Isovector & Isoscalar Densities in Nuclei

Pawel Danielewicz,¹ Pardeep Singh¹,² and Jenny Lee³

¹Natl Superconducting Cyclotron Lab, Michigan State U, ²Deenbandhu Chhotu Ram U Science & Techn, Murthal, India and ³U of Hong Kong

The 6th International Symposium on Nuclear Symmetry Energy

June 13-17, 2016, Tsinghua University, Beijing
Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under \( n \leftrightarrow p \) interchange

An isoscalar quantity \( F \) does not change under \( n \leftrightarrow p \) interchange. E.g. nuclear energy. Expansion in asymmetry \( \eta = (N - Z)/A \), for smooth \( F \), yields even terms only:

\[
F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \ldots
\]

An isovector quantity \( G \) changes sign. Example:
\[
\rho_{np}(r) = \rho_n(r) - \rho_p(r).
\]
Expansion with odd terms only:

\[
G(\eta) = G_1 \eta + G_3 \eta^3 + \ldots
\]

Note: \( G/\eta = G_1 + G_3 \eta^2 + \ldots \)

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in \( n-p \) space
Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under \( n \leftrightarrow p \) interchange

An isoscalar quantity \( F \) does not change under \( n \leftrightarrow p \) interchange. E.g. nuclear energy. Expansion in asymmetry \( \eta = (N - Z)/A \), for smooth \( F \), yields even terms only:

\[
F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \ldots
\]

An isovector quantity \( G \) changes sign. Example:
\[
\rho_{np}(r) = \rho_n(r) - \rho_p(r).
\]
Expansion with odd terms only:

\[
G(\eta) = G_1 \eta + G_3 \eta^3 + \ldots
\]

Note: \( G/\eta = G_1 + G_3 \eta^2 + \ldots \)

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in \( n-p \) space
Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under $n \leftrightarrow p$ interchange

An isoscalar quantity $F$ does not change under $n \leftrightarrow p$ interchange. E.g. nuclear energy. Expansion in asymmetry $\eta = (N - Z)/A$, for smooth $F$, yields even terms only:

$$F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \ldots$$

An isovector quantity $G$ changes sign. Example: $\rho_{np}(r) = \rho_n(r) - \rho_p(r)$. Expansion with odd terms only:

$$G(\eta) = G_1 \eta + G_3 \eta^3 + \ldots$$

Note: $G/\eta = G_1 + G_3 \eta^2 + \ldots$

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in $n-p$ space
Charge Symmetry & Charge Invariance

Charge symmetry: invariance of nuclear interactions under $n \leftrightarrow p$ interchange

An isoscalar quantity $F$ does not change under $n \leftrightarrow p$ interchange. E.g. nuclear energy. Expansion in asymmetry $\eta = (N - Z)/A$, for smooth $F$, yields even terms only:

$$F(\eta) = F_0 + F_2 \eta^2 + F_4 \eta^4 + \ldots$$

An isovector quantity $G$ changes sign. Example:

$$\rho_{np}(r) = \rho_n(r) - \rho_p(r).$$

Expansion with odd terms only:

$$G(\eta) = G_1 \eta + G_3 \eta^3 + \ldots$$

Note: $G/\eta = G_1 + G_3 \eta^2 + \ldots$

In nuclear practice, analyticity requires shell-effect averaging!

