上海交通大学
SHANGHAI JIAO TONG UNIVERSITY

Skyrme effective pseudo-potentials for heavy ion collisions

Rui Wang(王睿)
INPAC, Department of Physics and Astronomy,
Shanghai Jiao Tong University

Advisor: Prof. Lie-Wen Chen(陈列文)

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Background and motivation

Effective nuclear interactions:

Skyrme interaction

Momentum-dependent interaction(MDI)

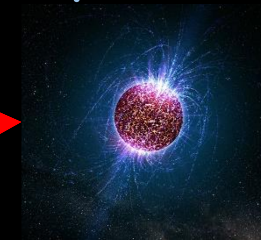
Gogny interaction

Relativistic mean-field

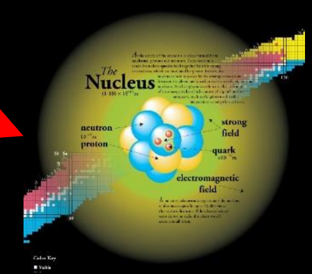
……

To higher energy effective pseudo-potential

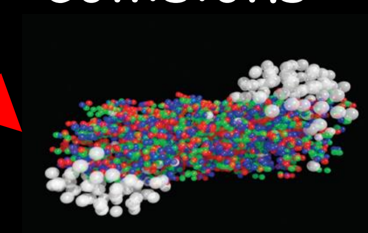
Compact stars



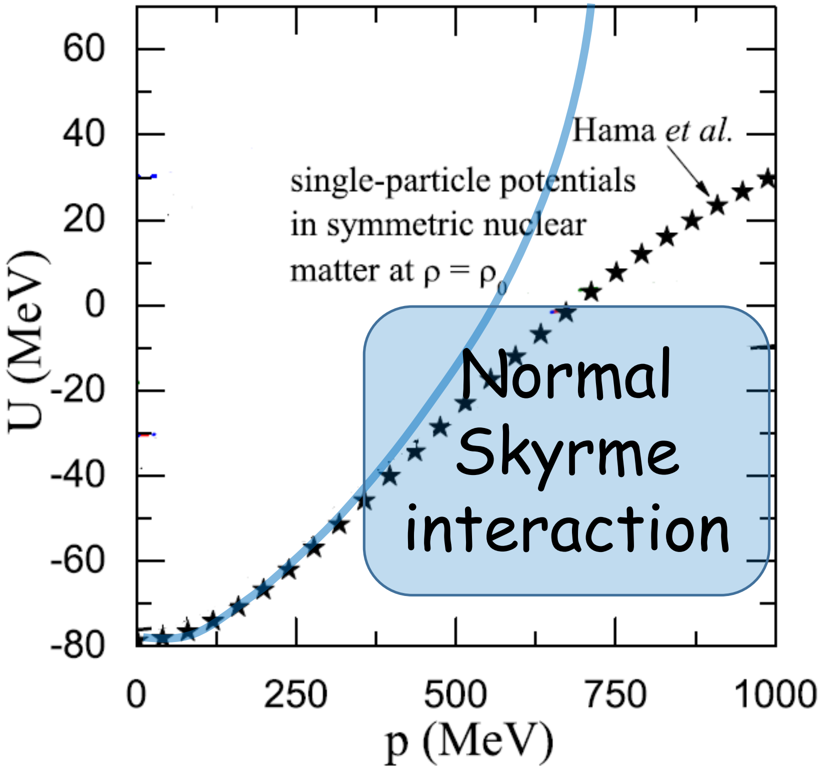
Finite nuclei



Heavy ion collisions



Background and motivation



Normal Skyrme interaction

Only capable for HICs less than 300MeV

local nuclear energy density functional up to N3LO
B.G. Carlsson et al
PRC78,044326 (2008)

