Pion production in heavy-ion collision by the AMD+JAM approach

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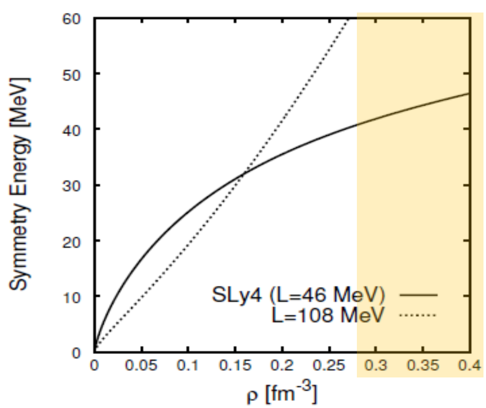
Symmetry energy and Heavy-ion collision

＊**Symmetry energy S (ρ)** :

EOS for asymmetric nuclear matter

$$\frac{E}{A}(\rho,\alpha) = \frac{E}{A}(\rho,0) + \mathbf{S(\rho)}[\alpha^2 + O(\alpha^4)]$$

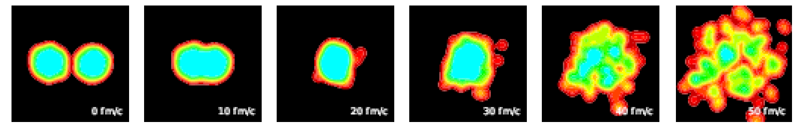


Interest: High density $\rho \sim 2\rho_0$

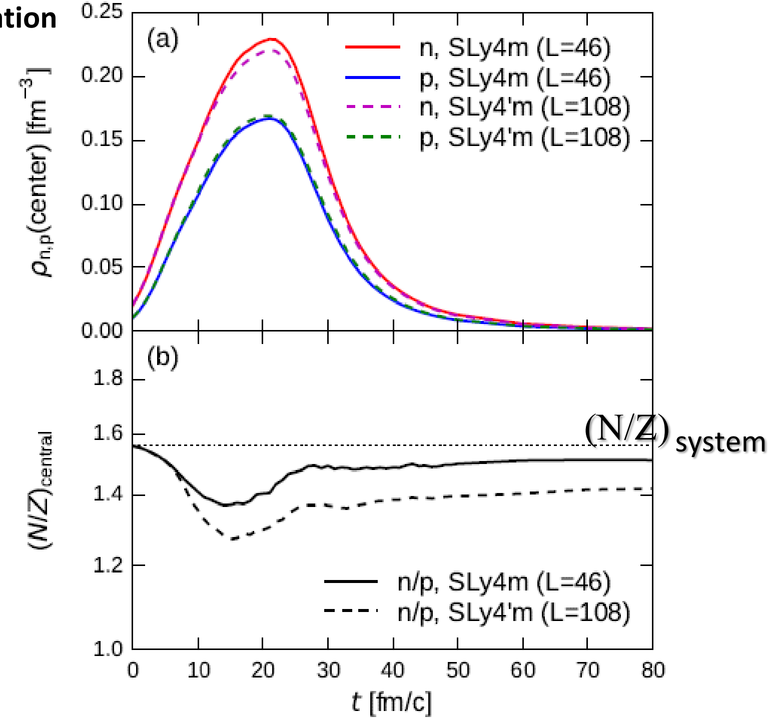
What is a sensitive observable for experiments to constrain S(ρ)?

→ **``Pion''** is a good probe !?

＊**Heavy-ion collisions (Neutron–rich system)**



AMD calculation ^{132}Sn + ^{124}Sn, $E/A = 300$ MeV, $b \sim 0$



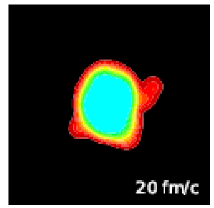
Clear difference of N/Z in high density region

due to different **S(ρ)**

Pion production in Heavy-ion collision

∗ **Pions, Δ resonances:**

Formation in NN collisions at early times in the compressed part of the system



π^- production (main reaction)

$$nn \rightarrow p\Delta^-$$
$$\Delta^- \rightarrow n\pi^-$$

π^+ production (main)

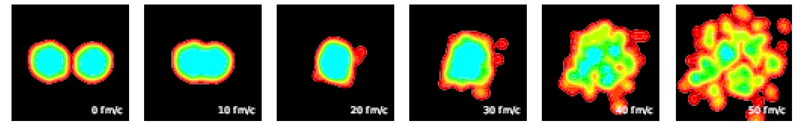
$$pp \rightarrow n\Delta^{++}$$
$$\Delta^{++} \rightarrow p\pi^+$$

Simple expectation :

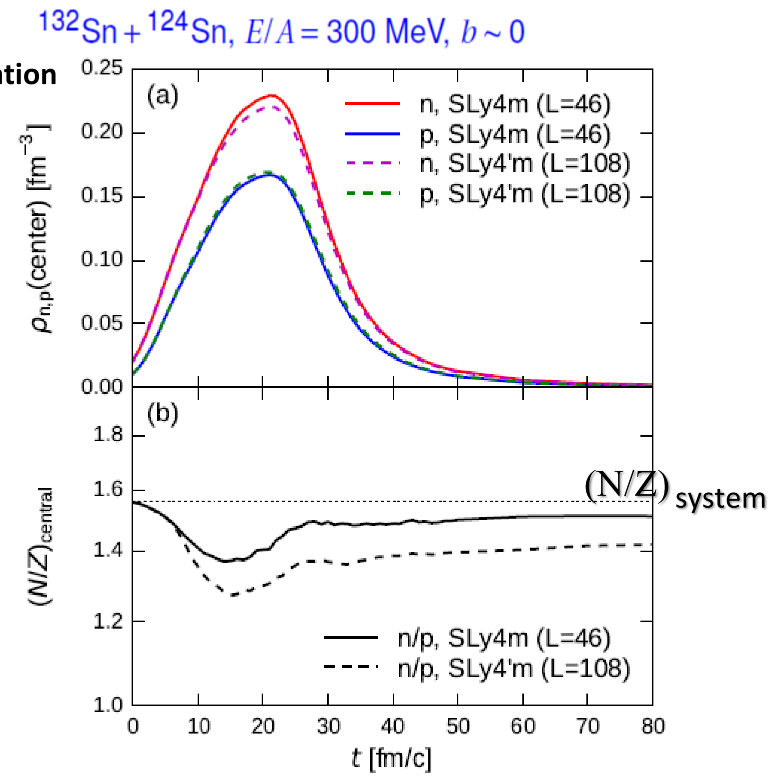
$$\left(\frac{\pi^-}{\pi^+}\right) \simeq \frac{5N^2 + NZ}{5Z^2 + NZ} \simeq \left(\frac{N}{Z}\right)^2$$

B. A. Li, PRL 88 (2002) 192701

∗ **Heavy-ion collisions (Neutron–rich system)**



AMD calculation

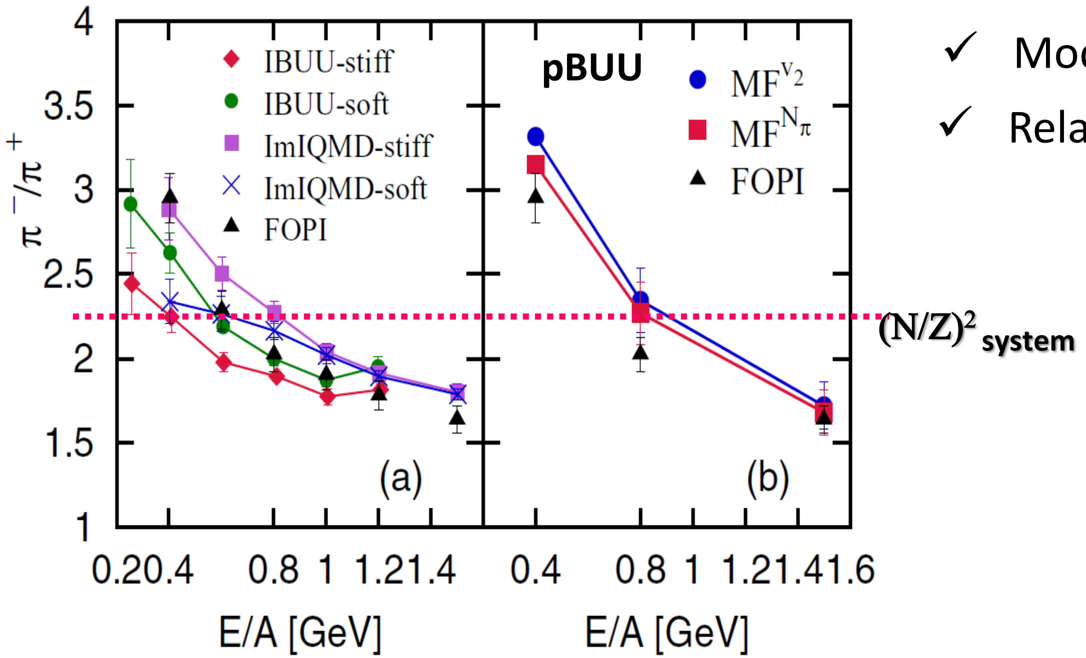


⇒ π^-/π^+ ratio is related to some kind of $(N/Z)^2$ ratio which is supposed to be sensitive to the symmetry energy at high densities.