Charge invariance: invariance of nuclear interactions under rotations in $n-p$ space
Charge Symmetry & Charge Invariance

Charge symmetry: $n \leftrightarrow p$ invariance

Charge invariance: symmetry under rotations in $n$-$p$ space

Isospin doublets
$p: (\tau, \tau_z) = (\frac{1}{2}, \frac{1}{2})$
$n: (\tau, \tau_z) = (\frac{1}{2}, -\frac{1}{2})$

Net isospin
\[
\vec{T} = \sum_{i=1}^{A} \vec{\tau}_i
\]

\[
T = \frac{3}{2}, \ldots, T = \frac{1}{2}, \frac{3}{2}, \ldots, T = \frac{3}{2}, \ldots
\]

Nuclear states: $(T, T_z), \ T \geq |T_z| = \frac{1}{2}|N-Z|

Isobars: Nuclei with the same $A$
Charge Symmetry & Charge Invariance

Charge symmetry: $n \leftrightarrow p$ invariance

Charge invariance: symmetry under rotations in $n-p$ space

Isospin doublets
- $p: (\tau, \tau_z) = (\frac{1}{2}, \frac{1}{2})$
- $n: (\tau, \tau_z) = (\frac{1}{2}, -\frac{1}{2})$

Net isospin

$$\vec{T} = \sum_{i=1}^{A} \vec{\tau}_i$$

Nuclear states: $(T, T_z), \quad T \geq |T_z| = \frac{1}{2}|N - Z|$
Charge Symmetry & Charge Invariance

Charge symmetry: $n \leftrightarrow p$ invariance

Charge invariance: symmetry under rotations in $n-p$ space

Isospin doublets

$p: (\tau, \tau_z) = (\frac{1}{2}, \frac{1}{2})$

$n: (\tau, \tau_z) = (\frac{1}{2}, -\frac{1}{2})$

Net isospin

$$\vec{T} = \sum_{i=1}^{A} \vec{\tau}_i$$

Nuclear states: $(T, T_z), \quad T \geq |T_z| = \frac{1}{2}|N - Z|$
Energy in Uniform Matter

\[
\frac{E}{A}(\rho_n, \rho_p) = \frac{E_0}{A}(\rho) + S(\rho) \left( \frac{\rho_n - \rho_p}{\rho} \right)^2 + \mathcal{O}(\ldots^4)
\]

Symmetric matter \quad (a)symmetry energy \quad \rho = \rho_n + \rho_p

**Known:**
- \( a_a \approx 16 \text{ MeV} \)
- \( K \approx 235 \text{ MeV} \)

**Unknown:**
- \( a_a^V ? \)
- \( L ? \)

---

**Symmetry Energy**

Danielewicz, Singh, Lee
Importance of Slope

\[
\frac{E}{A} = \frac{E_0}{A}(\rho) + S(\rho) \left( \frac{\rho_n - \rho_p}{\rho} \right)^2
\]

\[
S \simeq a_v^V + \frac{L}{3} \frac{\rho - \rho_0}{\rho_0}
\]

In neutron matter:
\[
\rho_p \approx 0 \quad \text{and} \quad \rho_n \approx \rho.
\]

Then, \( \frac{E}{A}(\rho) \approx \frac{E_0}{A}(\rho) + S(\rho) \)

Pressure:
\[
P = \rho^2 \frac{d}{d\rho} \frac{E}{A} \simeq \rho^2 \frac{dS}{d\rho} \simeq \frac{L}{3\rho_0} \rho^2
\]

Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given $A$. [Coulomb suppressed...]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A \rho_{np}(r)/(N - Z)$ isoscalar!

$A/(N - Z)$ normalizing factor global... Similar local normalizing factor, in terms of intense quantities, $2a^V_a/\mu_a$, where $a^V_a \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$$\rho_a(r) = \frac{2a^V_a}{\mu_a} [\rho_n(r) - \rho_p(r)]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on $\eta$!

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a^V_a} \rho_a(r)]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, $\rho$’s as dynamic vbles: Hohenberg-Kohn functional
Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given $A$. [Coulomb suppressed . . .]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A \rho_{np}(r)/(N - Z)$ isoscalar! $A/(N - Z)$ normalizing factor global . . . Similar local normalizing factor, in terms of intense quantities, $2a^V_a / \mu_a$, where $a^V_a \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$$\rho_a(r) = \frac{2a^V_a}{\mu_a} [\rho_n(r) - \rho_p(r)]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on $\eta$!

