Correlations between asymmetric nuclear matter EOS and properties of neutron stars

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Outline

1 Introduction

2 Tidal polarizability in SHF approach

3 Optimization and covariance analysis

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1. Introduction

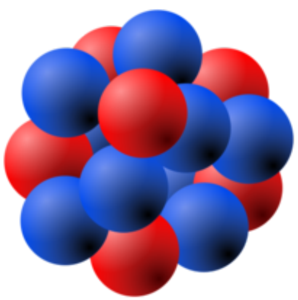
- EOS of isospin asymmetric nuclear matter (Parabolic law) :

$$E(\rho,\delta) = E_0(\rho) + E_{sym}(\rho)\delta^2 + O(\delta^4)$$

$\delta = (\rho_n - \rho_p)/\rho \longrightarrow$ Isospin asymmetry

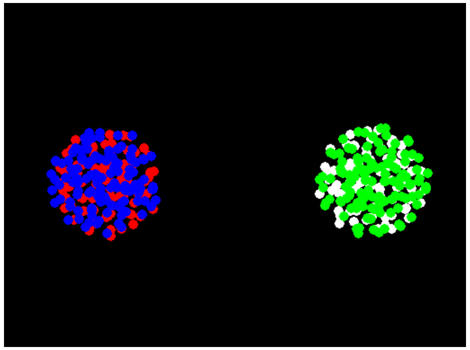
✓ EOS of symmetric nuclear matter(SNM) (well-known at ρ_0)

✓ Symmetry energy (poorly known)



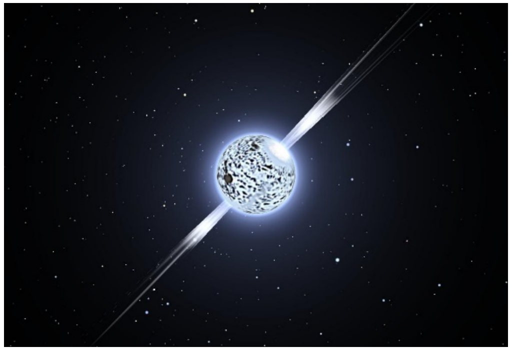
Finite nuclei

$\rho \sim \rho_0$



Heavy-ion collision

$2 \sim 3\rho_0$



Neutron star

$\rho > 3\rho_0$

Purpose: Studying the EOS via observables of neutron stars

- **EOS of SNM and Symmetry energy:**

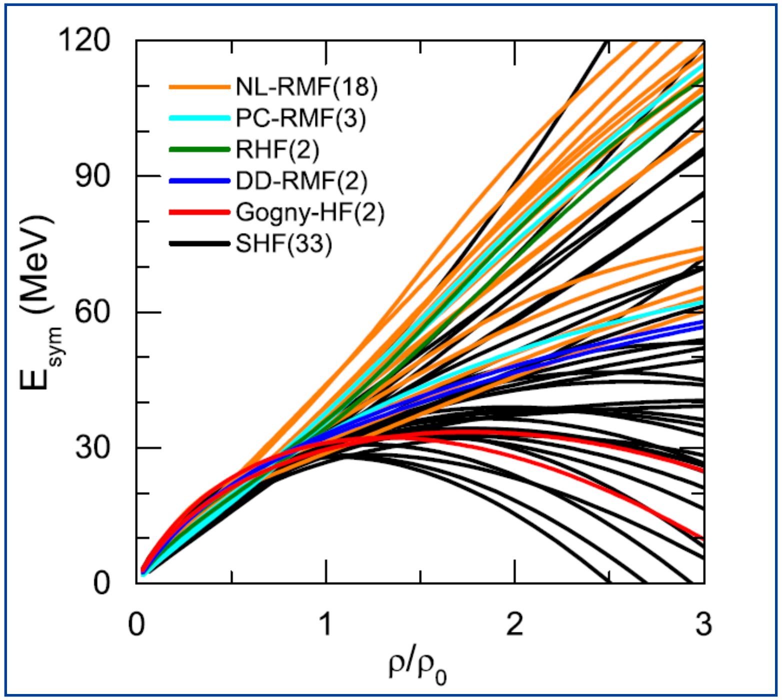
$$E_0(\rho) = E_0(\rho_0) + \frac{1}{2!}K_0 x^2 + \frac{1}{3!}J_0 x^3 + O(x^4), \qquad x = (\rho - \rho_0)/3\rho_0$$

$$E_{sym}(\rho) = E_{sym}(\rho_r) + L(\rho_r)x_r + \frac{K_{sym}(\rho_r)}{2!}x_r^2 + O(x_r^3), \qquad x_r = (\rho - \rho_r)/3\rho_r$$