Pion and Symmetry energy

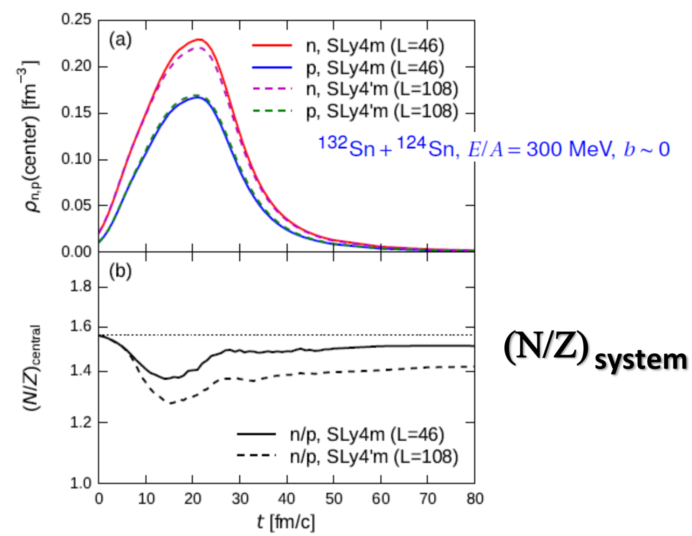
- **Pion calculations by some models**
  - B. A. Li, PRL 88 (2002) 192701 **: IBUU**
  - Z. Xiao, B. A. Li, L. W. Chen, G.-C. Yong, and M. Zhang, PRL102 (2009) 062502 **: IBUU04**
  - Z. Q. Feng and G. M. Jin, PLB 683 (2010) 140 **: ImIQMD**
  - J. Hong and P. Danielewicz , PRC90 (2014) 024605 **: pBUU**
  - Wen-Mei Guo, Gao-Chan Yong and Wei Zuo, PRC90 (2014) 044605 ... etc.

- Pion ratio in central Au+Au collisions: Theory vs. Exp. Data



J. Hong and P. Danielewicz , PRC90 (2014) 024605

- ✓ Model predictions **do not agree**
- ✓ Relation $\pi^-/\pi^+ \simeq (N/Z)^2$ **does not hold**



⇒We need more complete understanding of the relation between pion and symmetry energy

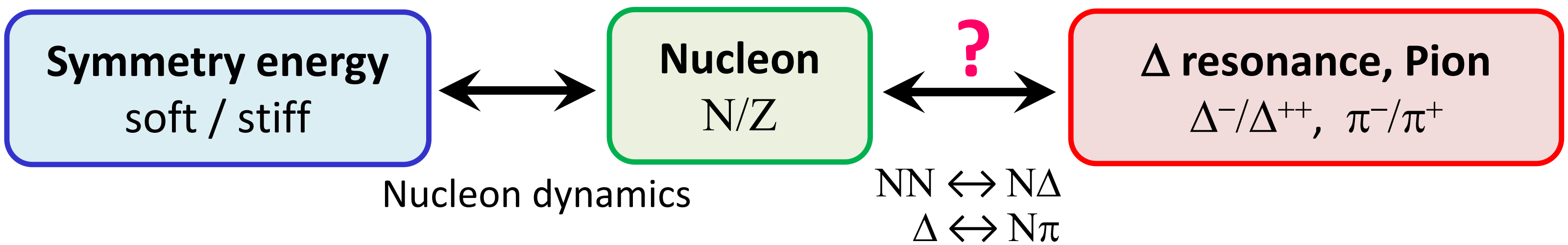
Our study

^{132}Sn + ^{124}Sn Collision @E/A=300MeV

- Experiment at RIKEN/RIBF T. Isobe, T. Murakami *et al.*
- Neutron rich system (N/Z) = 1.56 → $\pi^- > \pi^+$

➢ **Motivation:**

We like to understand the mechanism how Δ resonances and pions are produced, reflecting the dynamics of neutrons and protons.



➢ **Theoretical Model:** AMD + JAM

AMD
- Nucleon dynamics
- Treatment of cluster correlation

JAM
- π, Δ production in the reaction process
- hadronic cascade model

Transport model (AMD + JAM)

- Coupled equations for $f_\alpha(\mathbf{r}, \mathbf{p}, t)$ $(\alpha = N, \Delta, \pi)$

$$\frac{\partial f_N}{\partial t} + \frac{\partial h_N}{\partial \boldsymbol{p}} \cdot \frac{\partial f_N}{\partial \boldsymbol{r}} - \frac{\partial h_N[f_N, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_N}{\partial \boldsymbol{p}} = I_N[f_N, f_{\Delta,\pi}]$$

$$\frac{\partial f_{\Delta,\pi}}{\partial t} + \frac{\partial h_{\Delta,\pi}}{\partial \boldsymbol{p}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{r}} - \frac{\partial h_{\Delta,\pi}[f_N, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{p}} = I_{\Delta,\pi}[f_N, f_{\Delta,\pi}]$$

$I_N[f_N, f_{\Delta,\pi}]$:collision term

$N N \to N N$
$N N \to N \Delta$
$N \Delta \to N N$
$\Delta \to N \pi$
$N \pi \to \Delta$ ... etc.

Our model: JAM coupled with AMD

Perturbative treatment of pion and Δ particle production

$$I_N = I_N^{\mathrm{el}}[f_N, 0] + \lambda I'_N[f_N, f_{\Delta,\pi}]$$

$f_{\Delta,\pi} = O(\lambda)$: Δ and pion productions are rare

$f_N = f_N^{(0)} + \lambda f_N^{(1)} + \ldots$

- **Nucleon f_N : Zeroth order equation**

$$\frac{\partial f_N^{(0)}}{\partial t} + \frac{\partial h_N}{\partial \boldsymbol{p}} \cdot \frac{\partial f_N^{(0)}}{\partial \boldsymbol{r}} - \frac{\partial h_N[f_N^{(0)}, 0]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_N^{(0)}}{\partial \boldsymbol{p}} = I_N^{\mathrm{el}}[f_N^{(0)}, 0]$$

Solved by AMD

- **Δ particle f_Δ and pion f_π : First order equation**

$$\frac{\partial f_{\Delta,\pi}}{\partial t} + \frac{\partial h_{\Delta,\pi}}{\partial \boldsymbol{p}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{r}} - \frac{\partial h_{\Delta,\pi}[f_N^{(0)}, f_{\Delta,\pi}]}{\partial \boldsymbol{r}} \cdot \frac{\partial f_{\Delta,\pi}}{\partial \boldsymbol{p}} = I_{\Delta,\pi}[f_N^{(0)}, f_{\Delta,\pi}]$$

Solved by JAM for given $f_N^{(0)}$

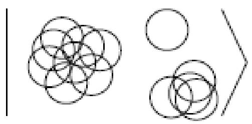
Transport model（AMD + JAM)

➢ AMD (Antisymmetrized Molecular Dynamics)

A. Ono, H. Horiuchi, T. Maruyama, and A. Ohnishi, PTP87 (1992) 1185

→ Ono-san's talk

- AMD wave function



$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = $(2.5\ \mathrm{fm})^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Solve the time evolution of the wave packet centroids Z

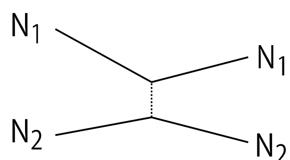
✓ Effective interaction

- Turn on/off Cluster correlation

- Without Cluster

N1 + N2 -> N1 + N2

N1, N2 : Colliding nucleons



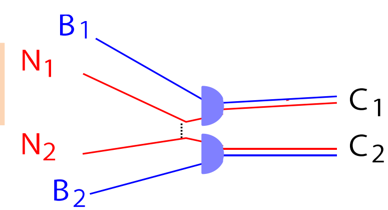
- With Cluster

N1 + B1 + N2 + B2 -> C1 + C2

N1, N2: Colliding nucleons

B1, B2: Spectator nucleons/clusters

C1, C2: N, (2N), (3N), (4N) (up to α cluster)



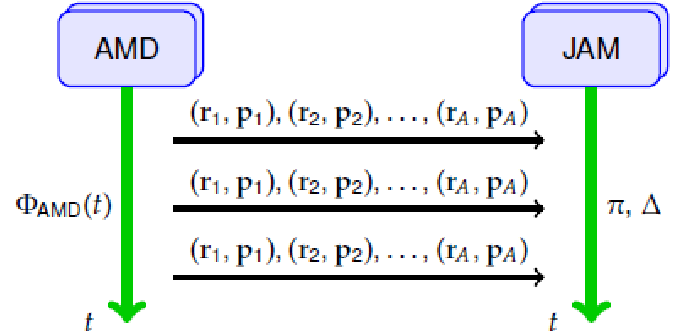
➢ JAM (Jet AA Microscopic transport model)

Y. Nara, N. Otuka, A. Ohnishi, K. Niita, S. Chiba, PRC61 (2000) 024901

- Applied to high-energy collisions (1 〜 158 A GeV)
- Hadron-Hadron reactions are based on experimental data and the detailed balance.
- No mean field (default)
- *s*-wave pion production ($NN \rightarrow NN\pi$) is turned off. … etc.