Corresponding Skyrme effective pseudo-potential up to N3LO
F. Raimondi et al
PRC83,054311 (2011)

high order terms of momentum

Skyrme effective pseudo-potential

$$
\begin{aligned}
v_{Sk} = \; & t_0^{(0)}(1 + x_0^{(0)}\hat{P}_\sigma) + \frac{1}{6}t_3^{(0)}(1 + x_3^{(0)}\hat{P}_\sigma)\rho^\alpha(\vec{R}) \\
& + \frac{1}{2}t_1^{(2)}(1 + x_1^{(2)}\hat{P}_\sigma)[\vec{k}'^2 + \vec{k}^2] + t_2^{(2)}(1 + x_2^{(2)}\hat{P}_\sigma)\vec{k}' \cdot \vec{k} \\
& + \frac{1}{4}t_1^{(4)}(1 + x_1^{(4)}\hat{P}_\sigma)[(\vec{k}'^2 + \vec{k}^2)^2 + 4(\vec{k}' \cdot \vec{k})^2] \\
& + t_2^{(4)}(1 + x_2^{(4)}\hat{P}_\sigma)(\vec{k}' \cdot \vec{k})(\vec{k}'^2 + \vec{k}^2) \\
& + \frac{1}{2}t_1^{(6)}(1 + x_1^{(6)}\hat{P}_\sigma)(\vec{k}'^2 + \vec{k}^2)^2[(\vec{k}'^2 + \vec{k}^2)^2 + 12(\vec{k}' \cdot \vec{k})^2] \\
& + t_2^{(6)}(1 + x_2^{(6)}\hat{P}_\sigma)(\vec{k}' \cdot \vec{k})[3(\vec{k}'^2 + \vec{k}^2)^2 + 4(\vec{k}' \cdot \vec{k})^2]
\end{aligned}
$$

Normal Skyrme

N2LO
4th order of momentum

N3LO
6th order of momentum

plus spin-orbit and tensor terms

an overall $\delta(\vec{r}_1 - \vec{r}_2)$ is implicit

Energy density functional

local nuclear energy density functional up to N3LO

B.G. Carlsson et al PRC78,044326 (2008)

$$H(\vec{r}) = f[\rho_{nLvJ}(\vec{r})] \quad \rho_{nLvJ}(\vec{r}) = \{[K_{nL}\rho_v(\vec{r},\vec{r}')]_J\}_{\vec{r}'=\vec{r}}$$

$$\rho_{00}(\boldsymbol{r},\boldsymbol{r}') = \rho(\boldsymbol{r},\boldsymbol{r}')$$

$$s_{1,\mu=\{-1,0,1\}}(\boldsymbol{r},\boldsymbol{r}') = -i\left\{\frac{1}{\sqrt{2}}[s_x(\boldsymbol{r},\boldsymbol{r}') - is_y(\boldsymbol{r},\boldsymbol{r}')], s_z(\boldsymbol{r},\boldsymbol{r}'), \frac{-1}{\sqrt{2}}[s_x(\boldsymbol{r},\boldsymbol{r}') + is_y(\boldsymbol{r},\boldsymbol{r}')]\right\}$$

$$\nabla_{1,\mu=\{-1,0,1\}} = -i\left\{\frac{1}{\sqrt{2}}(\nabla_x - i\nabla_y), \nabla_z, \frac{-1}{\sqrt{2}}(\nabla_x + i\nabla_y)\right\}$$

$$k_{1,\mu=\{-1,0,1\}} = -i\left\{\frac{1}{\sqrt{2}}(k_x - ik_y), k_z, \frac{-1}{\sqrt{2}}(k_x + ik_y)\right\}$$

No.	Tensor D_{nL}	Order n	Rank L
1	1	0	0
2	∇	1	1
3	$[\nabla\nabla]_0$	2	0
4	$[\nabla\nabla]_2$	2	2
5	$[\nabla\nabla]_0\nabla$	3	1
6	$[\nabla[\nabla\nabla]_2]_3$	3	3
7	$[\nabla\nabla]_0^2$	4	0
8	$[\nabla\nabla]_0[\nabla\nabla]_2$	4	2
9	$[\nabla[\nabla[\nabla\nabla]_2]_3]_4$	4	4
10	$[\nabla\nabla]_0^2\nabla$	5	1
11	$[\nabla\nabla]_0[\nabla[\nabla\nabla]_2]_3$	5	3
12	$[\nabla[\nabla[\nabla[\nabla\nabla]_2]_3]_4]_5$	5	5
13	$[\nabla\nabla]_0^3$	6	0
14	$[\nabla\nabla]_0^2[\nabla\nabla]_2$	6	2
15	$[\nabla\nabla]_0[\nabla[\nabla[\nabla\nabla]_2]_3]_4$	6	4
16	$[\nabla[\nabla[\nabla[\nabla[\nabla\nabla]_2]_3]_4]_5]_6$	6	6