ρ_0 satisfied with: $P = \rho^2 \frac{d\,E_0(\rho)}{d\rho}\Big|_{\rho=\rho_0} = 0$

$$K_0 = 9\rho_0^2 \frac{d^2 E_0(\rho)}{d\rho^2}\Big|_{\rho=\rho_0} \qquad L(\rho_r) = 3\rho_r \frac{dE_{sym}(\rho)}{d\rho}\Big|_{\rho=\rho_r}$$

$$J_0 = 27\rho_0^3 \frac{d^3 E_0(\rho)}{d\rho^3}\Big|_{\rho=\rho_0} \qquad K_{sym} = 9\rho_r^2 \frac{d^2 E_{sym}(\rho)}{d\rho^2}\Big|_{\rho=\rho_r}$$



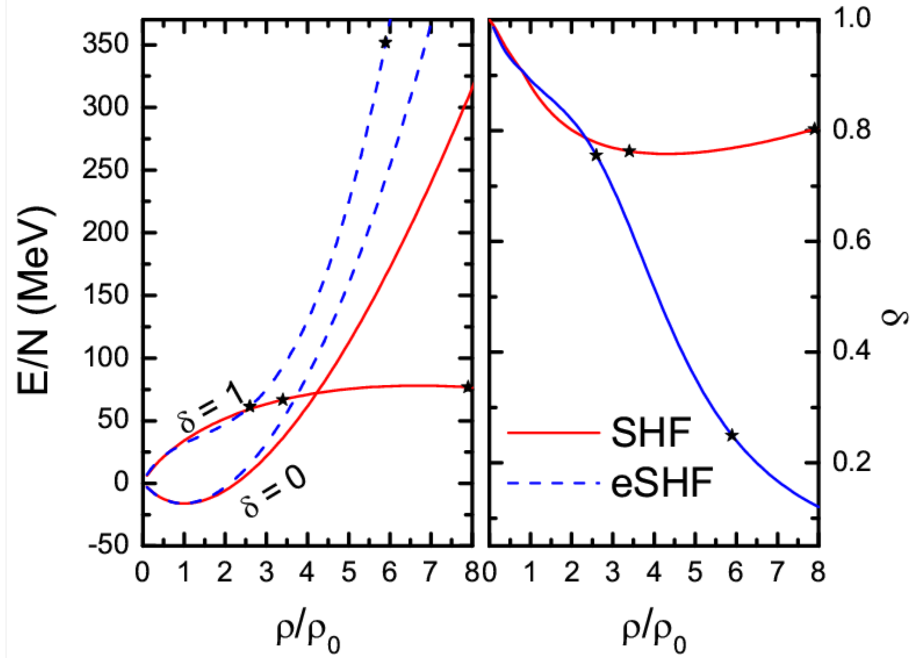
L.W. Chen, EPJ Web of Conferences, 88, 00017, (2015)