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a^V_a} \rho_a(r)]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, $\rho$’s as dynamic vbles: Hohenberg-Kohn functional.
Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given $A$. [Coulomb suppressed...]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A \rho_{np}(r)/(N - Z)$ isoscalar!

$A/(N - Z)$ normalizing factor global... Similar local normalizing factor, in terms of intense quantities, $2a^V_a/\mu_a$, where $a^V_a \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$$\rho_a(r) = \frac{2a^V_a}{\mu_a} \left[ \rho_n(r) - \rho_p(r) \right]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on $\eta$!

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2} \left[ \rho(r) \pm \frac{\mu_a}{2a^V_a} \rho_a(r) \right]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, $\rho$'s as dynamic vbls: Hohenberg-Kohn functional
Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given $A$. [Coulomb suppressed...]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A \rho_{np}(r)/(N - Z)$ isoscalar!

$A/(N - Z)$ normalizing factor global... Similar local normalizing factor, in terms of intense quantities, $2a^V_a/\mu_a$, where $a^V_a \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$$\rho_a(r) = \frac{2a^V_a}{\mu_a} [\rho_n(r) - \rho_p(r)]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on $\eta$!

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a^V_a} \rho_a(r)]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, $\rho$’s as dynamic vbles: Hohenberg-Kohn functional
Isoscalar and Isovector Densities

Net density $\rho(r) = \rho_n(r) + \rho_p(r)$ is isoscalar $\Rightarrow$ weakly depends on $(N - Z)$ for given $A$. [Coulomb suppressed...]

$\rho_{np}(r) = \rho_n(r) - \rho_p(r)$ isovector but $A \rho_{np}(r)/(N - Z)$ isoscalar! $A/(N - Z)$ normalizing factor global... Similar local normalizing factor, in terms of intense quantities, $2a^V_a/\mu_a$, where $a^V_a \equiv S(\rho_0)$

Isoscalar formfactor for isovector density:

$$\rho_a(r) = \frac{2a^V_a}{\mu_a} [\rho_n(r) - \rho_p(r)]$$

Normal matter: $\rho_a = \rho_0$. Both $\rho(r)$ & $\rho_a(r)$ weakly depend on $\eta$!

In any nucleus:

$$\rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a^V_a} \rho_a(r)]$$

where $\rho(r)$ & $\rho_a(r)$ have universal features! (subject to shell effects)

No shell-effects, $\rho$’s as dynamic vbles: Hohenberg-Kohn functional.
Isovector Density

\[
\rho_{n,p}(r) = \frac{1}{2} [\rho(r) \pm \frac{\mu_a}{2a_a^V} \rho_a(r)]
\]

Net density \( \rho \) usually parameterized w/Fermi function

\[
\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R_0}{d}\right)} \quad \text{with} \quad R = r_0 A^{1/3}
\]

Isovector density \( \rho_a \)?

Related to \( S(\rho) \)!

In uniform matter

\[
\mu_a = \frac{\partial E}{\partial (N-Z)} = \frac{\partial [S(\rho) \frac{\rho_{np}^2}{\rho}]}{\partial \rho_{np}} = \frac{2 S(\rho)}{\rho} \rho_{np}
\]

\[
\Rightarrow \quad \rho_a = \frac{2a_a^V}{\mu_a} \rho_{np} = \frac{a_a^V \rho}{S(\rho)}
\]

\( \rightarrow \) Hartree-Fock study of surface
Isovector Density

\[ \rho_{n,p}(r) = \frac{1}{2} \left[ \rho(r) \pm \frac{\mu_a}{2a_a^V} \rho_a(r) \right] \]

Net density \( \rho \) usually parameterized with Fermi function

\[ \rho(r) = \frac{\rho_0}{1 + \exp(\frac{r-R}{d})} \quad \text{with} \quad R = r_0 \frac{A^{1/3}}{d} \]

Isovector density \( \rho_a ?? \) Related to \( S(\rho) \)!