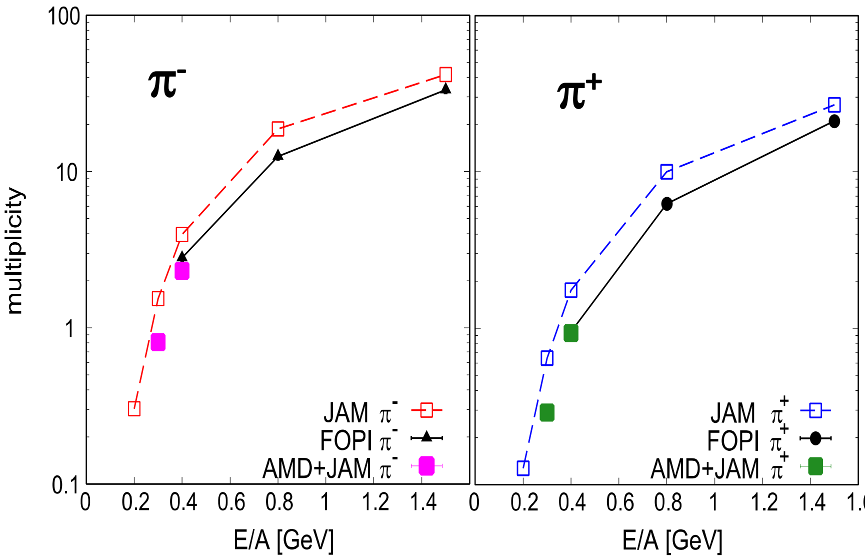
Transport model (AMD + JAM)

- We send **nucleon test particles** $(\mathbf{r}_1, \mathbf{p}_1), (\mathbf{r}_2, \mathbf{p}_2), \ldots, (\mathbf{r}_A, \mathbf{p}_A)$ from AMD to JAM at every 2 fm/c with corrections for the conservation of baryon number and charge.



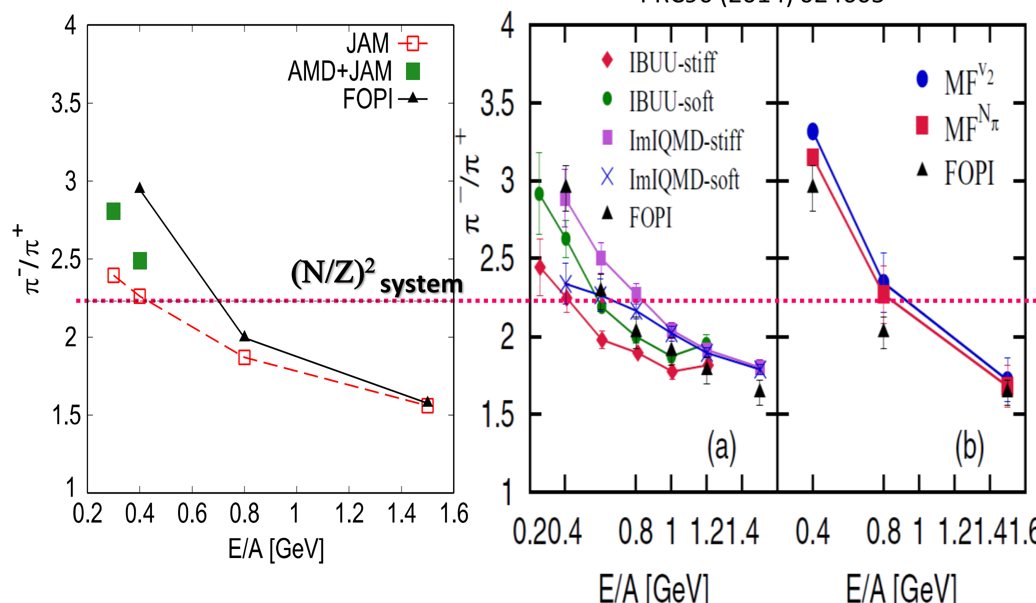
Pion Calcutions in central Au+Au collisions

- Pion multiplicity



- Pion ratio

J. Hong and P. Danielewicz , PRC90 (2014) 024605



- ✓ Our calculation almost reproduces the experimental data reasonably well
- ✓ Pion ratios are also larger than $(N/Z)^2_{system}$

Dynamics of neutrons and protons

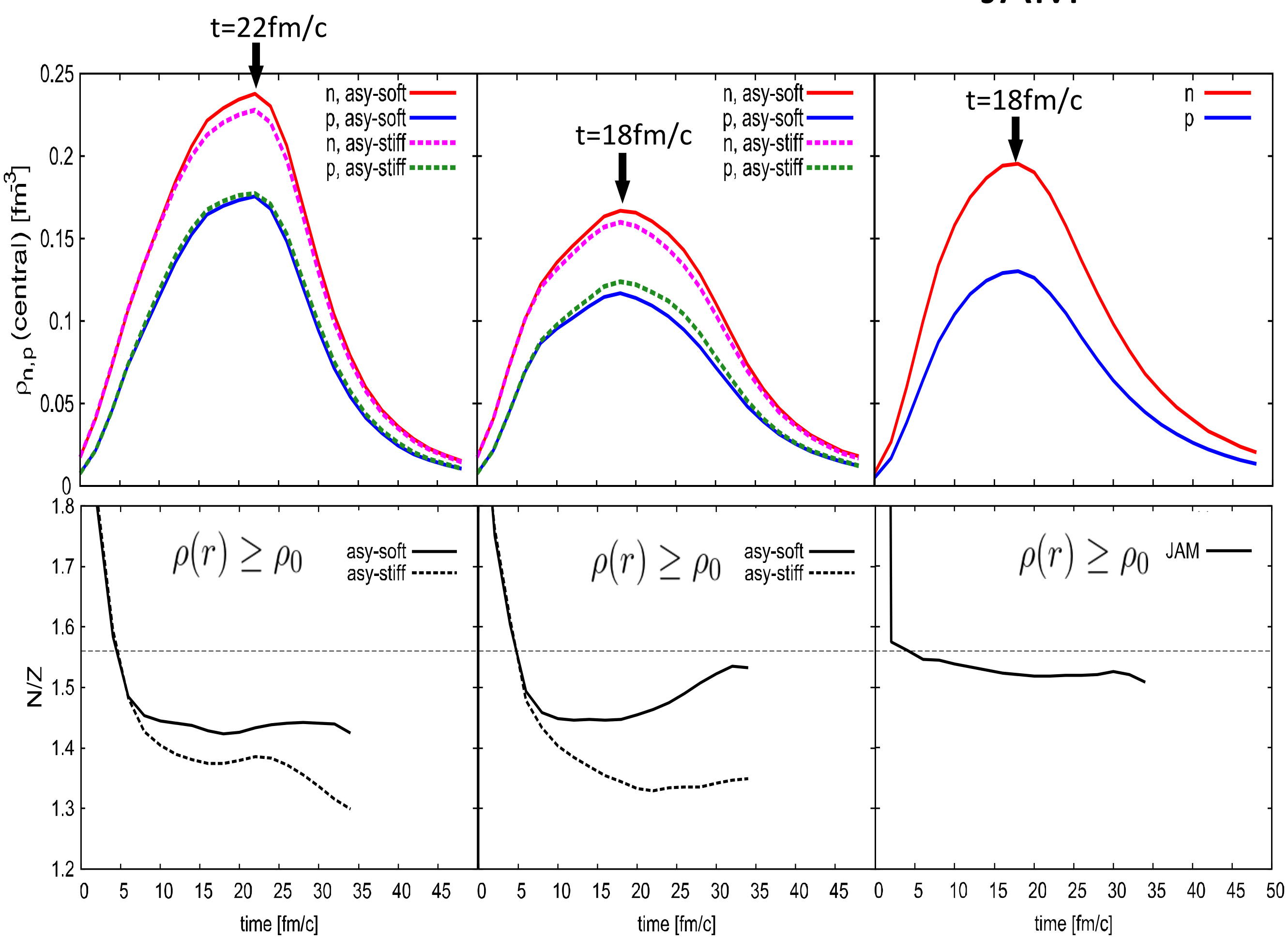
Calculation set:

