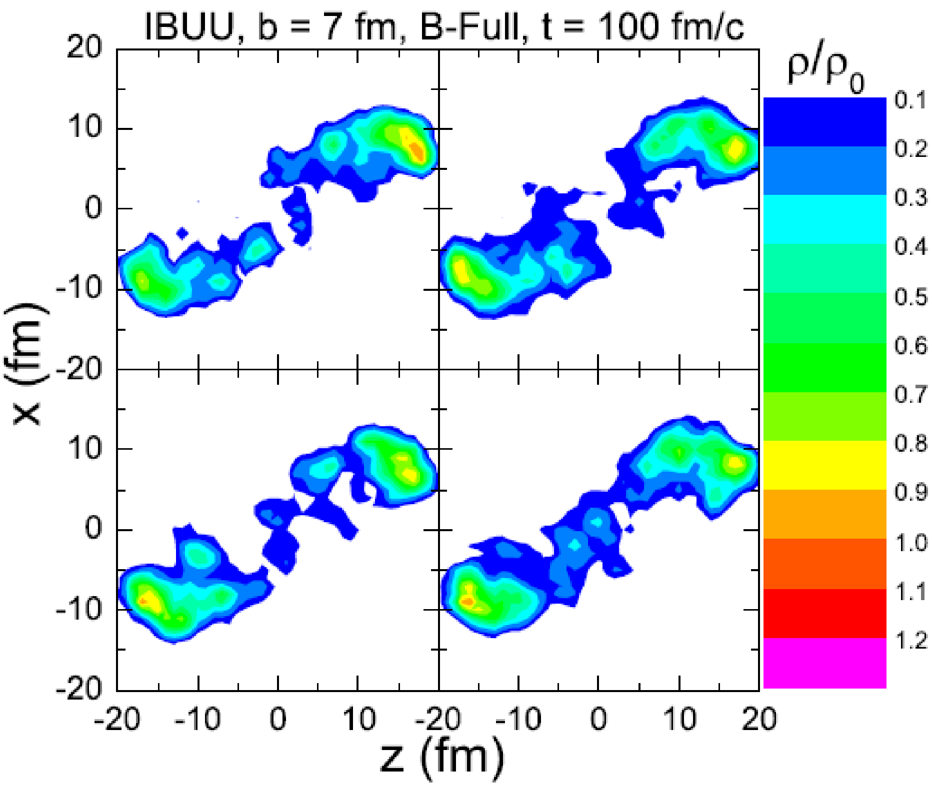
40 fm/c: maximum compression

60-80 fm/c: sideward flow developed

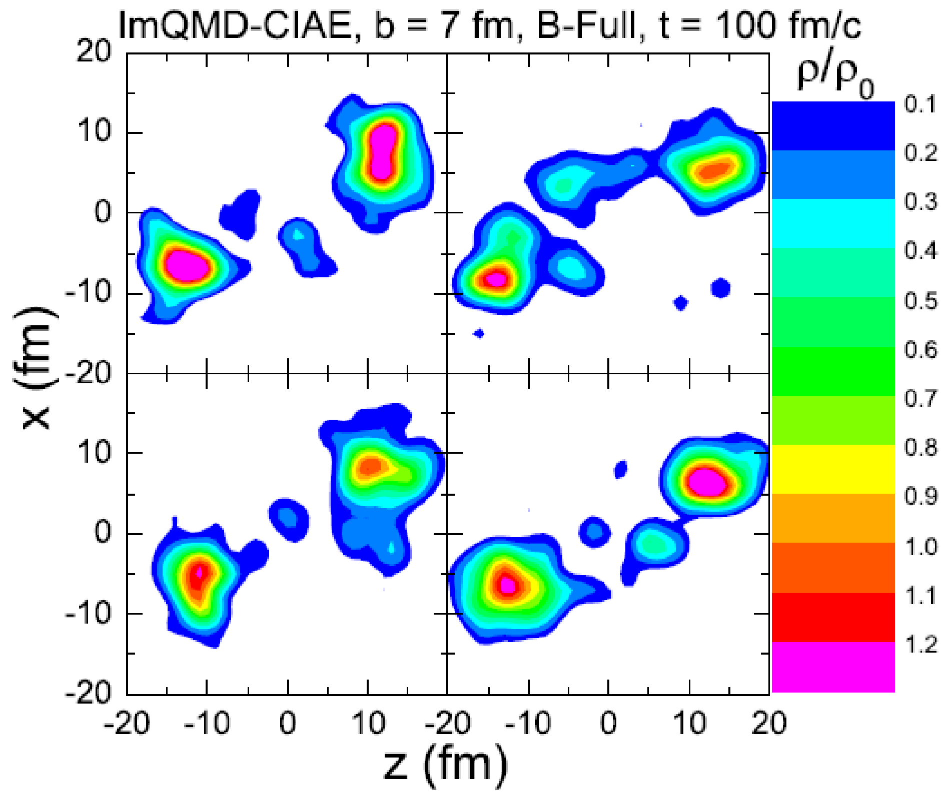
Around 100 fm/c: neck

Fluctuation in BUU and QMD

4 runs with 100 TPs per nucleon



4 individual events



Fluctuation related to N_{TP}

Fluctuation related to the Gaussian width

How are the collisions realized?
(in most transport models)