N3LO terms

B.G. Carlsson et al PRC78,044326 (2008)

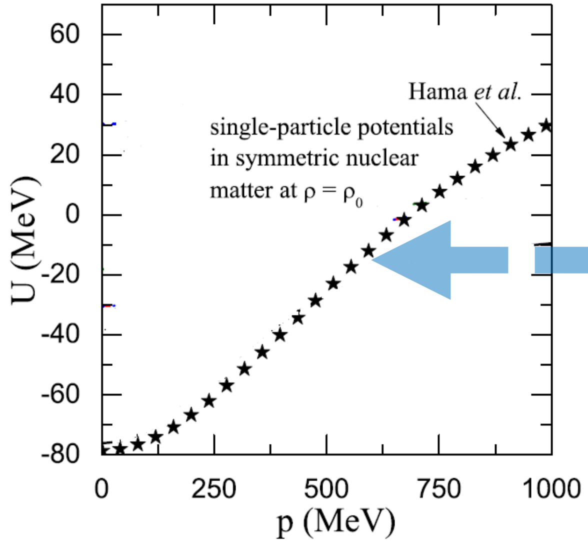
Background and motivation

local nuclear energy density functional up to N3LO-PRC78

Code on spherical nuclei
B.G. Carlsson et al
CPC181, 1641-1657 (2010)

Corresponding Skyrme effective pseudo-potential up to N3LO - PRC83

D. Davesne et al
developed various Skyrme effective pseudo-potentials up to N3LO which can be used to describe nuclear matter
PRC91,064303 (2015)
A&A585,A83 (2016)……

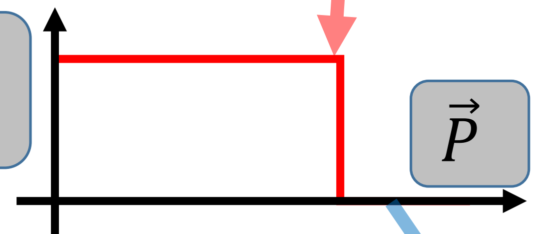


Landau Fermi liquid theory

Real particles → Qusai-particles → distribution of ground state

distribution of ground state unknown

Distribution $n(\vec{r},\vec{p})$



$$\varepsilon(\vec{r}, n_{p\sigma\tau}) = \varepsilon_0(\vec{r}) + \frac{1}{V}\sum_{p\sigma} \varepsilon^0_{p\sigma\tau}(\vec{r})\delta n_{p\sigma\tau}(\vec{r}) + high\ order\ terms$$

$$\int \varepsilon(\vec{r})d^3r = \sum_i \langle i|\frac{p^2}{2m}|i\rangle + \frac{1}{2}\sum_{i,j}\langle ij|v_{Sk}|ij\rangle \sim a\ function\ of\ n(\vec{r},\vec{p})$$

Single particle potential $\varepsilon^0_{p\sigma\tau} = V\frac{\delta E}{\delta n_{p\sigma\tau}}$