In uniform matter

\[ \mu_a = \frac{\partial E}{\partial (N-Z)} = \frac{\partial [S(\rho) \rho_{np}^2 / \rho]}{\partial \rho_{np}} = \frac{2 S(\rho)}{\rho} \rho_{np} \]

\[ \Rightarrow \quad \rho_a = \frac{2a_a^V}{\mu_a} \rho_{np} = \frac{a_a^V \rho}{S(\rho)} \]

\[ \rightarrow \text{Hartree-Fock study of surface} \]
Isovector Density

\[ \rho_{n,p}(r) = \frac{1}{2} \left[ \rho(r) \pm \frac{\mu_a}{2a_a^V} \rho_a(r) \right] \]

Net density \( \rho \) usually parameterized w/Fermi function

\[ \rho(r) = \frac{\rho_0}{1 + \exp \left( \frac{r-R}{d} \right)} \quad \text{with} \quad R = r_0 A^{1/3} \]

Isovector density \( \rho_a \)?? Related to \( S(\rho) \)!

In uniform matter

\[ \mu_a = \frac{\partial E}{\partial (N - Z)} = \frac{\partial [S(\rho) \rho_{np}^2/\rho]}{\partial \rho_{np}} = \frac{2}{\rho} \frac{S(\rho)}{\rho} \rho_{np} \]

\[ \Rightarrow \quad \rho_a = \frac{2a_a^V}{\mu_a} \rho_{np} = \frac{a_a^V}{S(\rho)} \rho \]

\[ \Rightarrow \quad \text{Hartree-Fock study of surface} \]

Danielewicz, Singh, Lee
Isovector Density

$$\rho_{n,p}(r) = \frac{1}{2} \left[ \rho(r) \pm \frac{\mu_a}{2a_a^V} \rho_a(r) \right]$$

Net density $\rho$ usually parameterized w/ Fermi function

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-R}{\rho}\right)} \quad \text{with} \quad R = r_0 A^{1/3}$$

Isovector density $\rho_a$?? Related to $S(\rho)$!

In uniform matter

$$\mu_a = \frac{\partial E}{\partial (N-Z)} = \frac{\partial [S(\rho) \rho^2_{np}/\rho]}{\partial \rho_{np}} = \frac{2S(\rho)}{\rho} \rho_{np}$$

$$\Rightarrow \quad \rho_a = \frac{2a_a^V}{\mu_a} \rho_{np} = \frac{a_a^V \rho}{S(\rho)}$$

⇒ Hartree-Fock study of surface
Half-Infinite Matter in Skyrme-Hartree-Fock

To one side infinite uniform matter & vacuum to the other

Wavefunctions: \( \Phi(\vec{r}) = \phi(z) e^{i \vec{k}_\perp \cdot \vec{r}_\perp} \)

matter interior/exterior:

\( \phi(z) \propto \sin (k_z z + \delta(\vec{k})) \)

\( \phi(z) \propto e^{-\kappa(\vec{k}) z} \)

Discretization in \( k \)-space. Set of 1D HF eqs solved using Numerov’s method until self-consistency:

\[
- \frac{d}{dz} \frac{\hbar^2}{2m^*(z)} \frac{d}{dz} \phi(z) + \left( \frac{\hbar^2 k^2}{2m^*(z)} + U(z) \right) \phi(z) = \epsilon(\vec{k}) \phi(z)
\]

Asymmetry Dependence of Net Density

Half-$\infty$ matter results for different Skyrme interactions and asymmetries

$$\eta = \frac{N - Z}{A}$$
Asymmetry Dependence of Asymmetric Density

\[
\rho_a = \frac{2a_a^V}{\mu_a} (\rho_n - \rho_p)
\]

Half-\(\infty\) matter results for different Skyrme interactions and asymmetries

PD&Lee
NP818(09)36
Sensitivity to $S(\rho)$

Results from different Skyrme interactions in half-$\infty$ matter.