AMD + JAM

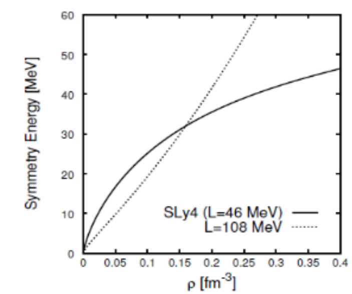
1. with cluster (asy-soft)
2. with cluster (asy-stiff)
3. without cluster (asy-soft)
4. without cluster (asy-stiff)

5. JAM (no mean field)

- with cluster
- without cluster
- JAM



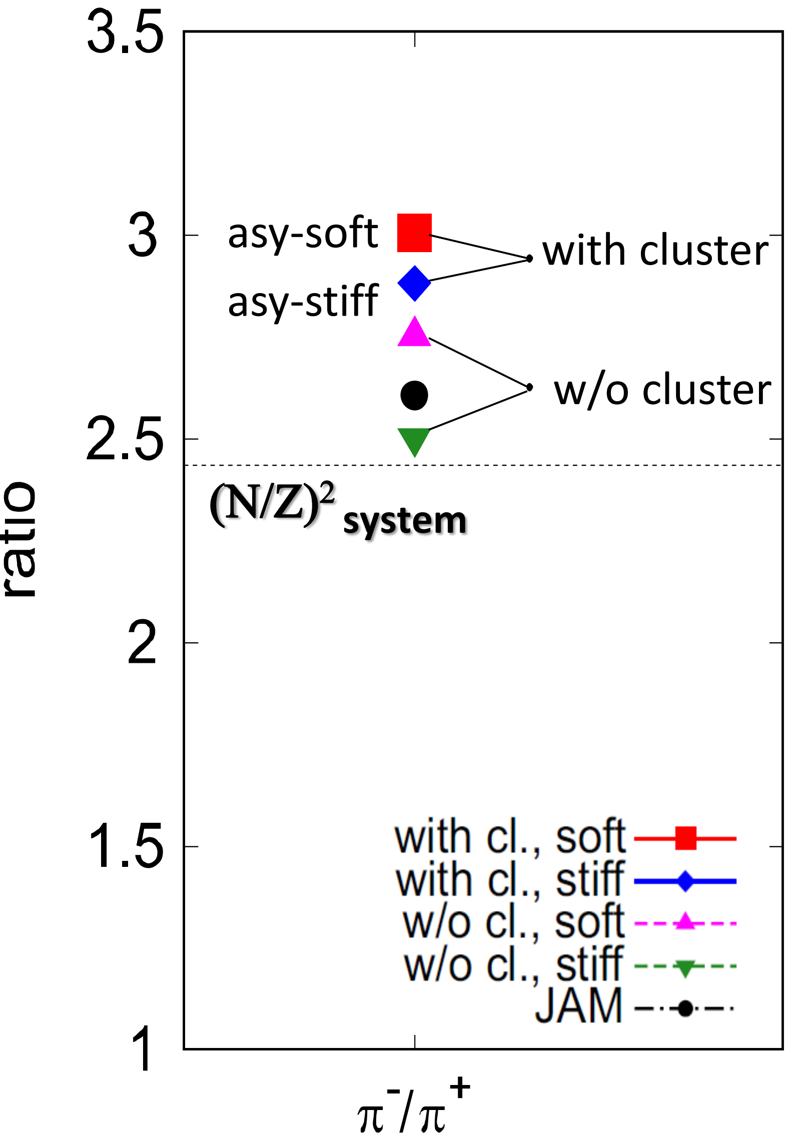
asy-soft : L=46 (SLy4)
asy-stiff : L=108



Effective interaction:
Skyrme force

- ✓ Density maximum is different for cases with or without cluster
- ✓ Clear difference of N/Z ratio due to different symmetry energy
- ✓ Especially symmetry energy effect is weaker if there is cluster correlation

Final π^- / π^+ ratio



1. Symmetry energy dependence S(ρ)
   π^-/π^+ ratio with soft S(ρ) is larger
   –> Similar result to IBUU

2. Model dependence of nucleon dynamics
   S(ρ) effect is weaker with cluster correlations

3. π^-/π^+ ratio > $(N/Z)^2$ system

⇒What is the origin of these behaviors?

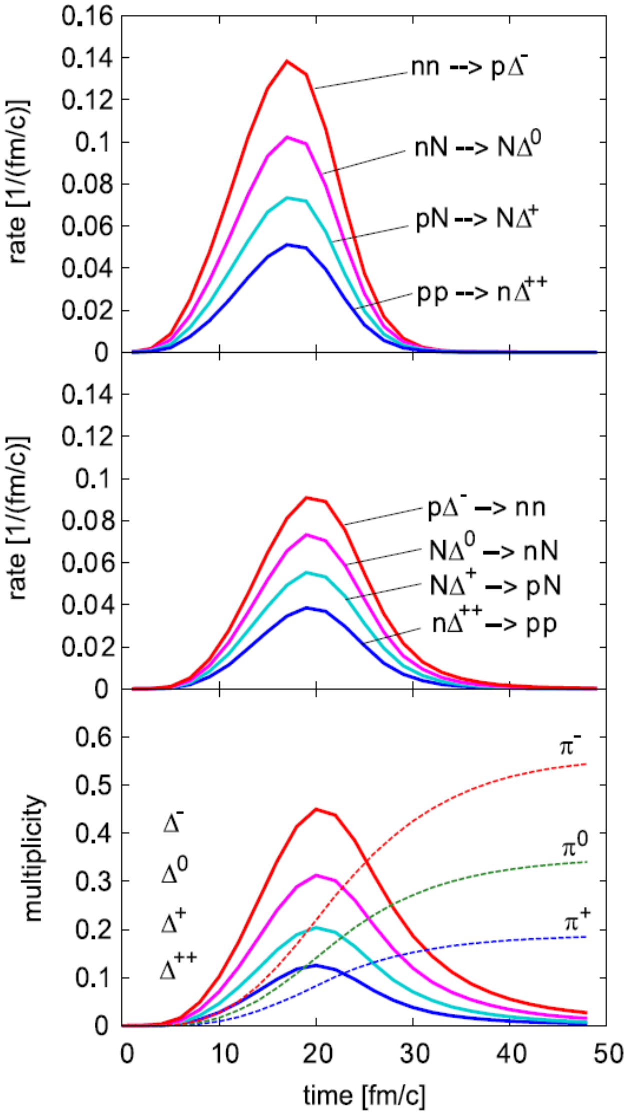
Δ resonance

Symmetry energy soft / stiff ⟷ **Nucleon** N/Z ⟷**?**⟷ **Δ resonance** Δ^-/Δ^{++} ⟷ **Pion** π^-/π^+

Nucleon dynamics

$NN \leftrightarrow N\Delta$

$\Delta \leftrightarrow N\pi$



∗ Δ production:

Reaction rate
$NN \rightarrow N\Delta$

∗ Δ absorption:

Reaction rate
$N\Delta \rightarrow NN$

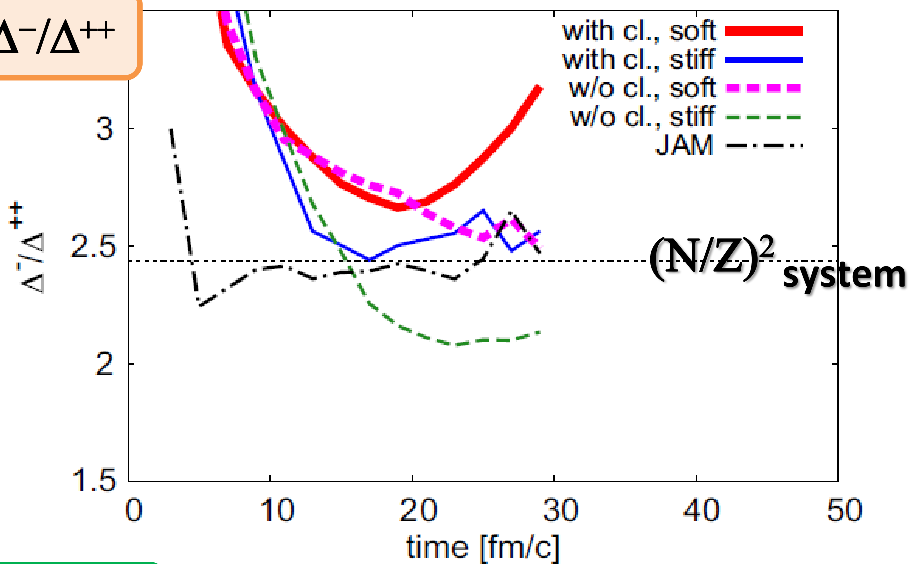
∗ Numbers of existing Δ and π

We study what kind of information of neutrons and protons is carried by Δ resonances.