Energy density of effective pseudo-potential

In the case of nuclear matter

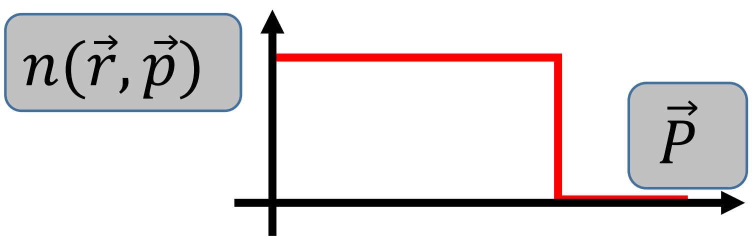
$$E(\vec{r}, n(\vec{r},\vec{p})) = \int dp \vec{p}^2 n(\vec{r},\vec{p}) + \sum_{i=2,4,6} C^{(i)} \int dp dp' (\vec{p}-\vec{p}')^i n(\vec{r},\vec{p}) n(\vec{r},\vec{p}')$$
$$+ D^{(i)} \int dp dp' (\vec{p}-\vec{p}')^i n_n(\vec{r},\vec{p}) n_n(\vec{r},\vec{p}')$$
$$+ D^{(i)} \int dp dp' (\vec{p}-\vec{p}')^i n_p(\vec{r},\vec{p}) n_p(\vec{r},\vec{p}')$$

$$C^{(i)} = \frac{1}{16\hbar^i}[t_1^{(i)}(2+x_1^{(i)}) + t_2^{(i)}(2+x_2^{(i)})]$$

$$D^{(i)} = \frac{1}{16\hbar^i}[-t_1^{(i)}(2x_1^{(i)}+1) + t_2^{(i)}(2x_2^{(i)}+1)]$$

Single particle potential

$$U(\vec{p},\rho,\tau) = \varepsilon^0_{p\sigma\tau} = V\frac{\delta E}{\delta n_{p\sigma\tau}}$$



Equation of state

$$E = E(\rho)$$

Equation of State of nuclear matter

Equation of state of asymmetry nuclear matter:

$$E(\rho,\delta) = E_0(\rho) + E_{sym}(\rho)\delta^2 + E_{sym,4}(\rho)\delta^4 + O(\delta^6)$$

Isospin asymmetry parameter

$\delta = (\rho_n - \rho_p)/(\rho_r + \rho_r)$

EoS of symmetric nuclear matter: $E_0(\rho) = E_0(\rho_0) + \frac{K_0}{2!}\chi^2 + O(\chi^3)$

Symmetry energy:

$$E_{sym}(\rho) = \frac{1}{2}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}\bigg|_{\delta=0}$$

$$E_{sym,4}(\rho_0) = \frac{1}{4!}\frac{\partial^2 E(\rho,\delta)}{\partial\delta^2}\bigg|_{\rho=\rho_0,\delta=0}$$

Where $\chi = (\rho - \rho_r)/3\rho_r$

$$K_0 = 9\rho^2\frac{\partial^2 E(\rho,\delta=0)}{\partial\rho^2}\bigg|_{\rho=\rho_0}$$

Characteristic parameter of $E_{sym}(\rho)$:

$$E_{sym}(\rho_r) = E_{sym}(\rho=\rho_r) \qquad L(\rho_r) = 3\rho_r\frac{\partial E_{sym}(\rho)}{\partial\rho}\bigg|_{\rho=\rho_r}$$

Single particle potential

In non-relativistic framework, the single nucleon energy $E(\rho, \delta, \vec{k})$ can be expressed generally using the following dispersion relation:

$$E_{n/p}(\rho, \delta, \vec{k}) = \frac{\vec{k}^2}{2m} + U_{n/p}(\rho, \delta, \vec{k}, n_{n/p}(\vec{r}, \vec{k}))$$

Single particle potential

Distribution function

$$U_{n/p}(\rho, \delta, \vec{k}, f(\vec{r}, \vec{k}))$$

$$= U_0(\rho, \vec{k}, f(\vec{r}, \vec{k})) + \sum_{i=1} U_{sym,i}(\rho, \vec{k}, n_{n/p}(\vec{r}, \vec{k}))\, \tau_3{}^i \delta^i$$

Symmetry potential

Effective mass $$\frac{\hbar^2}{2m^*_{n/p}} = \frac{\hbar^2}{2m} + \frac{1}{p}\frac{\partial U_{n/p}(\rho, \delta, \vec{k}, n_{n/p}(\vec{r}, \vec{k}))}{\partial p}$$

