# Clustering Effects on Nuclear Symmetry Energy at Low Densities

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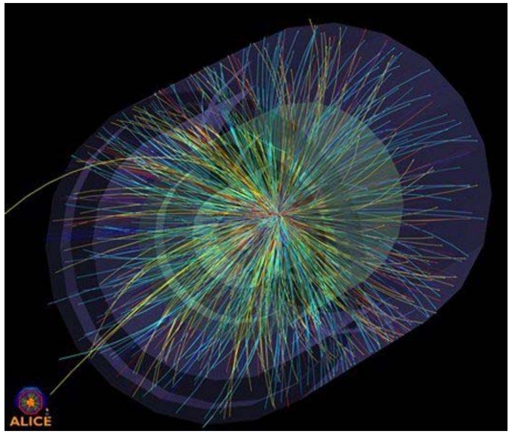
# OUTLINE

- Background & Motivation
- Nonlinear RMF model with clusters
- Binding energy of clusters
- Results and discussion
- Conclusion

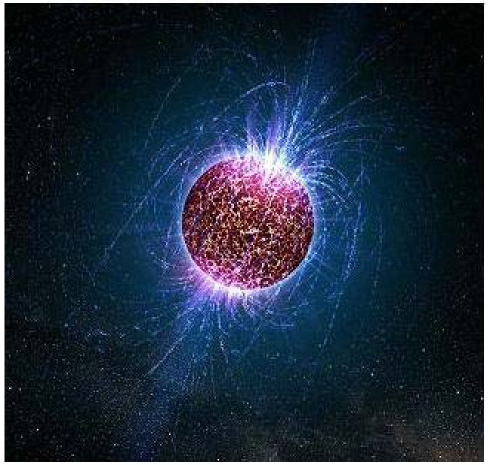
# Background

- Equation of state (EOS) of nuclear matter is very important in astrophysics and nuclear physics.

supernova explosion




heavy-ion collision



neutron star formation

baryon density

temperature

isospin asymmetry

# Background

- empirical parabolic law

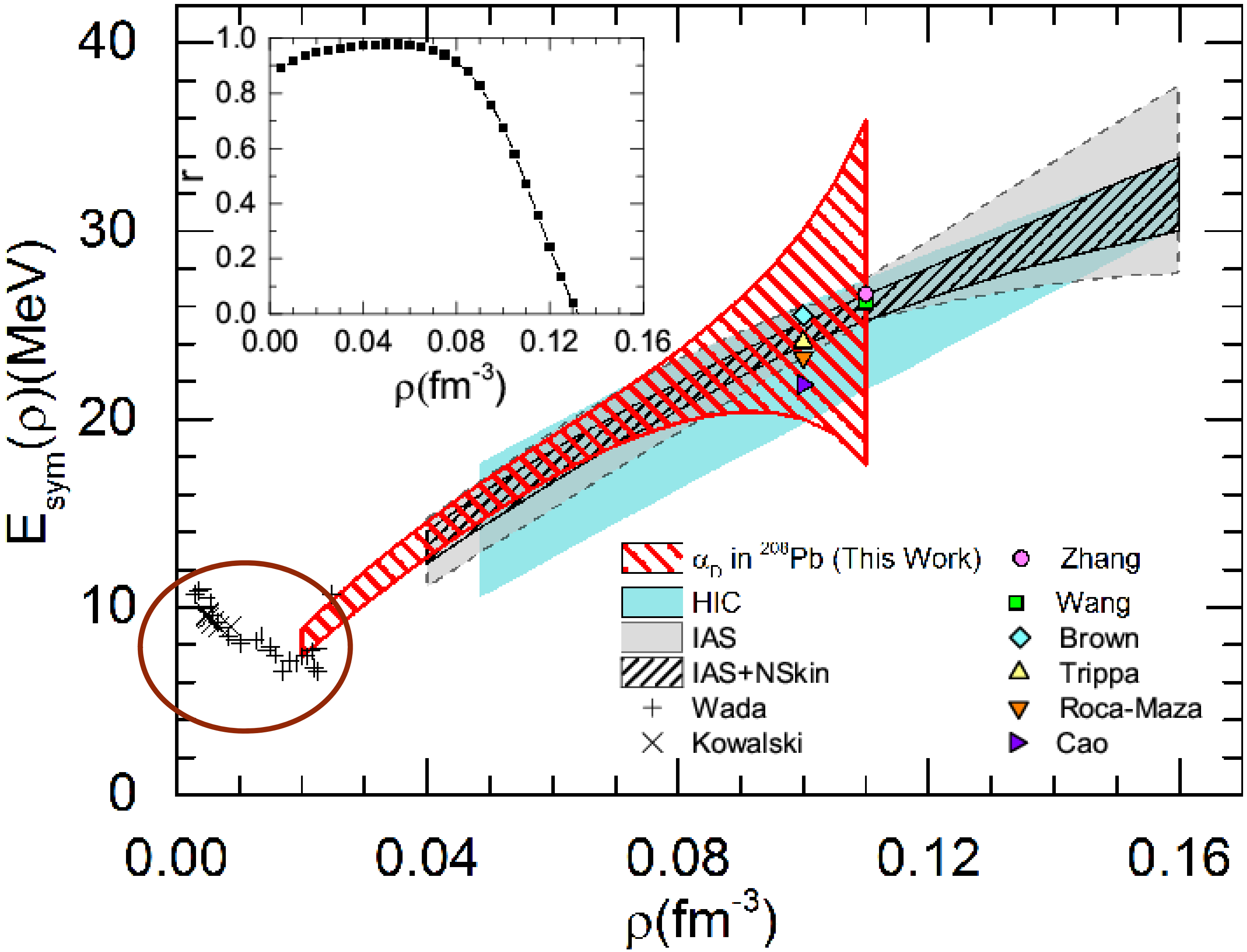
$$E_{\mathrm{int}}(n,\delta,T) = E_{\mathrm{int}}(n,0,T) + E_{\mathrm{sym}}(n,T)\delta^2 + O(\delta^4)$$