Relation between N/Z and Δ^-/Δ^{++}

Nucleon N/Z ↔ ? **Δ resonance** Δ^-/Δ^{++}

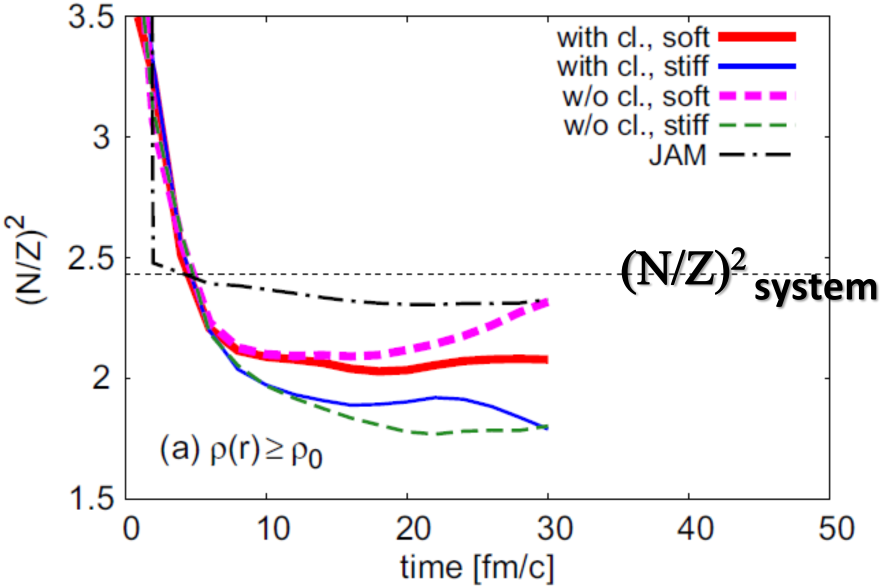
Δ^-/Δ^{++}



Simple expectation: $\Delta^-/\Delta^{++} \sim (N/Z)^2$

$$\frac{\Delta^-}{\Delta^{++}} = \frac{\mathrm{Rate}(nn \to n\Delta^-)}{\mathrm{Rate}(pp \to p\Delta^{++})}$$

$(N/Z)^2_\rho$ Nucleons in the sphere $\rho(r) \geqq \rho_0$ centered at CM.

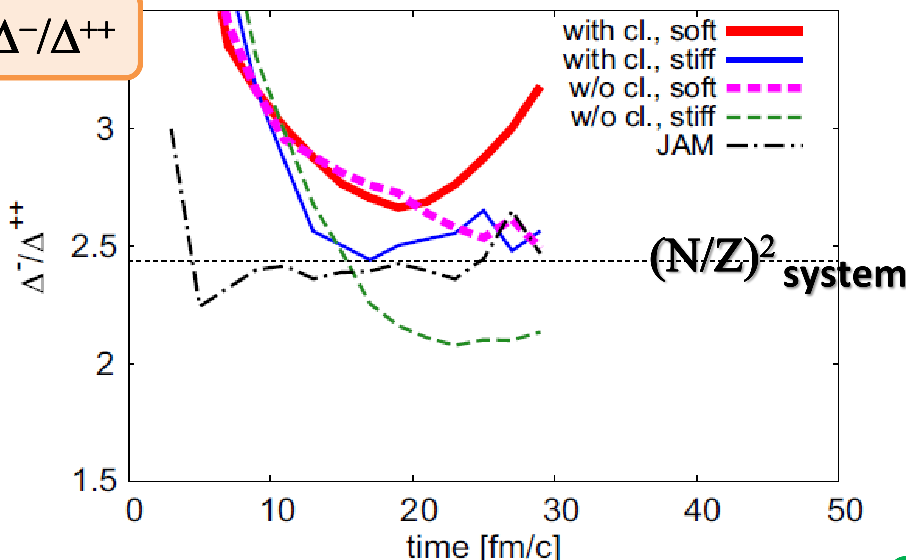


$\Delta^-/\Delta^{++} \neq (N/Z)^2$

Relation between N/Z and Δ^-/Δ^{++}

Nucleon N/Z ↔ ? ↔ Δ resonance Δ^-/Δ^{++}

Δ^-/Δ^{++}



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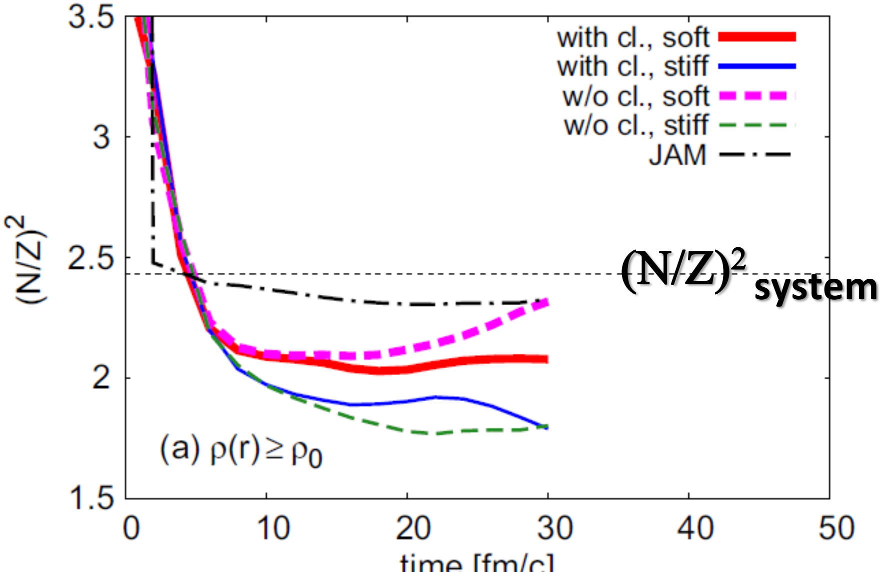
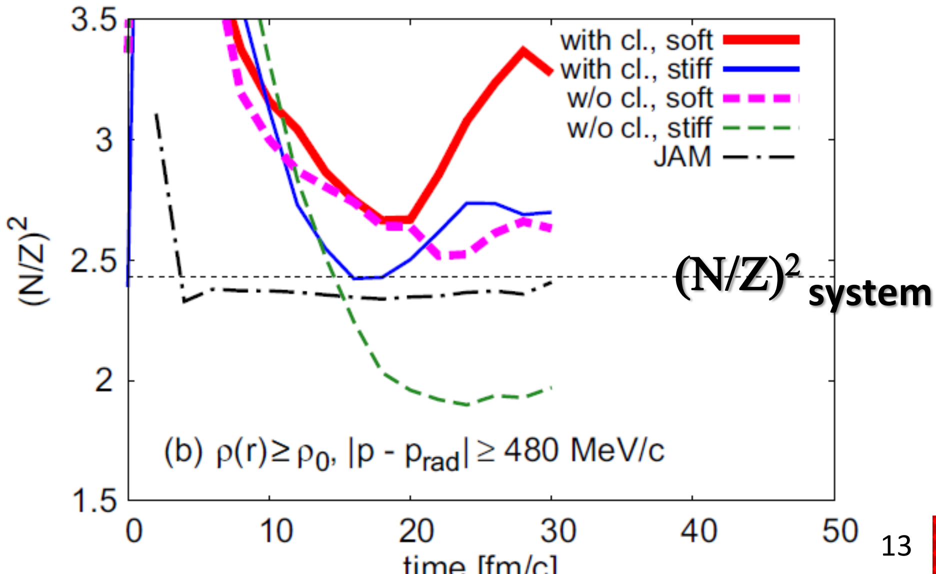