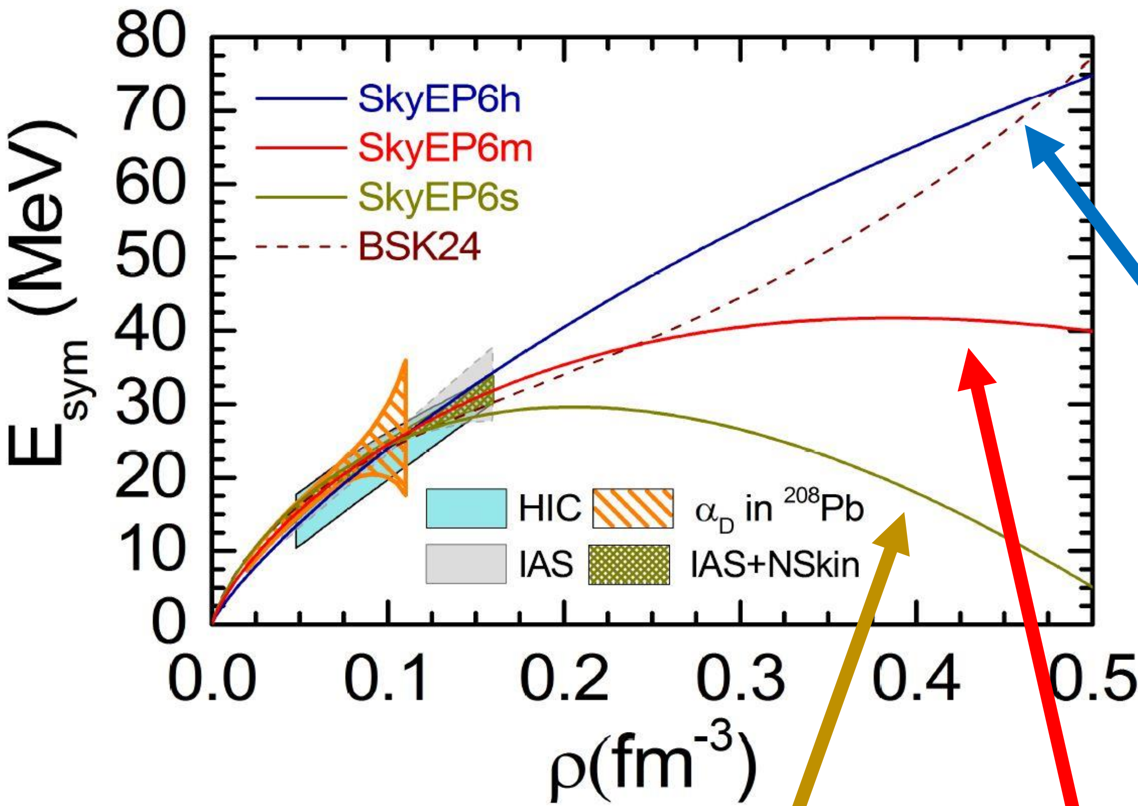
The requirements of the new interactions

- Satisfies all possible constrains from the nuclear matter;

- Realistic single particle potential or Hama potential to higher momentum ~ $1.5\ GeV/c^2$ (corresponding to incident energy of 1 GeV), at $\rho = \rho_0$;

- Qualitatively reproduce the first order symmetry potential calculated by microscopic calculation up to momentum ~ $1.5\ GeV/c^2$, at $\rho = \rho_0$;