$$E_{\mathrm{sym}}(n,T) \approx \underline{E_{\mathrm{int}}(n,1,T)} - \underline{E_{\mathrm{int}}(n,0,T)}$$

pure neutron matter

symmetric nuclear matter

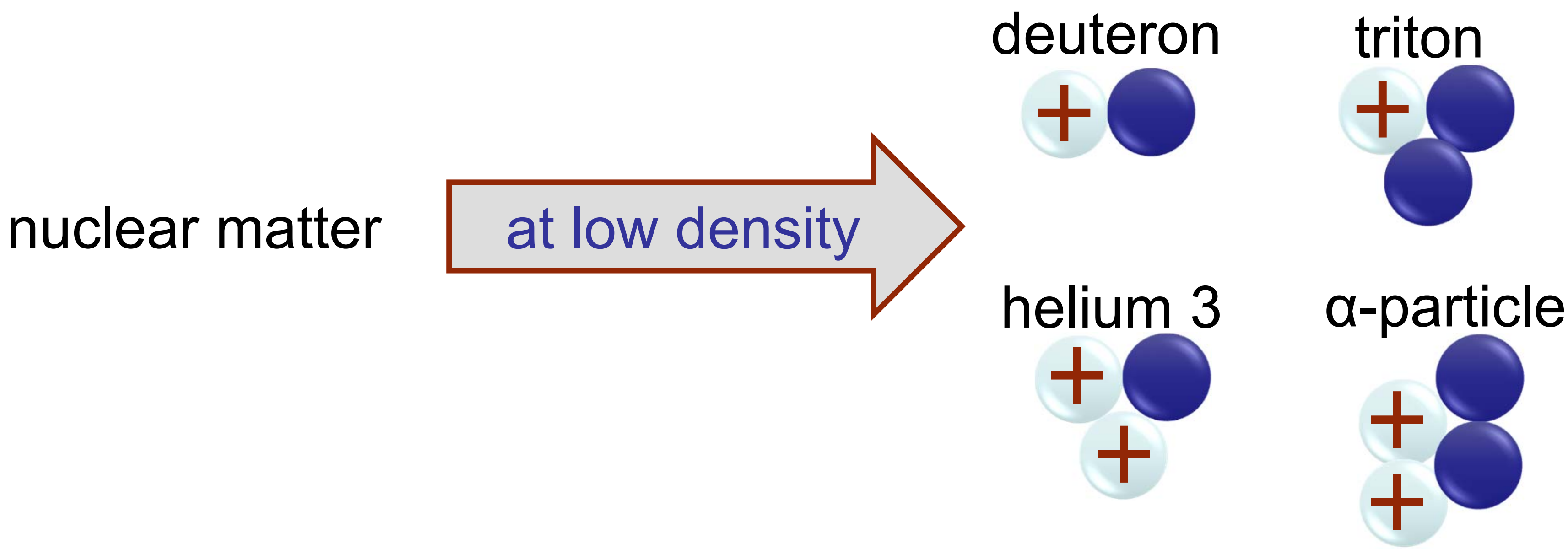
# Background




Z. Zhang and L. W. Chen, Phys. Rev. C **92**, 031301(R) (2015)

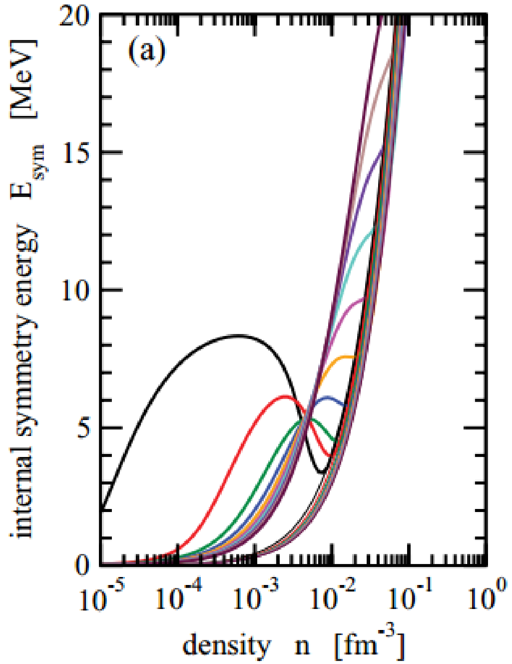
# Background

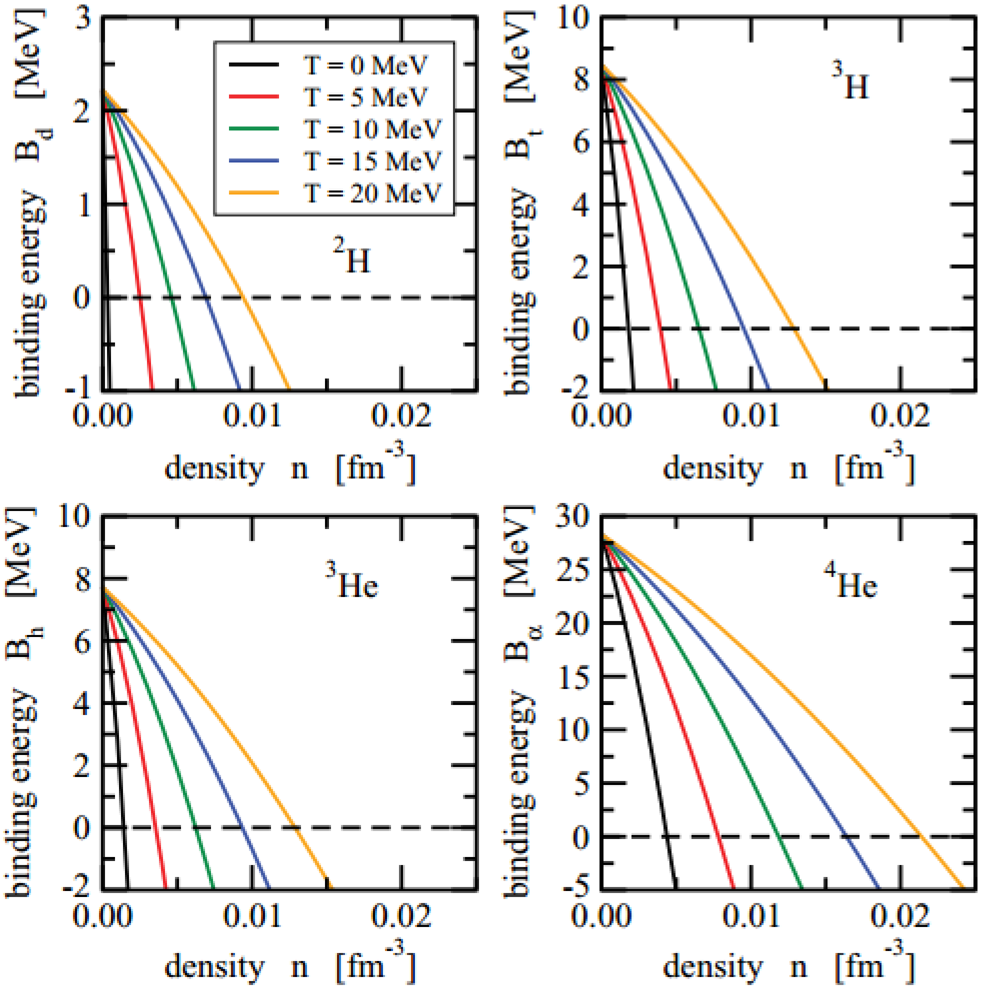
- clustering effect



# Background

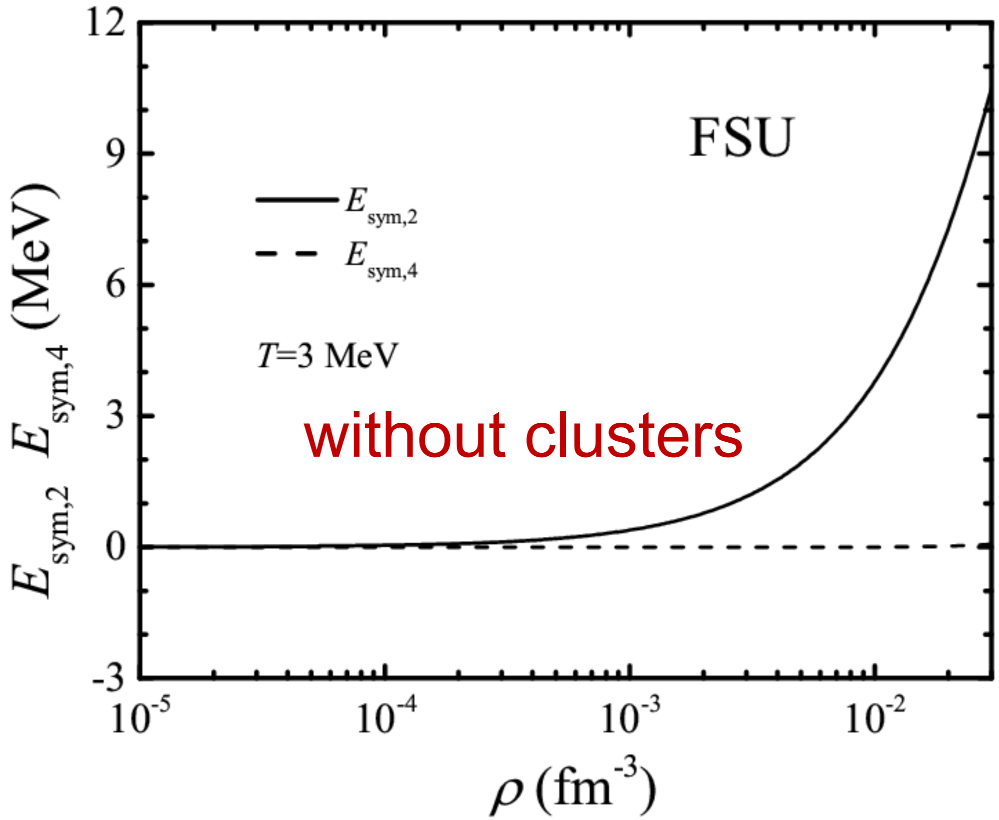
- generalized relativistic mean-field model with light clusters




S. Typel, G. Röpke, T. Klähn, D. Blaschke, and H. H.Wolter, Phys. Rev. C **81**, 015803 (2010)