- Bertsch's approach (Phys. Rep. 160, 189 (1988))
  - Go into the C.M. frame of the two nucleons
  - Collision can happen if

$$b = \sqrt{(\Delta \boldsymbol{r})^2 - (\Delta \boldsymbol{r} \cdot \boldsymbol{p}/p)^2} < \sqrt{\sigma/\pi}$$ and $$\left|\frac{\Delta \boldsymbol{r} \cdot \boldsymbol{p}}{p}\right| < \left(\frac{p}{\sqrt{p^2 + m_1^2}} + \frac{p}{\sqrt{p^2 + m_2^2}}\right) \delta t/2$$

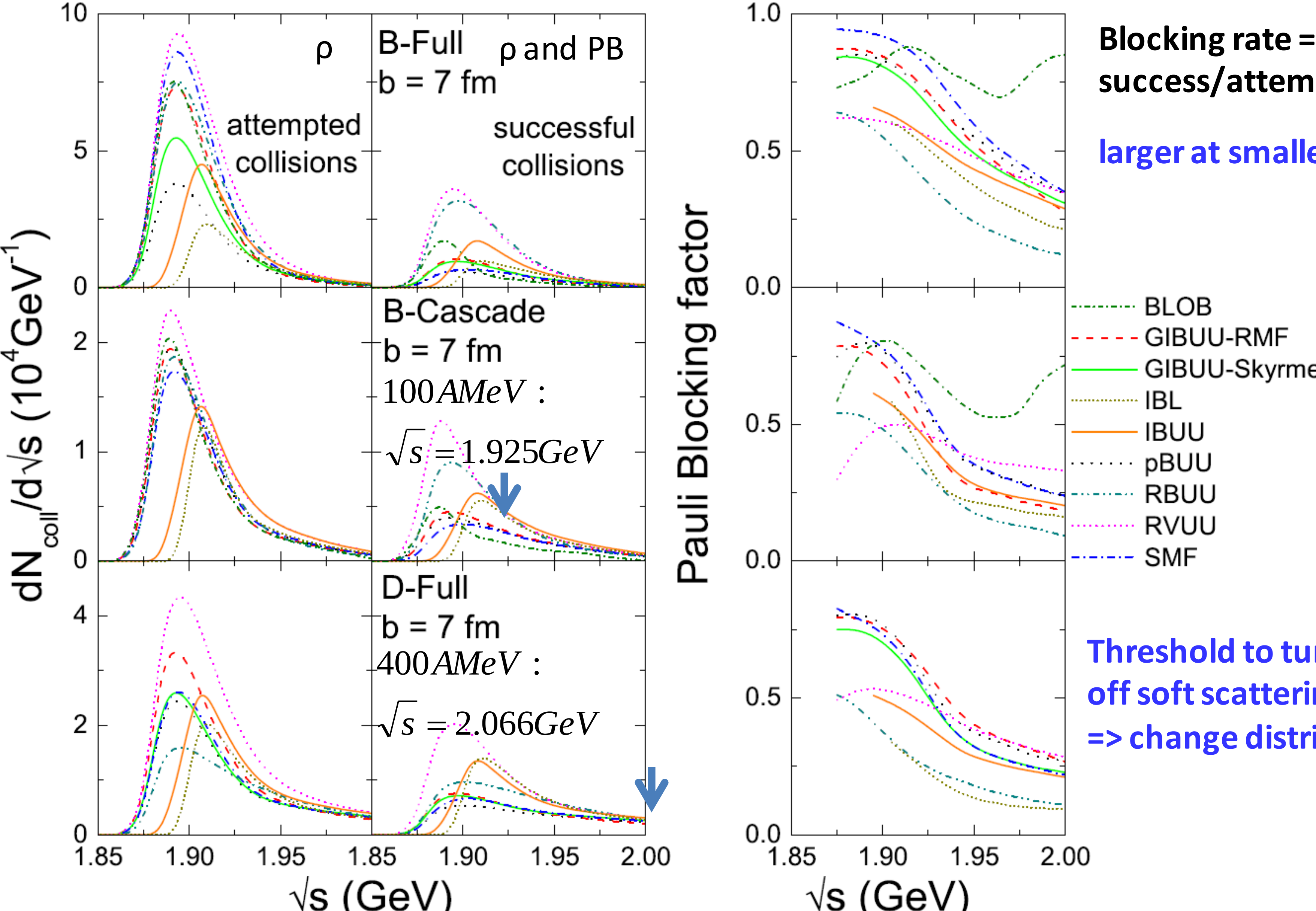
  - If collision happen, change the direction of P_{cm} in the C.M. frame
  - Boost the momenta of the two nucleons to lab frame
  - Check phase space density; if Pauli blocked, return to the initial momenta

How is Pauli blocking realized?