Isoscalar ($\rho = \rho_n + \rho_p$; blue) & isovector ($\rho_n - \rho_p$; green) densities displaced relative to each other.

As $S(\rho)$ changes, so does displacement.
Strategies for Independent Densities

Jefferson Lab
Direct: $\sim p$
Interference: $\sim n$

PD
elastic: $\sim p + n$
charge exchange: $\sim n - p$
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!

Not suppressed by low $(N - Z)/A$!

Nucleon optical potential in isospin space:

$$U = U_0 + \frac{4\tau T}{A} U_1$$

isoscalar potential $U_0 \propto \rho$, isovector potential $U_1 \propto (\rho_n - \rho_p)$

In elastic scattering $U = U_0 \pm \frac{N-Z}{A} U_1$

In quasielastic charge-exchange (p,n) to IAS: $U = \frac{4\tau - T_+}{A} U_1$

Elastic scattering dominated by $U_0$

Quasielastic governed by $U_1$

Geometry usually assumed the same for $U_0$ and $U_1$

e.g. Koning & Delaroche NPA713(03)231

?Isovector skin $\Delta R$ from comparison of elastic and quasielastic (p,n)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low $(N - Z)/A$!

Nucleon optical potential in isospin space:

\[ U = U_0 + \frac{4\tau T}{A} U_1 \]

isoscalar potential \( U_0 \propto \rho \), isovector potential \( U_1 \propto (\rho_n - \rho_p) \)

In elastic scattering \( U = U_0 \pm \frac{N-Z}{A} U_1 \)

In quasielastic charge-exchange (p,n) to IAS: \( U = \frac{4\tau - T_+}{A} U_1 \)

Elastic scattering dominated by \( U_0 \)
Quasielastic governed by \( U_1 \)

Geometry usually assumed the same for \( U_0 \) and \( U_1 \)
e.g. Koning & Delaroche NPA713(03)231

?Isovector skin \( \Delta R \) from comparison of elastic and quasielastic (p,n)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N − Z)/A\)!

Nucleon optical potential in isospin space:

\[ U = U_0 + \frac{4\tau T}{A} U_1 \]

Isoscalar potential \( U_0 \propto \rho \), isovector potential \( U_1 \propto (\rho_n - \rho_p) \)

In elastic scattering \( U = U_0 \pm \frac{N-Z}{A} U_1 \)
In quasielastic charge-exchange \((p,n)\) to IAS: \( U = \frac{4\tau - T_+}{A} U_1 \)
Elastic scattering dominated by \( U_0 \)
Quasielastic governed by \( U_1 \)
Geometry usually assumed the same for \( U_0 \) and \( U_1 \)
e.g. Koning & Delaroche NPA713(03)231
?
Isovector skin \( \Delta R \) from comparison of elastic and quasielastic \((p,n)\)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N - Z)/A\)!

Nucleon optical potential in isospin space:

\[
U = U_0 + \frac{4\tau T}{A} U_1
\]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)

In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)

In quasielastic charge-exchange \((p,n)\) to IAS: \(U = \frac{4\tau - T_+}{A} U_1\)

Elastic scattering dominated by \(U_0\)
Quasielastic governed by \(U_1\)

Geometry usually assumed the same for \(U_0\) and \(U_1\)

e.g. Koning & Delaroche NPA713(03)231

?Isovector skin \(\Delta R\) from comparison of elastic and quasielastic \((p,n)\)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N - Z)/A\)!

Nucleon optical potential in isospin space:

\[
U = U_0 + \frac{4\tau T}{A} U_1
\]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)

In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)

In quasielastic charge-exchange \((p,n)\) to IAS: \(U = \frac{4\tau - T_+}{A} U_1\)
Elastic scattering dominated by \(U_0\)
Quasielastic governed by \(U_1\)

Geometry usually assumed the same for \(U_0\) and \(U_1\)
e.g. Koning & Delaroche NPA713(03)231

\(\Delta R\) from comparison of elastic and quasielastic \((p,n)\)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N - Z)/A\)!