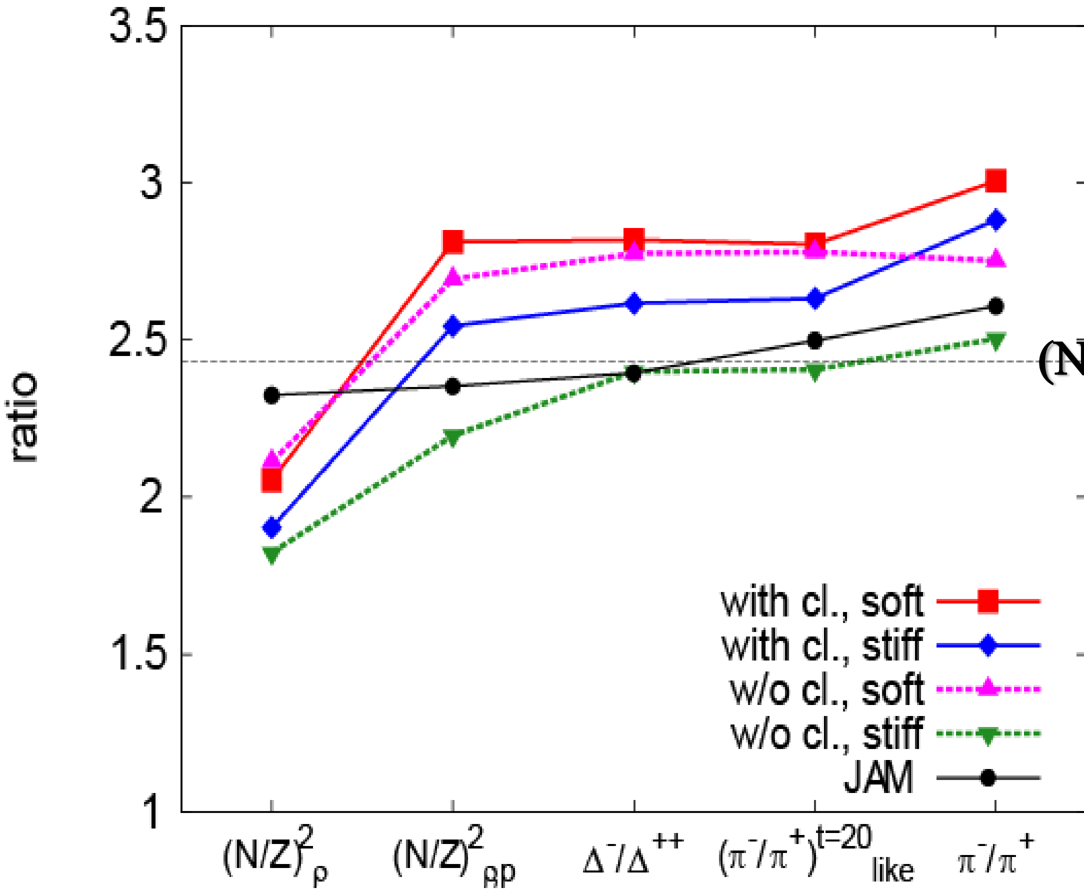
(The collective radial momentum $\boldsymbol{p}_{rad}$ is subtracted)

$(N/Z)^2_{\rho,p}$ Nucleons in the sphere $\rho(r) > \rho_0$ with **high momentum**

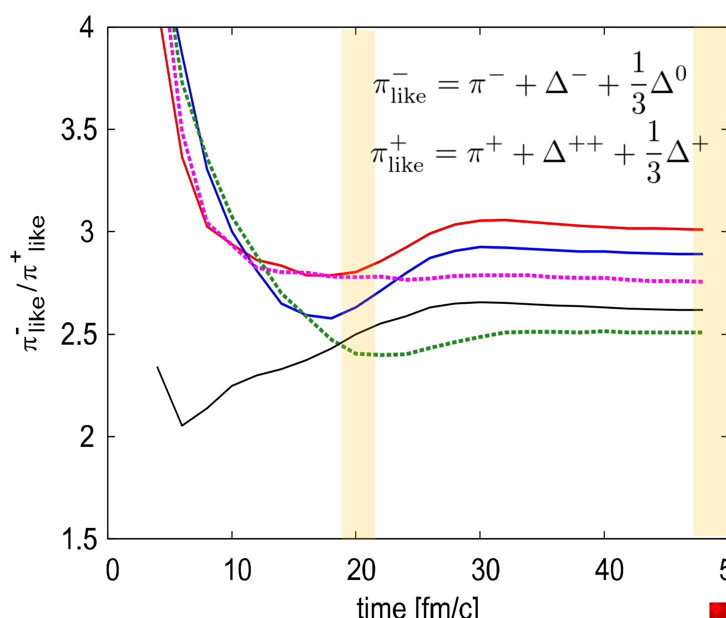


Final π^- / π^+ ratio

- From nucleons to pion ratios



- $(N/Z)^2_{\rho,\,p} \sim \Delta^-/\Delta^{++} \sim (\pi^-/\pi^+)^{t=20}_{\text{like}}$
- Final stage:
  π^-/π^+ is modified from $(\pi^-/\pi^+)^{t=20}_{\text{like}}$
  - ✓ S(ρ) effect: 30% weaker
  - ✓ Cluster correlation → π^-/π^+ up



Representative ratios:

$$\left(\frac{N}{Z}\right)^2 = \frac{\int_0^\infty N(t)^2 dt}{\int_0^\infty Z(t)^2 dt}$$

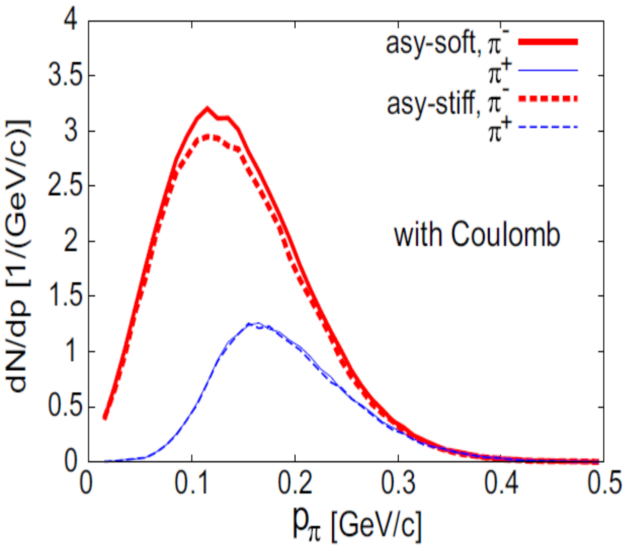
$$\frac{\Delta^-}{\Delta^{++}} = \frac{\int_0^\infty (nn \to p\Delta^-) dt}{\int_0^\infty (pp \to n\Delta^{++}) dt}$$

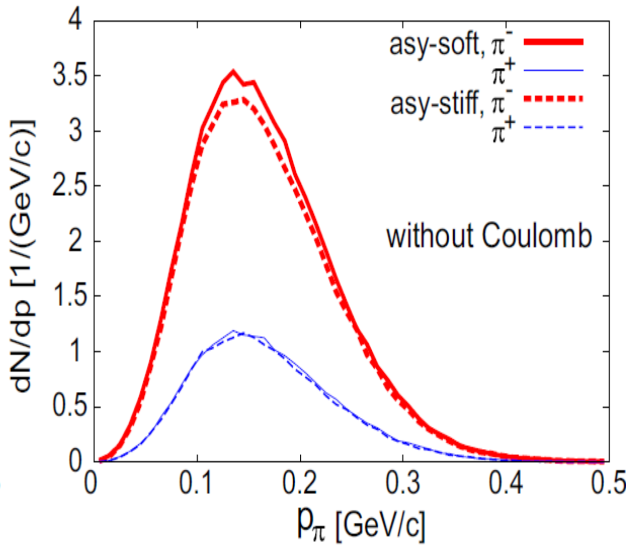
N(t), Z(t) : Numbers of nucleon which satisfy the conditions