(in most transport models)

- Update the occupation probability f_i at each time step (f_i is calculated differently for each code, and can be different for BUU and QMD)
- When a collision is attempted, check the Pauli blocking probability $1-(1-f_i)(1-f_j)$
- Additional constraint if any (phase-space constraint …)

NN scattering & Pauli blocking - BUU

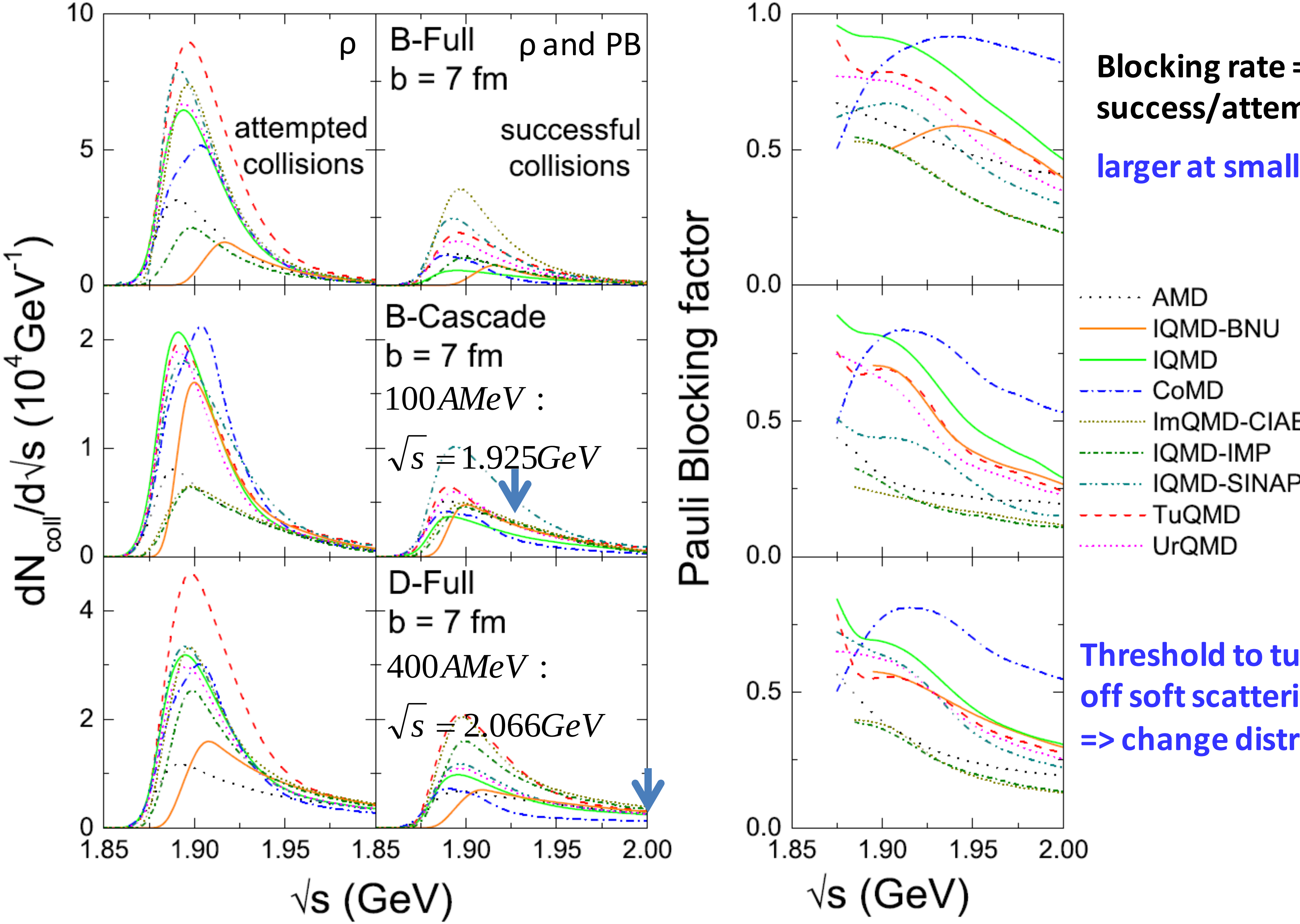


Blocking rate = 1-success/attempted

larger at smaller √s

Threshold to turn off soft scatterings => change distribution

NN scattering & Pauli Blocking - QMD



Blocking rate = 1-success/attempted

larger at smaller √s

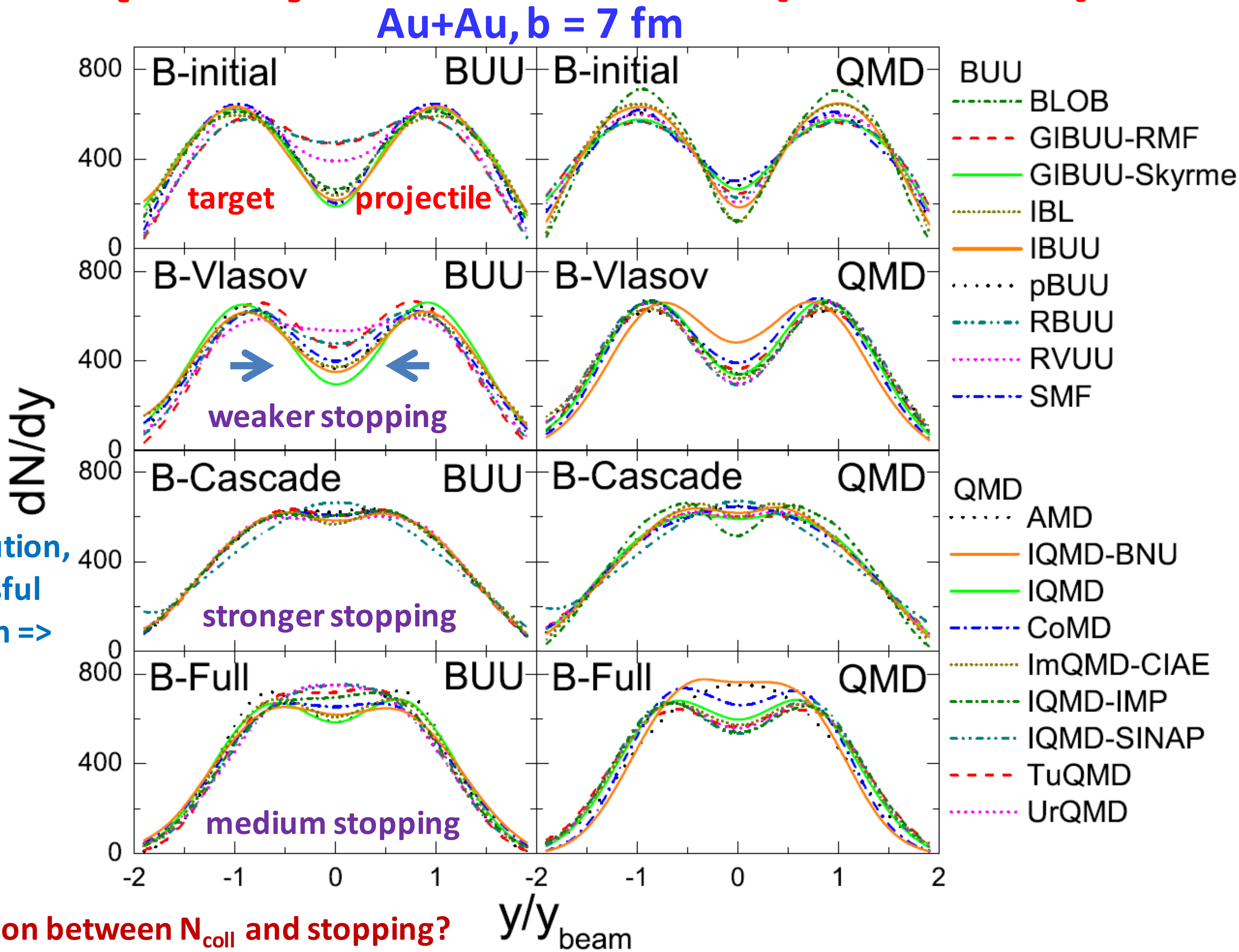
Threshold to turn off soft scatterings => change distribution

Code name	Occupation probability f_i	Blocking probability[a]	Additional constraints
AMD	Antisymmetrized wave packets[b]	Physical wave packet[b]	No
IQMD-BNU	f_i in h^3	$1-(1-f_i)(1-f_j)$	Yes[c]
IQMD	f_i in h^3	$1-(1-f_i)(1-f_j)$	Yes[d]
CoMD	f_i in h^3	$f_i', f_j' < f_{\max} = 1.05 - 1.1$	Yes[e]
ImQMD-CIAE	f_i in h^3	$1-(1-f_i)(1-f_j)$	No
IQMD-IMP	f_i in phase-space cell with $dx = 3.367$ fm, $dp = 89.3$ MeV/c	$1-(1-f_i)(1-f_j)$	No