Nucleon optical potential in isospin space:

\[
U = U_0 + \frac{4\tau T}{A} U_1
\]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)

In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)

In quasielastic charge-exchange \((p,n)\) to IAS: \(U = \frac{4\tau T_\pm}{A} U_1\)

Elastic scattering dominated by \(U_0\)
Quasielastic governed by \(U_1\)
Geometry usually assumed the same for \(U_0\) and \(U_1\)
e.g. Koning & Delaroche NPA713(03)231

?Isovector skin \(\Delta R\) from comparison of elastic and quasielastic \((p,n)\)-to-IAS scattering?
Why Isovector Rather than Neutron Skins

Isovector skin: in no-curvature, no-shell-effect, no-Coulomb limit, the same for every nucleus!
Not suppressed by low \((N - Z)/A\)!

Nucleon optical potential in isospin space:

\[
U = U_0 + \frac{4\tau T}{A} U_1
\]

isoscalar potential \(U_0 \propto \rho\), isovector potential \(U_1 \propto (\rho_n - \rho_p)\)
In elastic scattering \(U = U_0 \pm \frac{N-Z}{A} U_1\)
In quasielastic charge-exchange (p,n) to IAS: \(U = \frac{4\tau_+ T_+}{A} U_1\)
Elastic scattering dominated by \(U_0\)
Quasielastic governed by \(U_1\)
Geometry usually assumed the same for \(U_0\) and \(U_1\)
e.g. Koning & Delaroche NPA713(03)231

?Isovector skin \(\Delta R\) from comparison of elastic and quasielastic (p,n)-to-IAS scattering?
Expectations on Isovector Skin?

Much Larger Than Neutron!

Surface radius $R \sim \sqrt{\frac{5}{3} \langle r^2 \rangle^{1/2}}$

$$\langle r^2 \rangle_{\rho_n}^{1/2} - \langle r^2 \rangle_{\rho_p}^{1/2} \sim 2 \frac{N - Z}{A} \left[ \langle r^2 \rangle_{\rho_n - \rho_p}^{1/2} - \langle r^2 \rangle_{\rho_n + \rho_p}^{1/2} \right]$$

Estimated $\Delta R \sim 3 \left( \langle r^2 \rangle_{\rho_n}^{1/2} - \langle r^2 \rangle_{\rho_p}^{1/2} \right)$ for $^{48}\text{Ca}/^{208}\text{Pb}$!

Even before consideration of Coulomb effects that further enhances difference!
Direct Reaction Primer

DWBA:

\[ \frac{d\sigma}{d\Omega} \propto \left| \int dr \, \psi_f^* U_1 \psi_i \right|^2 \]

- Oscillations: 2-side interference/source size
- Fall-off: softness of source
- Filling of minima: imaginary/real contributions, spin-orbit
Potentials Fit to Elastic in Quasielastic

E.g. Koning-Delaroche NPA713(03)231 same radii for neutrons/protons, isoscalar/isovector, focus on p elastic

p Elastic Scattering

\[ U_0 + \frac{N-Z}{A} U_1 \]

QuasiElastic (p,n)

\[ U_1 \text{ only?} \]
Effect of Changing Isovector Radius

Koning-Delaroche
NPA713(03)231
same radii $R$ for $U_0$ & $U_1$!

$$U_1(r) \propto \frac{U_{01}}{1 + \exp \frac{r-R}{a}}$$

$R \rightarrow R + \Delta R_1$

charge-exchange cs oscillations grow

Elastic

Charge Exchange

$p,p$ Elastic

QE $(p,n)$

$E_p = 35$ MeV $^{48}$Ca

effect of $U_1$ radii growing in increments of 0.25fm
Effect of Changing Isoscalar Radius