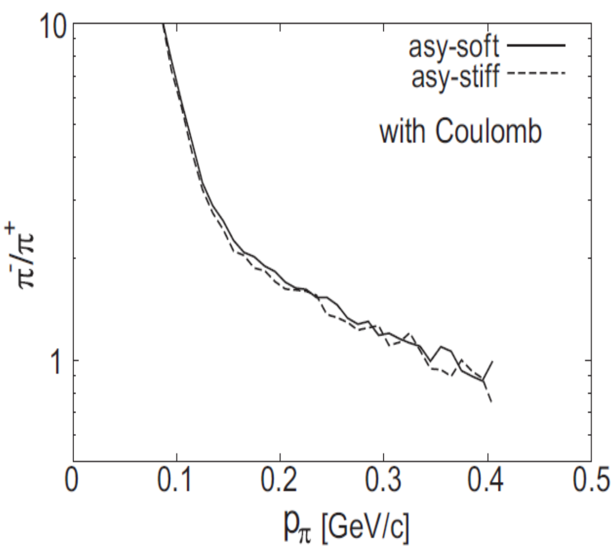
Pion spectra

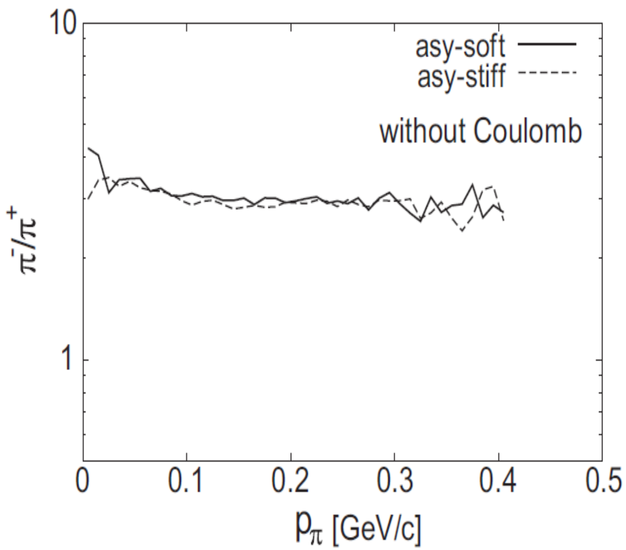
AMD + JAM with cluster (asy-soft)

- With Coulomb
- Without Coulomb









- Coulomb effect:
  Acceleration of π^+
  Deceleration of π^-

→ Changes of pion spectra

	π^-	π^+	π^-/π^+
with Coulomb	0.577	0.192	**3.01(1)**
w/o Coulomb	0.582	0.193	**3.02(1)**

→ Coulomb effect has almost no effect on the pion multiplicities and the pion ratio.

Clusters at high density?

In the calculation, cluster correlation played important roles for the pions.
But, in the high density region, should cluster correlations really exist?

3 Options: Treatment of cluster correlations

1. With cluster

   Clusters are formed at any density.

2. Without cluster

   Clusters are not formed at all.

NEW 3. With cluster ($\rho < 0.16\ \mathrm{fm}^{-3}$)

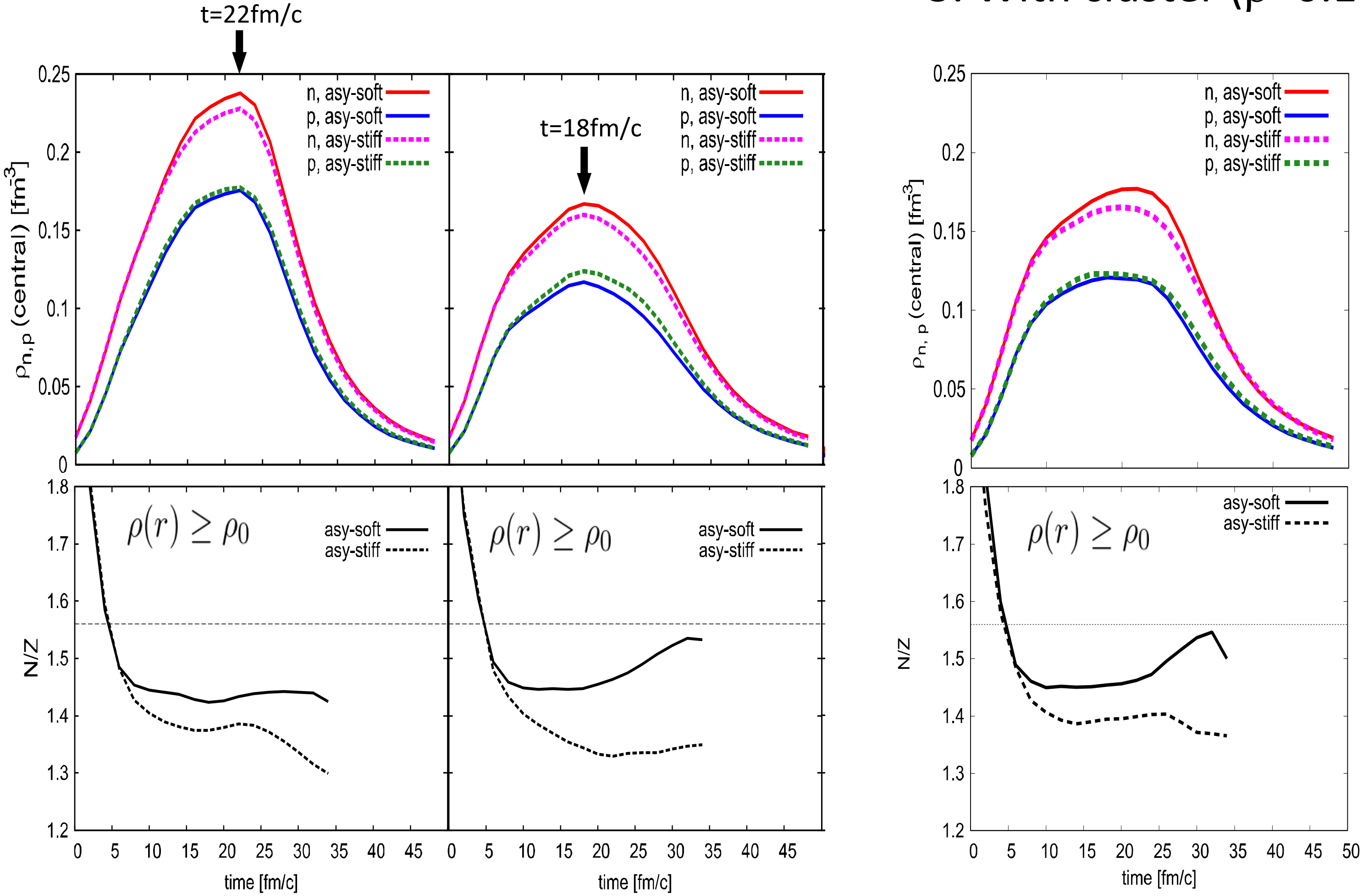
   Clusters are formed in the low density region ($\rho < 0.16\ \mathrm{fm}^{-3}$)

   Clusters are not formed in the high density region ($\rho > 0.16\ \mathrm{fm}^{-3}$)

Preliminary result with cluster (ρ < 0.16 fm^{-3})

➢ Dynamics of neutrons and protons

1. with cluster　　2. without cluster　　3. With cluster (ρ<0.16fm^{-3})