IQMD-SINAP	$f_i = \sum_k e^{-(\vec{r}_k-\vec{r}_i)^2/[2(\Delta x)^2]} e^{-(\vec{p}_k-\vec{p}_i)^2\cdot 2(\Delta x)^2/\hbar^2}$	$1-(1-f_i)(1-f_j)$	No
TuQMD	f_i in spherical phase-space cell with $dx = 3.0$ fm, $dp = 240$ MeV/c[f]	$1-(1-f_i)(1-f_j)$	Yes[g]
UrQMD	$f_i = \sum_k e^{-(\vec{r}_k-\vec{r}_i)^2/[2(\Delta x)^2]} e^{-(\vec{p}_k-\vec{p}_i)^2\cdot 2(\Delta x)^2/\hbar^2}$	$1-(1-f_i)(1-f_j)$	Yes[h]
BLOB	f_i in sphere with radius 3.5 fm with Gaussian weight in momentum space[i]	$1-(1-f_i)(1-f_j)$	Yes[j]
GIBUU-RMF	f_i in phase-space cell with $dx = 1.4$ fm, $dp = 68$ MeV/c	$1-(1-f_i)(1-f_j)$	No
GIBUU-Skyrme	f_i in phase-space cell with $dx = 1.4$ fm, $dp = 68$ MeV/c	$1-(1-f_i)(1-f_j)$	No
IBL	f_i in h^3	$1-(1-f_i)(1-f_j)$	yes[k]
IBUU	f_i in phase-space cell with $dx = 2.73$ fm, $dp = 187$ MeV/c	$1-(1-f_i)(1-f_j)$	No
pBUU	f_i in same and neighboring spatial cell[l]	$1-(1-f_i)(1-f_j)$	No
RBUU	f_i in phase-space cell with $dx = 1.4$ fm, $dp = 64$ MeV/c	$1-(1-f_i)(1-f_j)$	No
RVUU	f_i in phase-space cell with $dx = 1.14$ fm, $dp = 331$ MeV/c[m]	$1-(1-f_i)(1-f_j)$	No
SMF	f_i in sphere with radius 2.53 fm with Gaussian weight in momentum space[n]	$1-(1-f_i)(1-f_j)$	No