2. Tidal polarizability in SHF approach

- **SHF and eSHF interactions:**

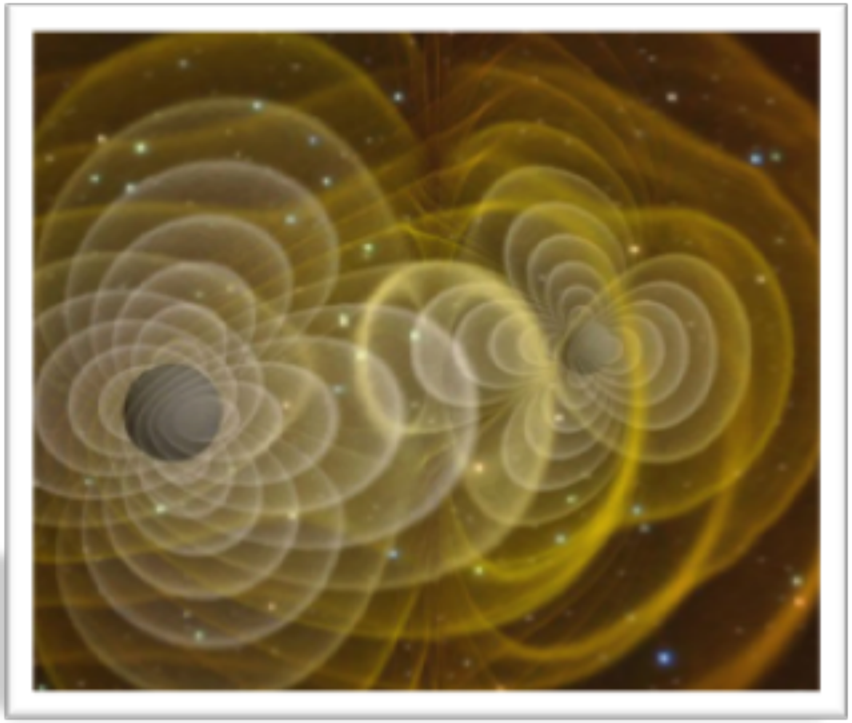
$$
\begin{aligned}
v_{i,j} =& t_0(1 + x_0 P_\sigma)\delta(\mathbf{r}) \\
& + \frac{1}{2} t_1 (1 + x_1 P_\sigma)[\overleftarrow{\mathbf{k}}^2 \delta(\mathbf{r}) + \delta(\mathbf{r}) \overrightarrow{\mathbf{k}}^2] \\
& + t_2 (1 + x_2 P_\sigma) \overleftarrow{\mathbf{k}} \cdot \delta(\mathbf{r}) \overrightarrow{\mathbf{k}} \\
& + \frac{1}{6} t_3 (1 + x_3 P_\sigma) \rho(\mathbf{R})^\alpha \delta(\mathbf{r}) \\
& + i W_0 (\sigma_i + \sigma_j) \overleftarrow{\mathbf{k}} \cdot \delta(\mathbf{r}) \overrightarrow{\mathbf{k}}
\end{aligned}
$$

$$
\begin{aligned}
v'_{i,j} =& v_{i,j} + \frac{1}{2} t_4 (1 + x_4 P_\sigma) \\
& [\overleftarrow{\mathbf{k}}^2 \rho(\mathbf{R})^\beta \delta(\mathbf{r}) + \delta(\mathbf{r}) \rho(\mathbf{R})^\beta \overrightarrow{\mathbf{k}}^2] \\
& + t_5 (1 + x_5 P_\sigma) \overleftarrow{\mathbf{k}} \cdot \rho(\mathbf{R})^\gamma \delta(\mathbf{r}) \overrightarrow{\mathbf{k}}
\end{aligned}
$$



Different density functional

- Tidal polarizability
  (oscillation response coefficient λ)



$$Q_{ij} = \lambda \varepsilon_{ij}$$

Q_{ij}: Quadrupole moment

ε_{ij}: Tidal field of companion

$$\lambda = \frac{2}{3} k_2 R^5$$

K_2: the rigidity of star

Éanna É. Flanagan and Tanja Hinderer, Phys.Rev.D **77**, 021502(R) (2008)
F.J. Fattoyev, J. Carvajal, W.G. Newton, and Bao-An Li, Phys. Rev. C **87**, 015806 (2013)