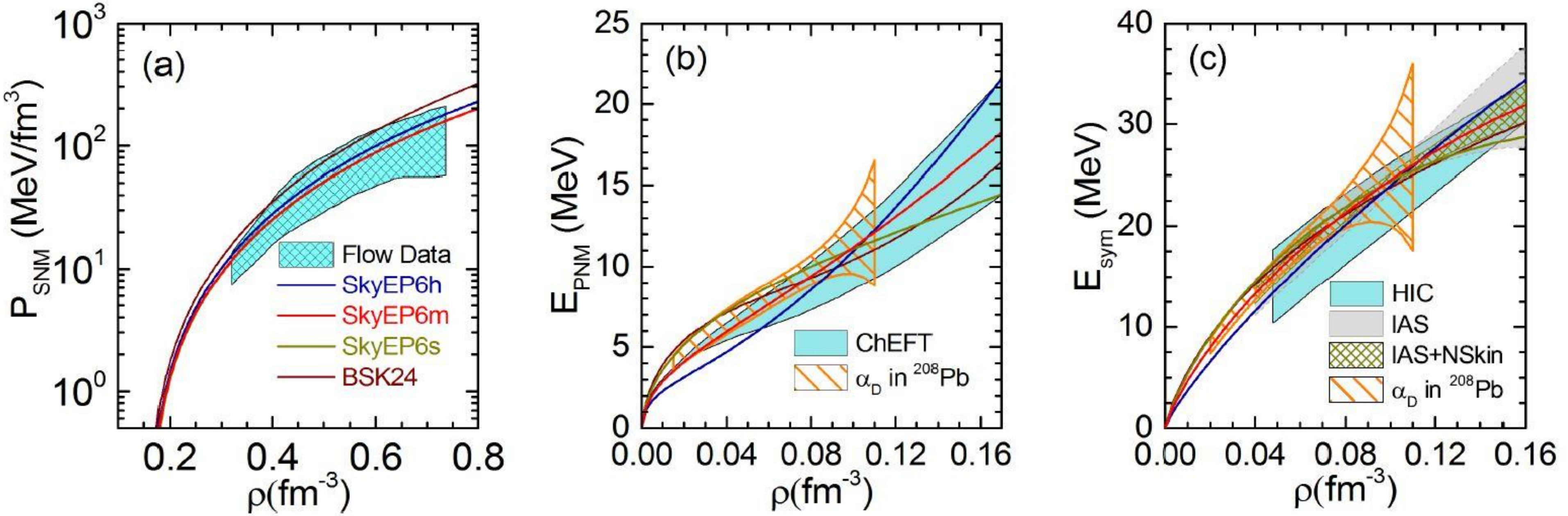
New Skyrme effective pseudo-potentials



New interaction SkyEP6s SkyEP6m and SkyEP6h
Skyrme **E**ffective **P**seudo-potential up to N3LO(**6**th order in momentum)