Code name	Attempted collisions	First collisions within same nucleus
AMD	$p = \alpha e^{-\nu R_{ij}^2} v_{ij} \Delta t$	Yes
IQMD-BNU	Bertsch approach[d]	No
IQMD	Bertsch approach	Yes
CoMD	$p = 1 - e^{\Delta t/\tau}$	Yes
ImQMD-CIAE	Bertsch approach	Yes
IQMD-IMP	Bertsch approach	Yes
IQMD-SINAP	Bertsch approach	Yes
TuQMD	Bertsch approach	Yes
UrQMD	Collision time table[e]	Yes

Code name	Attempted collisions	First collisions within same nucleus
BLOB	$p = \sigma^{\text{med}} \frac{(\rho_i+\rho_j)}{2} v_{ij} \Delta t$	Yes
GIBUU-RMF	Bertsch approach	Yes
GIBUU-Skyrme	Bertsch approach	Yes
IBL	Bertsch approach	No
IBUU	Bertsch approach	Yes
pBUU	Cell[h]	Yes
RBUU	Bertsch approach	Yes
RVUU	Bertsch approach	Yes
SMF	$p = \sigma^{\text{med}} \frac{(\rho_i+\rho_j)}{2} v_{ij} \Delta t$	Yes

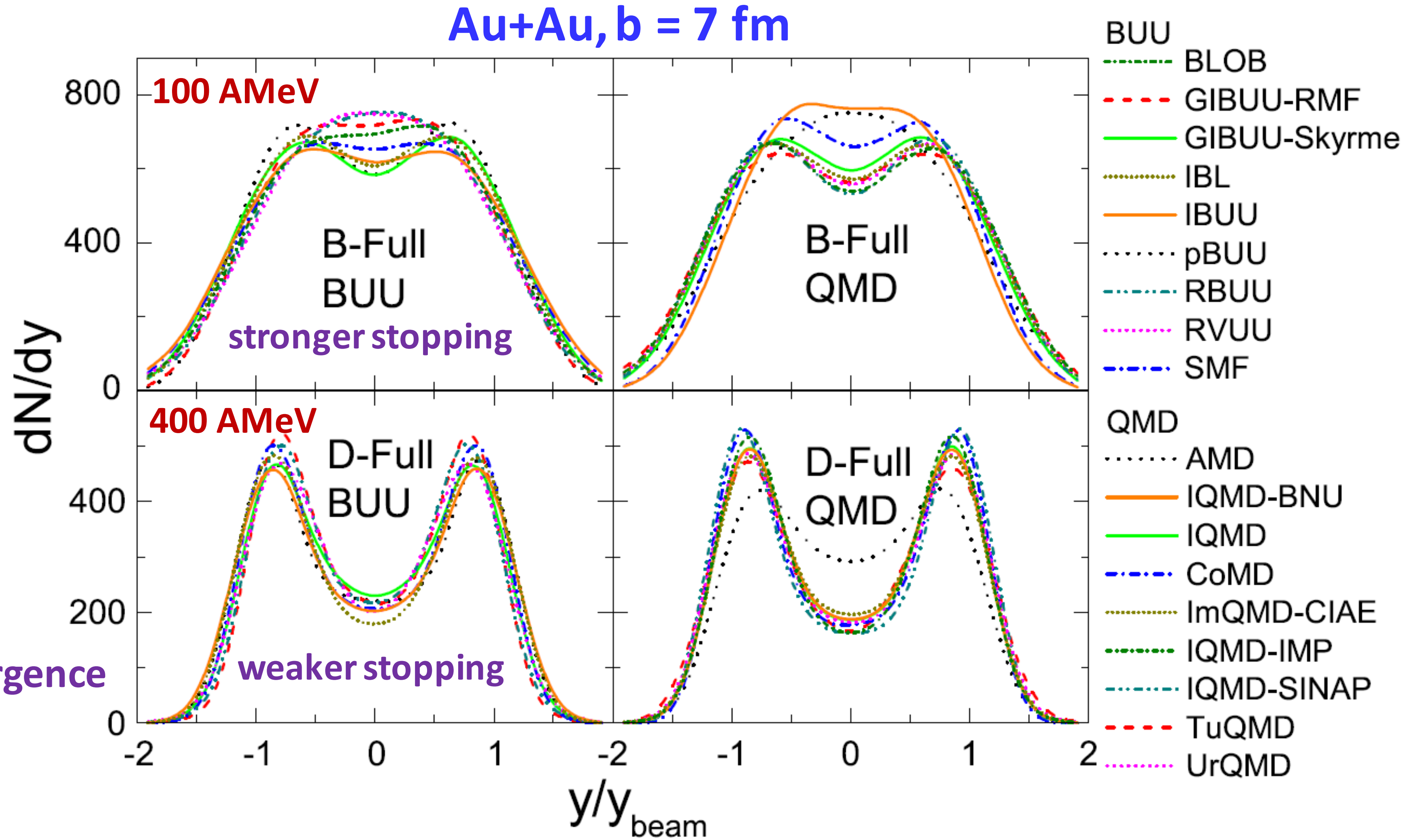
Rapidity distribution (B-mode)