EOS +

$$\frac{dy}{dr} = -\frac{1}{r}[y^2 + yF(p,\varepsilon) + r^2 Q(p,\varepsilon)]$$

$$\frac{dp}{dr} = -\frac{(\varepsilon + p)(m + 4\pi r^3 p)}{r(r-2m)}$$

$$\frac{dm}{dr} = 4\pi r^2 \varepsilon$$

⇨

M

R

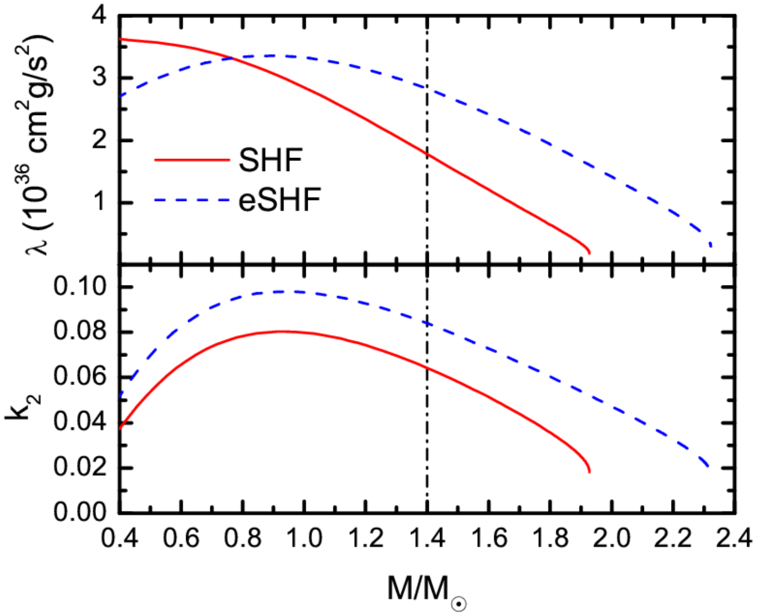
$y_R \equiv y(R)$

⇩

$$k_2 = \frac{1}{20}\left(\frac{R_s}{R}\right)^5\left(1-\frac{R_s}{R}\right)^2\left[2-y_R+(y_R-1)\frac{R_s}{R}\right]\left\{\frac{R_s}{R}\left(6-3y_R+\frac{3R_s}{2R}(5y_R-8)\right)\right.$$
$$+\frac{1}{4}\left(\frac{R_s}{R}\right)^3\left[26-22y_R+\frac{R_s}{R}(3y_R-2)+\left(\frac{R_s}{R}\right)^2(y_R+1)\right]$$
$$\left.+3\left(1-\frac{R_s}{R}\right)^2\left[2-y_R+(y_R-1)\frac{R_s}{R}\right]\ln\left(1-\frac{R_s}{R}\right)\right\}^{-1}$$

⇩

$$\lambda = \frac{2}{3}k_2 R^5$$



For earth: $0.304 < k_2 < 0.312$

3. Optimization and covariance analysis

- **Minimizing the Chi-square $\chi^2(p)$:**

$$\chi^2(P) = \sum_{n=1}^{N}\left(\frac{O_n^{(th)}(P) - O_n^{(exp)}}{\Delta O_n}\right)^2$$

Properties of finite nuclei

Nucleus	B(MeV)	Rch(fm)	E_{GMR}(MeV)	error	spin-orbit
$^{20}_{8}O$	-127.61723	2.701	–	–	6.30(1np)
					6.10(1pp)
$^{40}_{20}Ca$	-342.03360	3.478	–	–	–
$^{48}_{20}Ca$	-415.97198	3.479	–	–	–
$^{56}_{28}Ni$	-483.95050	3.750	–	–	–
$^{68}_{28}Ni$	-590.36648	–	–	–	–
$^{88}_{38}Sr$	-768.38296	4.220	–	–	–
$^{90}_{40}Zr$	-783.79526	4.269	17.81	0.35	–
$^{100}_{50}Sn$	-824.62947	–	–	–	–
$^{116}_{50}Sn$	-988.51881	4.626	15.90	0.07	–
$^{132}_{50}Sn$	-1102.68603	–	–	–	–
$^{144}_{62}Sm$	-1195.46065	4.960	15.25	0.11	–
$^{208}_{82}Pb$	-1635.89266	5.504	14.18	0.11	1.42(2dp)
					0.90(3pn)
					1.77(2fn)

Fitting →

Quantity	MSL0	P_0
ρ_0	0.16	0.1608
E_0(MeV)	-16.0	-16.0313
K_0(MeV)	230.0	228.3431
$m^*_{s,0}/m$	0.80	1.0281
$m^*_{v,0}/m$	0.70	1.0119
$E_{sym}(\rho_0)$(MeV)	30.0	33.7289
L(MeV)	60	63.8086
G_S(MeVfm^5)	132.0	113.4911
G_V(MeVfm^5)	5.0	15.8508
W_0(MeVfm^5)	133.3	108.7601

Quantity	eMSL09	P_0
ρ_0	0.1585	0.1600
E_0(MeV)	-16.046	-16.0365
K_0(MeV)	230.1	229.0086
J_0(MeV)	-353.7	-395.0170
G_S(MeVfm^5)	92.9	100.7536
G_V(MeVfm^5)	46.2	49.7946
G_{SV}(MeVfm^5)	-8.4	15.9162
$G'_0(\rho_0)$	0.25	0.3079
$m^*_{s,0}/m$	0.9	1.0156
$m^*_{v,0}/m$	0.75	0.7542
$E_{sym}(\rho_0)$	32.8	32.5187
$L(\rho_0)$	53.7	57.8552
$K_{sym}(\rho_0)$	-100.2	-76.2283
W_0(MeVfm^5)	103.78	109.2549