Properties of Nuclear Matter

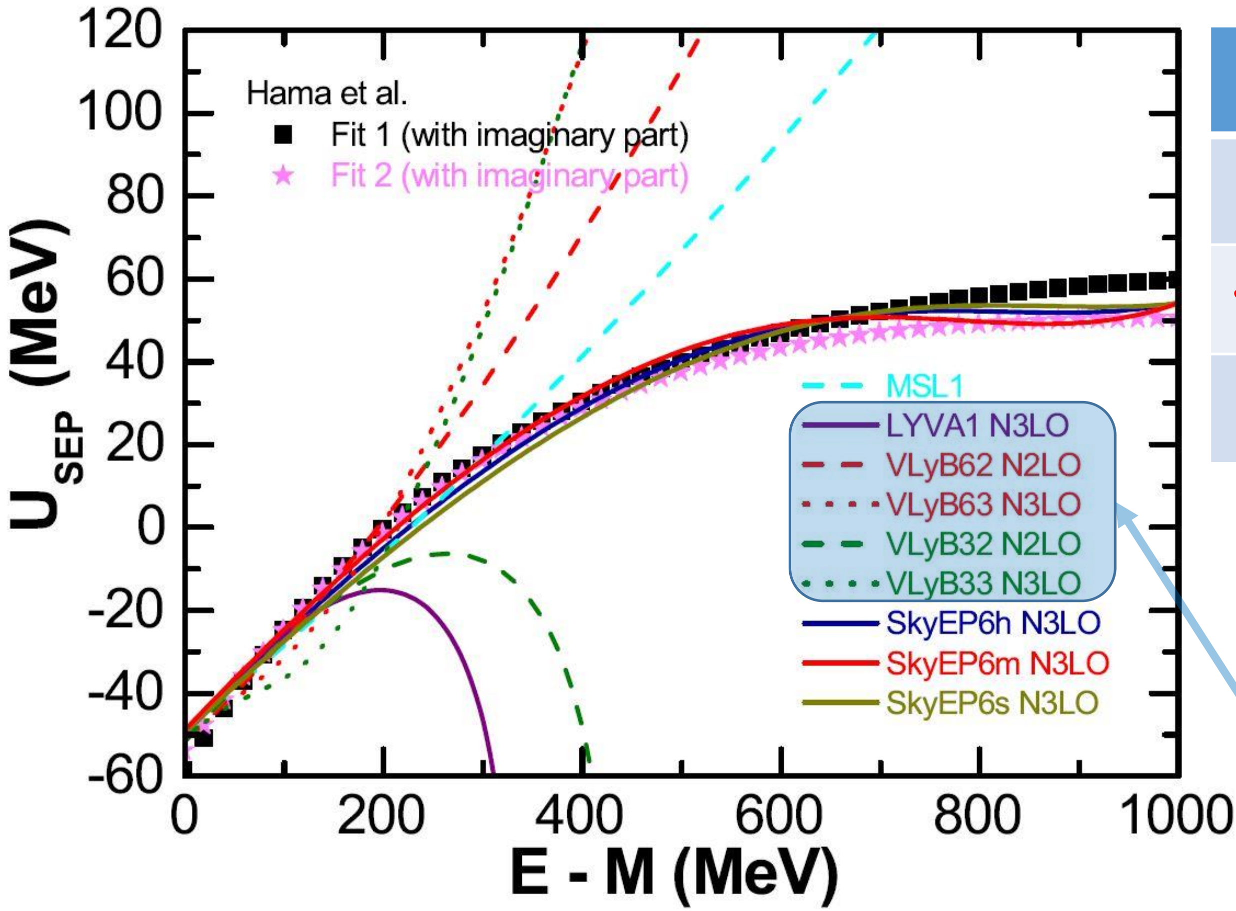
flow data of SNM | EoS for PNM | symmetry energy



	ρ_0	E_0	K_0	$E_{sym}(\rho_0)$	$L(\rho_0)$	$E_{sym}(\rho_{sc})$	$L(\rho_{sc})$	$E_{sym,4}(\rho_0)$
	fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV
SkyEP6s	**0.163**	**-16.1**	**216.9**	**28.84**	**18.28**	**25.60**	**31.42**	**≈0.94**
SkyEP6m	**0.162**	**-16.0**	**216.1**	**32.02**	**47.89**	**26.09**	**45.94**	**≈0.92**
SkyEP6h	**0.161**	**-15.9**	**230.7**	**34.54**	**77.62**	**25.97**	**59.92**	**≈0.96**

Single particle properties

The main improvement of the new interactions SkyEP6s SkyEP6m and SkyEP6h is the realistic single particle potential to about 1 GeV incident energy



	$m^*_{s,0}/m$	$m^*_{v,0}/m$
SkyEP6s	0.78	0.66
SkyEP6m	0.75	0.65
SkyEP6h	0.77	0.65