Initial distribution, successful collision =>

Correlation between N_{coll} and stopping?

Rapidity distribution (Full mode)

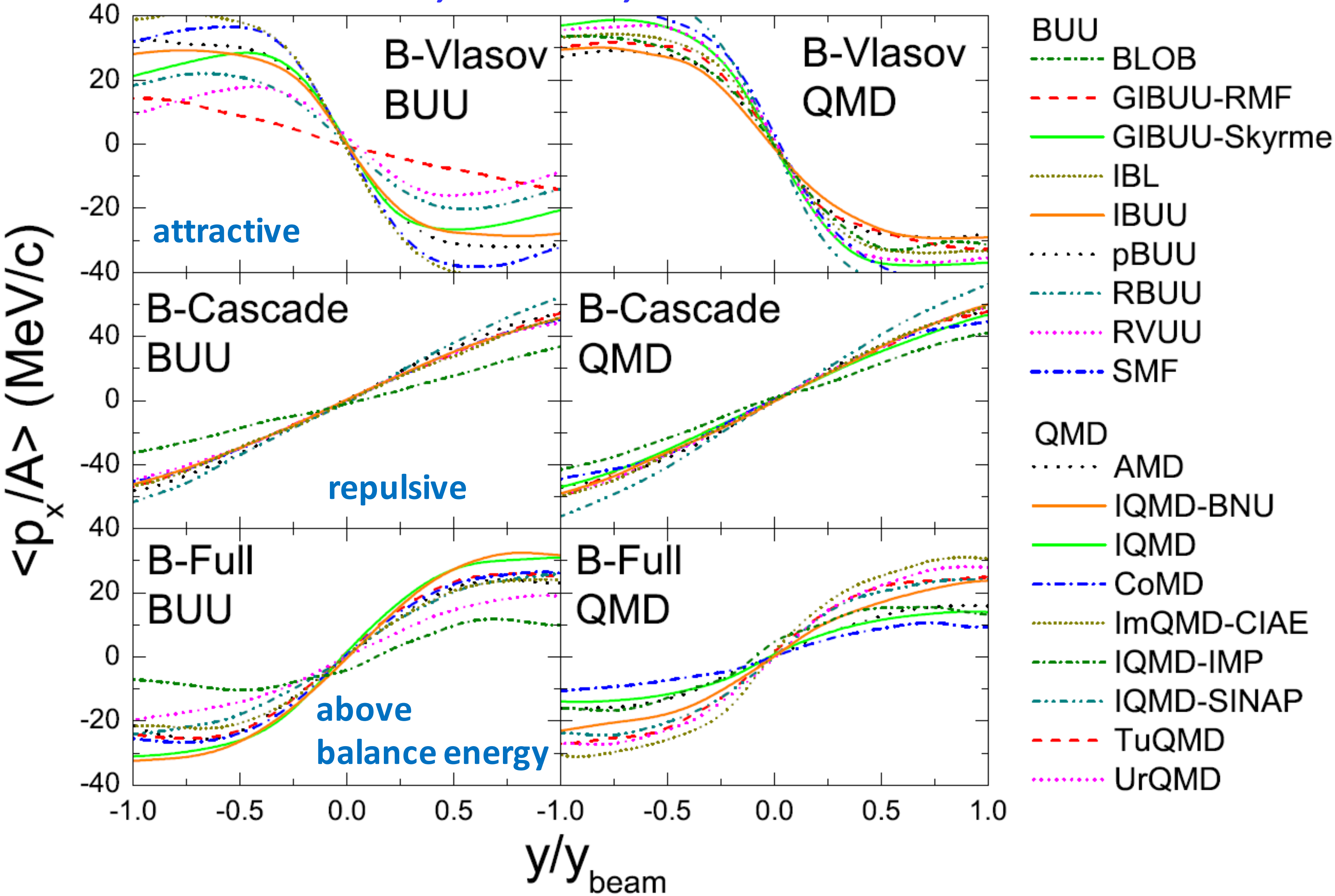


Good convergence

400AMeV: weaker Pauli blocking, less sensitive to initialization, good convergence of N_{coll} at larger $\sqrt{s}$