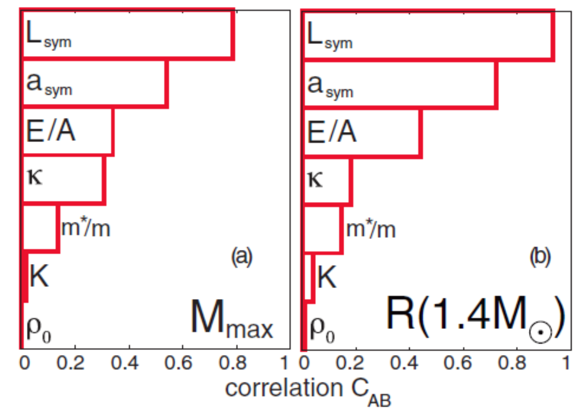
M. Wang *et al.*, Chinese Physics C, **36**, 12 (2012) experimental data
M. Kortelainen *et al.*, Phys.Rev.C **82**, 024313 (2010) Fitting example

• Covariance analysis:

The expansion of $\chi^2(p)$ around the point p_0 :

$$\chi^2(P) = \chi^2(P_0) + \frac{1}{2}\sum_{i,j=1}^{F}(P-P_0)_i(P-P_0)_j\,\partial_i\partial_j\chi^2(P_0) + \cdots$$

$$\Delta\chi^2(\mathrm{x}) \equiv \chi^2(P) - \chi^2(P_0) = \mathrm{x}^T \mathbf{M}\ \mathrm{x} \qquad \left(\mathrm{x} = \frac{P-P_0}{P}\right)$$



(Color online) Correlation C_{AB} between selected nuclear matter properties and (a) the maximum mass of a neutron star and (b) the radius of a $1.4M_\odot$ neutron star obtained for the Skyrme functional SV-min.

Phys.Rev.C **87**, 044320 (2013)

Covariance and correlation coefficient:

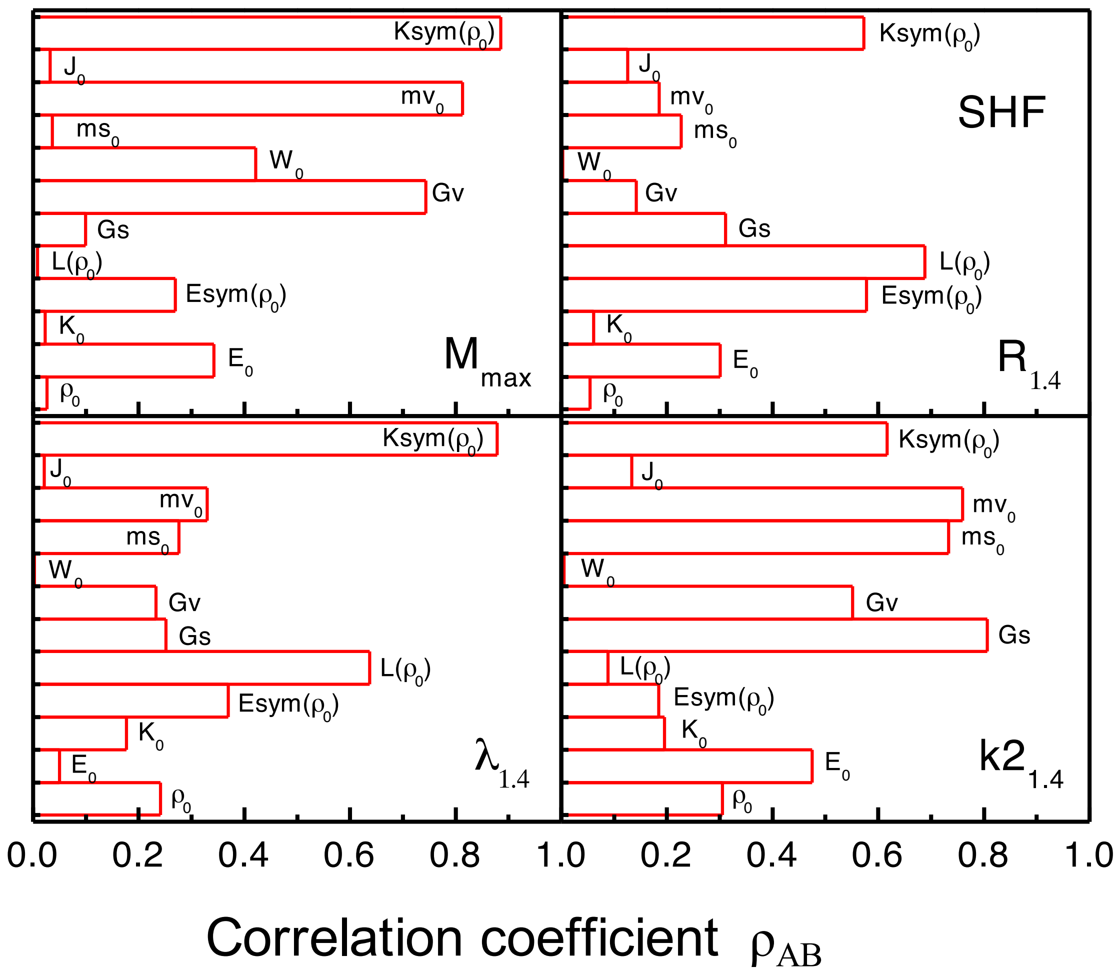
$$\mathrm{Cov}(A,B) = \sum_{i,j=1}^{F}\frac{\partial A}{\partial \mathrm{x}_i}\mathbf{M}_{ij}^{-1}\frac{\partial B}{\partial \mathrm{x}_j}$$