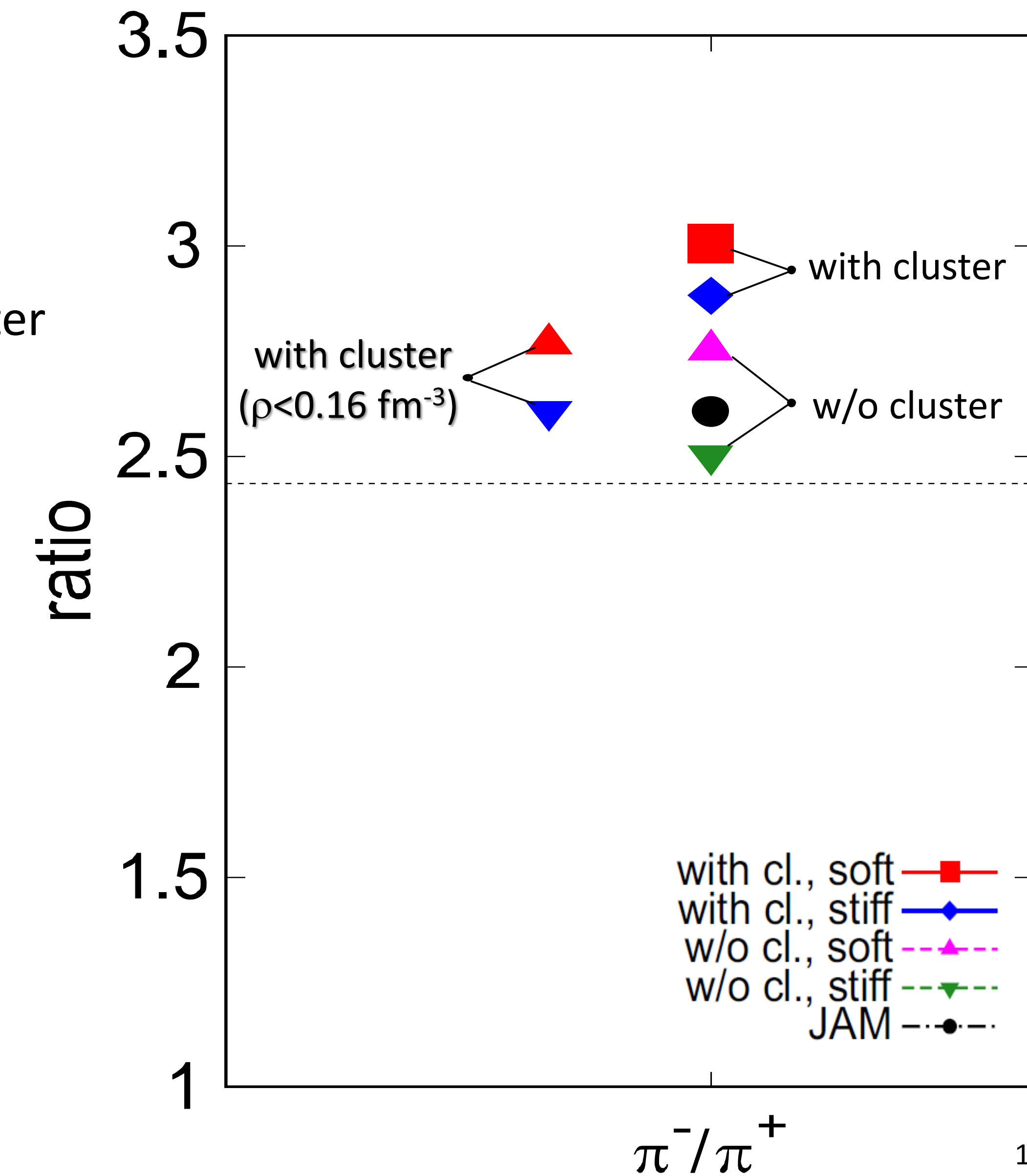
✓ Density maximum is not as high as the case with cluster

Preliminary result with cluster ($\rho < 0.16$ fm^{-3})

- Final π^-/π^+ ratio

  - With cluster ($\rho < 0.16$fm^{-3})

    Closer to the case without cluster



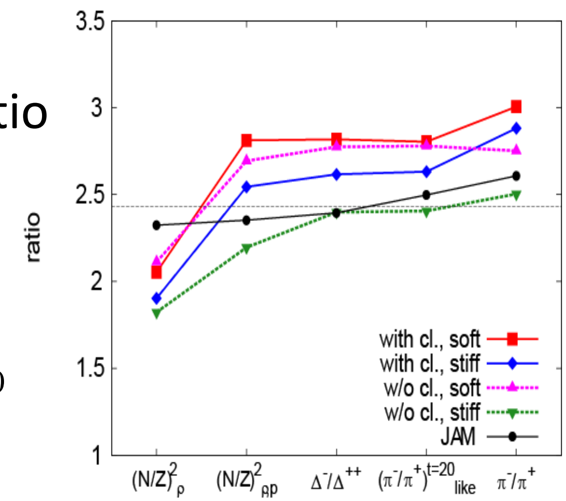
Summary: Pion production in $^{132}Sn+^{124}Sn$ collisions @E/A = 300MeV

- Motivation: To understand the mechanism how pions are produced reflecting the dynamics of neutrons and protons

  Calculation: - Transport model combining AMD and JAM
  - Effective interaction: soft/stiff symmetry energy
  - Turn on/off cluster correlation

Pion ratio certainly carries the information on neutrons and protons at the dynamical stage of collisions

- ✓ The π^-/π^+ and Δ^-/Δ^{++} ratios are related to the $(N/Z)^2$ ratio in high-density and high-momentum region.
- ✓ The π^-/π^+ ratio with soft E_{sym} is larger
- ✓ E_{sym} effect is weaker with cluster correlations
- ✓ In the final stage, π^-/π^+ ratio is modified from $(\pi^-/\pi^+)^{t=20}_{like}$



Future work:

-> We need to investigate not only pions but also other observables (cluster correlation)
  - Δ resonance production threshold

Potential for Δ and pion

In JAM, reaction thresholds are the same as in free space.

(The production and absorption reactions for Δ and pions occur in the JAM calculation as in the free space)

Nucleons feel potential in the AMD calculation.

Therefore AMD+JAM assumes

$$\mathrm{NN} \leftrightarrow \mathrm{N}\Delta \qquad\qquad \Delta \leftrightarrow \mathrm{N}\pi$$

$$U_{\tau_1}^{(N)} + U_{\tau_2}^{(N)} = U_{\tau_3}^{(N)} + U_{\tau_4}^{(\Delta)}, \qquad U_{\tau_1}^{(\Delta)} = U_{\tau_3}^{(N)} + U_{\tau_4}^{(\pi)} \qquad \text{for } \tau_1(+\tau_2) = \tau_3 + \tau_4$$

This is equivalent to the choice in the pBUU calculation

c.f. Hong and Danielewicz, PRC 90 (2014) 024605

$$\begin{aligned} v_{asy}(\Delta^-) &= 2v_{asy}(n) - v_{asy}(p) = 3v_{asy}(n), \\ v_{asy}(\Delta^0) &= v_{asy}(n), \\ v_{asy}(\Delta^+) &= v_{asy}(p) = -v_{asy}(n), \\ v_{asy}(\Delta^{++}) &= 2v_{asy}(p) - v_{asy}(n) = -3v_{asy}(n). \end{aligned}$$

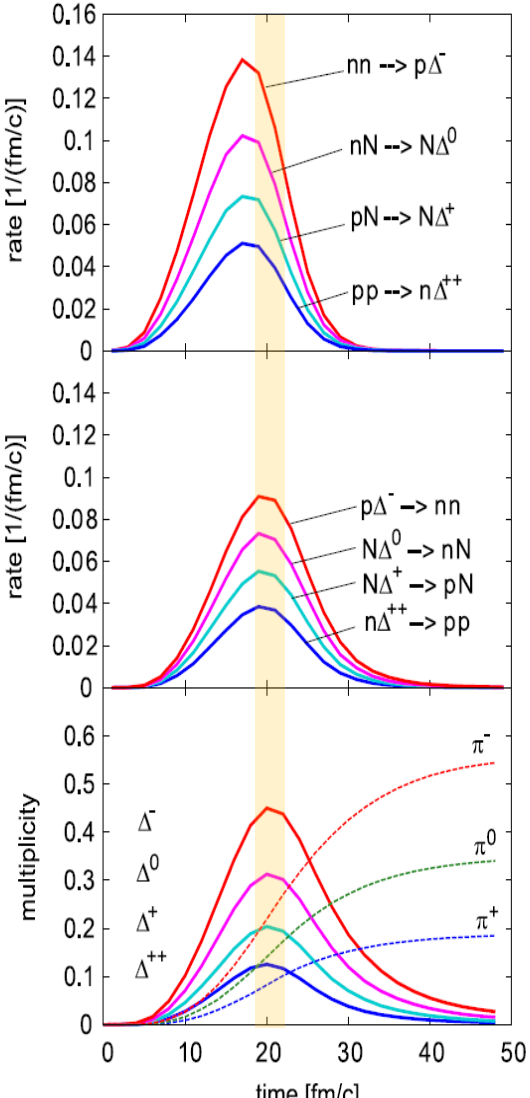
* Different choice,

cf. Bao-An Li

$$\begin{aligned} v_{asy}(\Delta^-) &= v_{asy}(n), \\ v_{asy}(\Delta^0) &= \frac{2}{3}v_{asy}(n) + \frac{1}{3}v_{asy}(p) = \frac{1}{3}v_{asy}(n), \\ v_{asy}(\Delta^+) &= \frac{1}{3}v_{asy}(n) + \frac{2}{3}v_{asy}(p) = -\frac{1}{3}v_{asy}(n), \\ v_{asy}(\Delta^{++}) &= v_{asy}(p) = -v_{asy}(n). \end{aligned}$$

Δ resonance

$$\Delta^-/\Delta^{++} \simeq e^{3(\mu_n-\mu_p)/T} \simeq (N/Z)^3_{\rho,p}$$



$$(\pi^-/\pi^+)'_{\rm like} = (\Delta^- + \tfrac{1}{3}\Delta^0)/(\Delta^{++} + \tfrac{1}{3}\Delta^+)$$

∗ Δ production:

Reaction rate
NN→NΔ

Δ^-/Δ^{++}

$(N/Z)^2_{\rho,p}$

∗ Δ absorption:

Reaction rate
NΔ→NN

If the system were in chemical equilibrium,

∗ Numbers of existing Δ and π