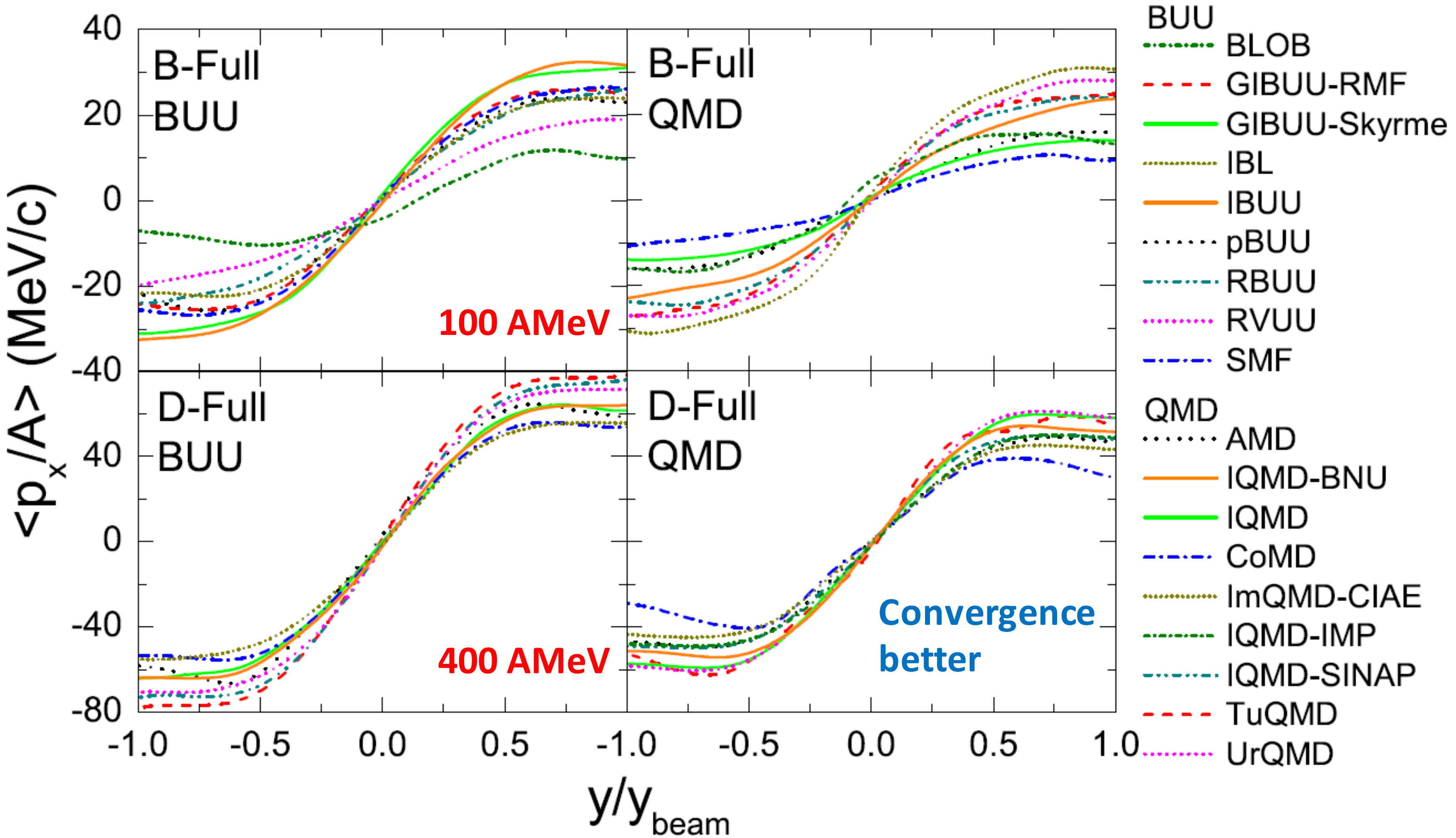
Transverse flow (B-mode)