$$\rho_{AB} = \frac{\mathrm{Cov}(A,B)}{\sqrt{D(A)}\sqrt{D(B)}}$$

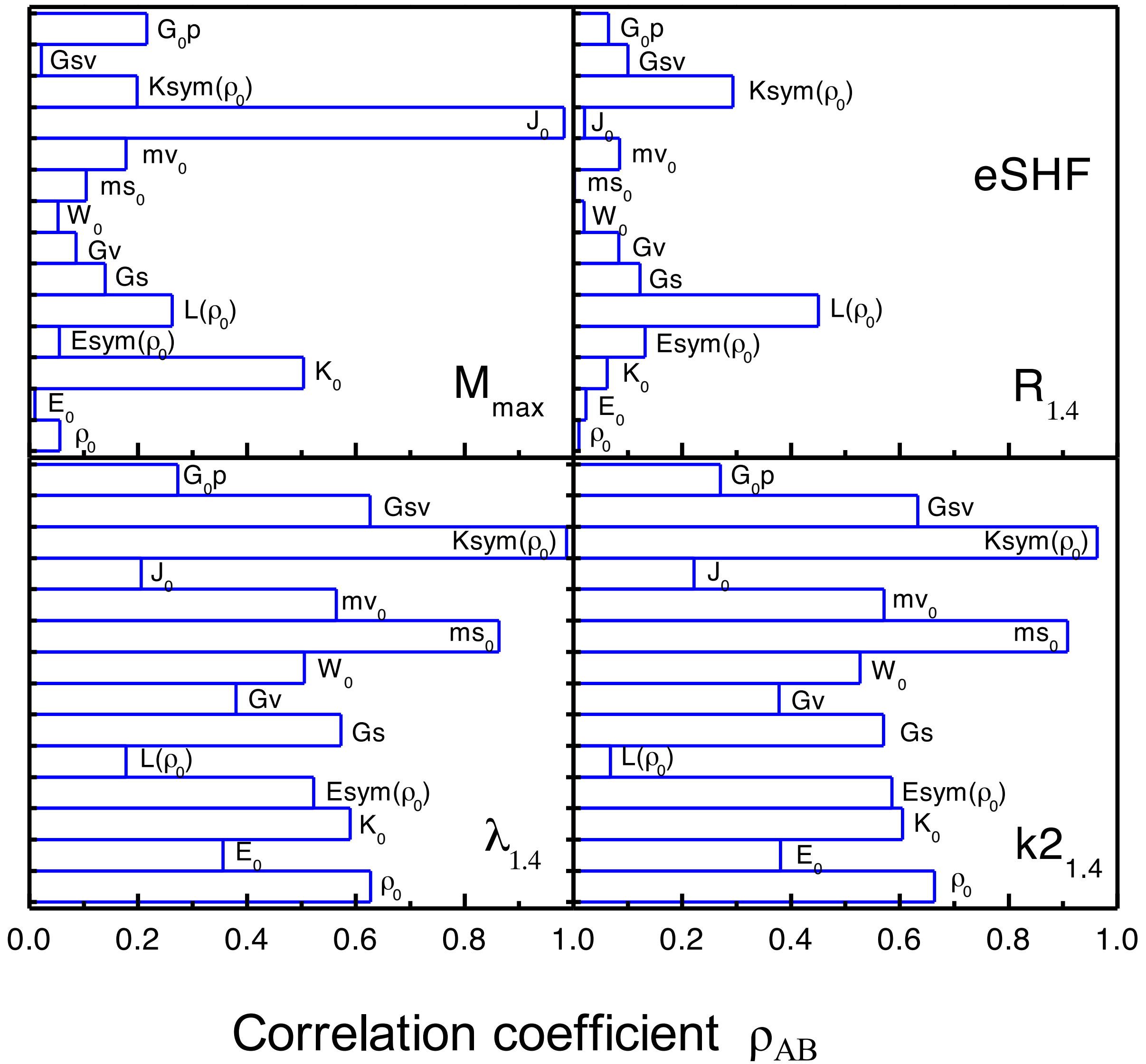
F. J. Fattoyev and J. Piekarewicz, Phys.Rev.C **84**, 064302 (2011)
Wei-Chia Chen and J. Piekarewicz, Phys.Rev.C **90**, 044305 (2014)