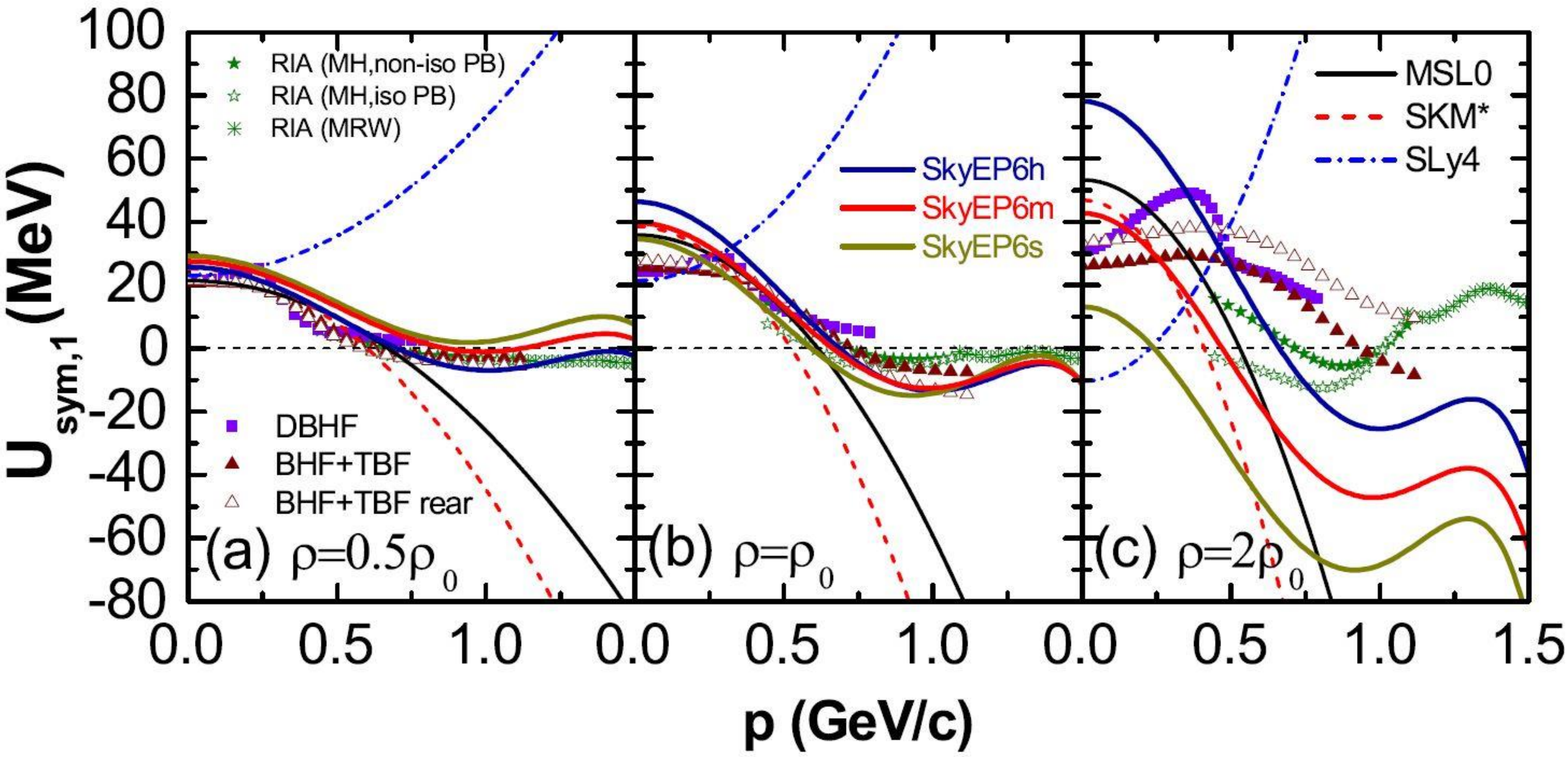
Previous Skyrme effective pseudo-potentials
D. Davesne et al

Single particle properties

The comparison of first order symmetry potential to microscopic calculations

relativistic impulse approximation (RIA)

Brueckner-Hartree-Fock (BHF)



Summery and outlook

- We have developed three Skyrme effective pseudo-potentials up to N3LO (6^{th} order in momentum, with D-wave component included), the main improvement of these new interaction is its realistic single particle behavior; apart from that they also satisfies constrains from nuclear matter.

- With the realistic single particle potential, Skyrme interactions can be adopted to simulations of heavy ion collisions of higher energy region(~1GeV/A) in the future, then we may expect more crosschecks for Skyrme interaction from finite nuclei and heavy ion collisions.

- These new Skyrme effective pseudo-potentials can be used to probe the symmetry energy behavior at supra-saturation density (2 – 3 times of saturation density).

Thank you