Koning-Delaroche
NPA713(03)231
same radii $R$ for $U_0$ & $U_1$!

$$U_0(r) \propto \frac{U_{00}}{1 + \exp \frac{r-R}{a}}$$

$R \to R + \Delta R_0$

charge-exchange cs oscillations shrink

Elastic

Charge Exchange

$E_p = 35$ MeV $^{48}$Ca
effect of $U_0$ radii growing in increments of 0.25 fm

Symmetry Energy Danielewicz, Singh, Lee
Impact of $U$-Radii on $(p,n)$ Cross Section

**DWBA**

$$\frac{d\sigma}{d\Omega} \propto \left| \int dr \, \psi_p^*(r) \, U_1(r) \, \psi_n(i) \right|^2$$

**Symmetry Energy**

- Isoscalar radius responsible for holes in wavefunctions $\psi$
- Isovector radius responsible for region where $(p,n)$ conversion can occur
Modified Koning-Delaroche Fits: $^{48}$Ca

In Koning-Delaroche:  
\[ R_{0,1} = R + \Delta R_{0,1} \quad a_{0,1} = a + \Delta a_{0,1} \]
Modified Koning-Delaroche Fits: $^{90}$Zr

In Koning-Delaroche:

$R_{0,1} = R + \Delta R_{0,1}$  
$a_{0,1} = a + \Delta a_{0,1}$
Modified Koning-Delaroche Fits: $^{120}$Sn

In Koning-Delaroche: $R_{0,1} = R + \Delta R_{0,1}$, $a_{0,1} = a + \Delta a_{0,1}$
Modified Koning-Delaroche Fits: $^{208}\text{Pb}$

In Koning-Delaroche:

\[ R_{0,1} = R + \Delta R_{0,1} \]

\[ a_{0,1} = a + \Delta a_{0,1} \]

Symmetry Energy Danielewicz, Singh, Lee
Size of Isovector Skin

Large \sim 0.9 \text{ fm skins!} \sim \text{Independent of } A...
Difference in Surface Diffuseness

Sharper isovector surface!
Constraints on Symmetry-Energy Parameters (Outdated)
Constraints on $S(\rho)$ (Outdated)
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.

- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.

- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.

- Such an analysis produces large isovector skins $\Delta R \sim 0.9$ fm!

- Symmetry energy is stiff!

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al
US PHY-1403906 + Indo-US Grant
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.
- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.
- Such an analysis produces large isovector skins $\Delta R \sim 0.9 \text{ fm}!$
- Symmetry energy is stiff!

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh *et al*
US PHY-1403906 + Indo-US Grant
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.
- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.
- Such an analysis produces large isovector skins $\Delta R \sim 0.9$ fm!
- Symmetry energy is stiff!

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al
US PHY-1403906 + Indo-US Grant
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.
- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.
- Such an analysis produces large isovector skins $\Delta R \sim 0.9$ fm!
- Symmetry energy is stiff!

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al
US PHY-1403906 + Indo-US Grant
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.
- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.
- Such an analysis produces large isovector skins $\Delta R \sim 0.9$ fm!
- Symmetry energy is stiff!

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al
US PHY-1403906 + Indo-US Grant
Conclusions

- Symmetry-energy polarizes nuclear densities, pushing isovector density out to region of low isoscalar density.
- For large $A$, displacement of isovector relative to isoscalar surface is expected to be roughly independent of nucleus and depend on slope of symmetry energy.
- Surface displacement can be studied in comparative analysis of data on elastic scattering and quasielastic charge-exchange reactions.
- Such an analysis produces large isovector skins $\Delta R \sim 0.9$ fm!
- Symmetry energy is stiff!

PD&Lee NPA818(09)36 NPA922(14)1; PD, Singh et al
US PHY-1403906 + Indo-US Grant