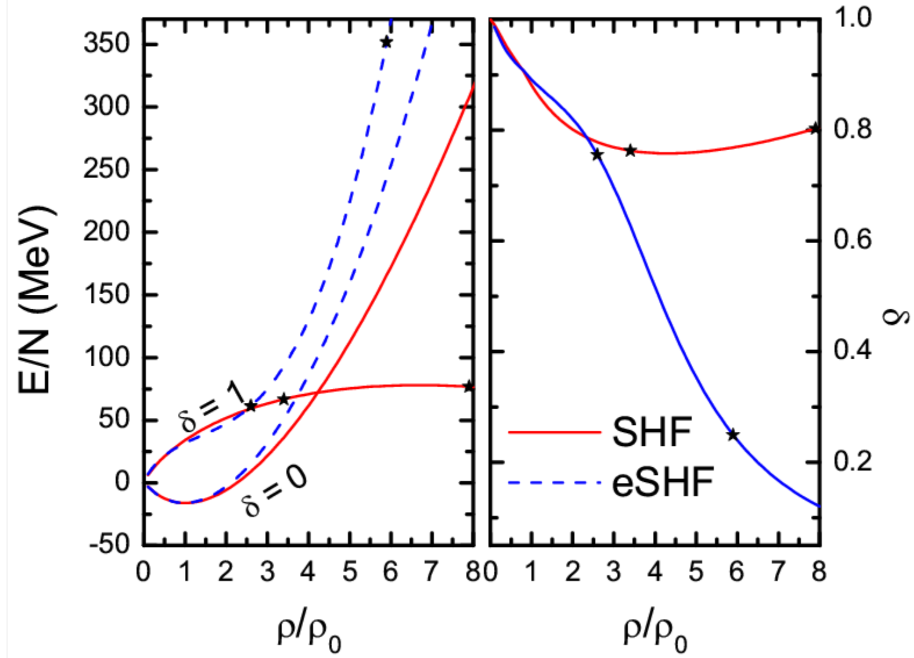
4. Results and discussions

- **SHF:**



- **eSHF:**


eSHF
M_{max}
$R_{1.4}$
$\lambda_{1.4}$
$k2_{1.4}$
G_0p
Gsv
$Ksym(\rho_0)$
J_0
mv_0
ms_0
W_0
Gv
Gs
$L(\rho_0)$
$Esym(\rho_0)$
K_0
E_0
ρ_0
0.0 0.2 0.4 0.6 0.8 1.0 0.2 0.4 0.6 0.8 1.0
Correlation coefficient ρ_{AB}