Au+Au, b = 7 fm, 100 AMeV

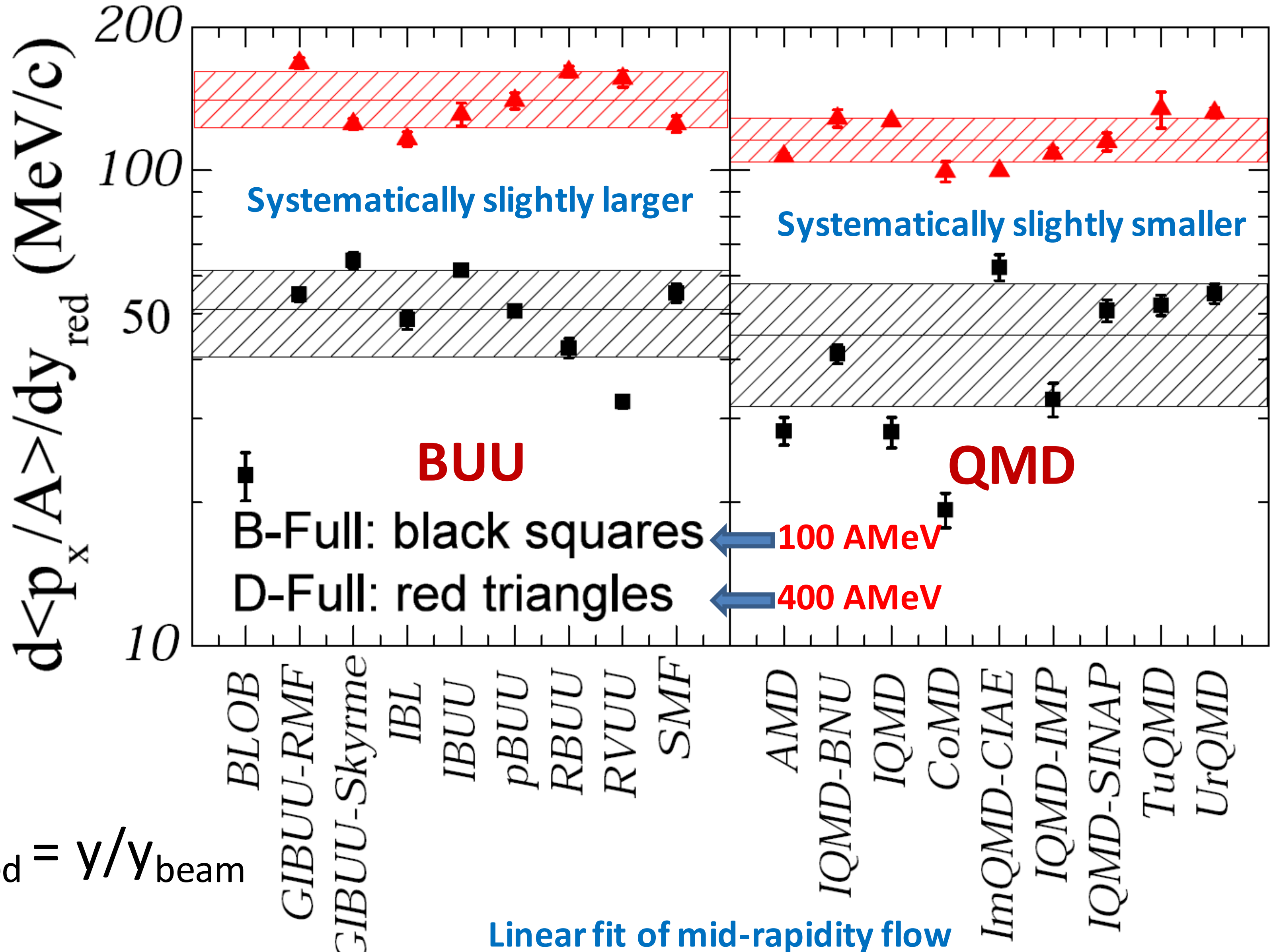


Transverse flow (Full-mode)

Au+Au, b = 7 fm



Uncertainties: initialization, Pauli blocking



$y_{red} = y/y_{beam}$

Theoretical uncertainties of flow parameter:

about 30% at 100 AMeV, 13% at 400 AMeV

conclusions and outlook

- **Initialization** and **Pauli blocking** are the two main sources of uncertainties in heavy-ion simulations

- Theoretically error bar for collective flow is about **13%** at 400 AMeV and **30%** at 100 AMeV (need to be reduced!)

- Future test: **particle production**, **fluctuation and fragmentation**, **momentum dependence**, …

An agreement in treating each part of the simulation

Towards an accurate description of heavy-ion collisions!

Transport2016 - box calculation

the Box Simulation Organizing Committee
Maria Colonna
Akira Ono
Yongjia Wang
Jun Xu
Yingxun Zhang

Initial configuration well controlled
Theoretical answer available
collision rate, blocking rate, mean field

Local organizers:
Chair: Fengshou Zhang (BNU, China)

Lie-Wen Chen (SJTU, China), Qingfeng Li (HZU, China), Yongjia Wang (HZU, China), Jun Xu (SINAP, China), Yingxun Zhang(CIAE, China), Zhigang Xiao (TSHU, China), Fengshou Zhang (BNU, China)

Date: June 18th, 2016, right after Nusym16
http://info.phys.tsinghua.edu.cn/enpg/nusym16/htmls/transport-model.html

Thank you!

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