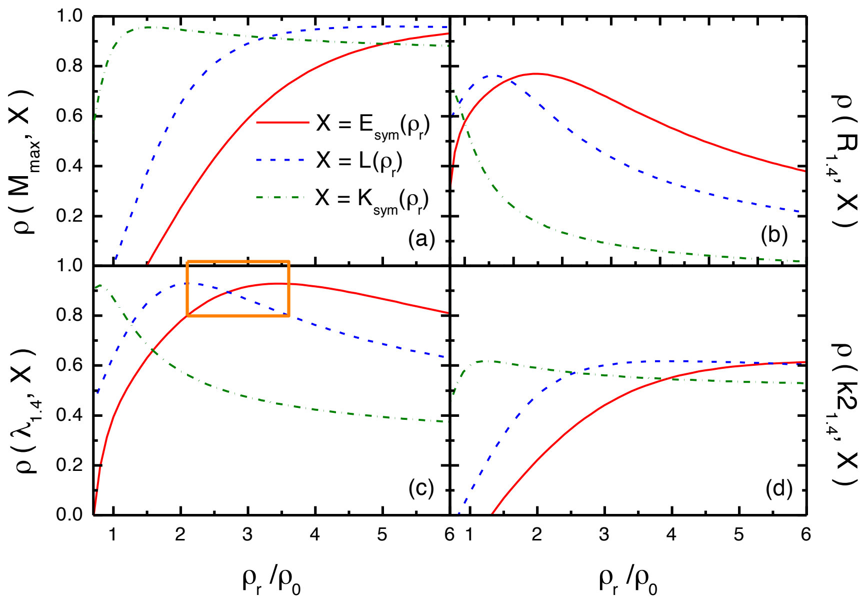
$$E(\rho,\delta) = E_0(\rho) + E_{sym}(\rho)\delta^2 + O(\delta^4)$$

$$E_0(\rho) = E_0(\rho_0) + \frac{1}{2!}K_0 x^2 + \frac{1}{3!}J_0 x^3 + O(x^4), \qquad x = (\rho - \rho_0)/3\rho_0$$

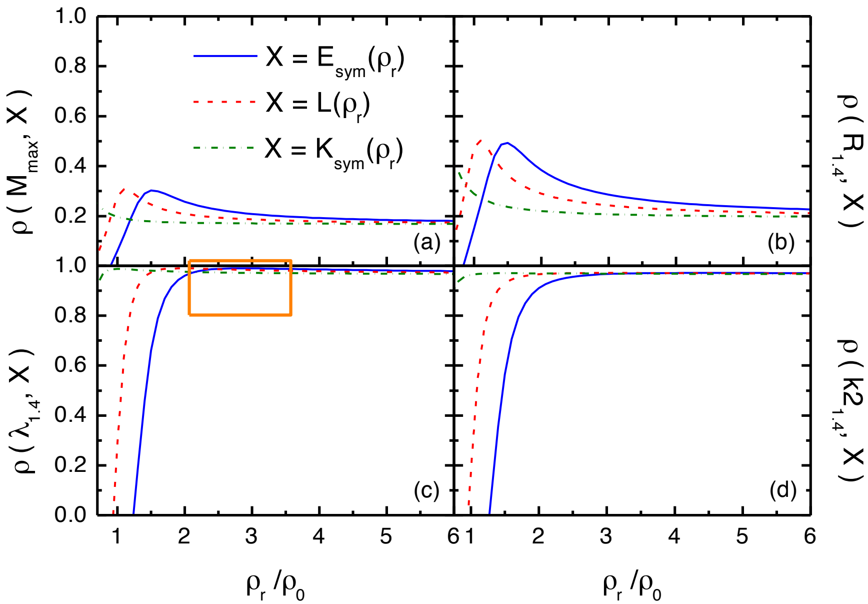
$$E_{sym}(\rho) = E_{sym}(\rho_r) + L(\rho_r)x_r + \frac{K_{sym}(\rho_r)}{2!}x_r^2 + O(x_r^3), \qquad x_r = (\rho - \rho_r)/3\rho_r$$

- **Correlation coefficients as function of ρ_r :**

SHF:



eSHF:



5. Conclusion

- At the saturation density, the $\lambda_{1.4}$ has strongest correlation with $K_{sym}(\rho_0)$.
- In supra-saturation density region ($2.1\rho_0 < \rho_r < 3.6\rho_0$) , the $\lambda_{1.4}$ is sensitive to $E_{sym}(\rho_r)$ and $L(\rho_r)$, it is $\rho_{AB} > 0.8$.

Thank you for your attention!