# Motivation




- relativistic mean-field model with nonlinear meson terms considering the clustering effect

# Nonlinear RMF with clusters

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$$\mathcal{L} = \sum_{i=p,n,t,h} \mathcal{L}_i + \mathcal{L}_\alpha + \mathcal{L}_d + \mathcal{L}_{\text{meson}}$$

- fermions

$$\mathcal{L}_i = \overline{\Psi}_i \left( \gamma_\mu i D_i^\mu - M_i^* \right) \Psi_i$$

- bosons

$$\mathcal{L}_\alpha = \frac{1}{2} \left( i D_\alpha^\mu \phi_\alpha \right)^* \left( i D_{\mu\alpha} \phi_\alpha \right) - \frac{1}{2} \phi_\alpha^* \left( M_\alpha^* \right)^2 \phi_\alpha$$

$$\mathcal{L}_d = \frac{1}{4} \left( i D_d^\mu \phi_d^\upsilon - i D_d^\upsilon \phi_d^\mu \right)^* \left( i D_{d\mu} \phi_{d\mu} - i D_{d\upsilon} \phi_{d\upsilon} \right) - \frac{1}{2} \phi_d^{\mu *} \left( M_d^* \right)^2 \phi_{d\mu}$$

- mesons

$$\mathcal{L}_\sigma = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4$$

$$\mathcal{L}_\omega = -\frac{1}{4} W_{\mu\upsilon} W^{\mu\upsilon} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} c_3 \left( \omega_\mu \omega^\mu \right)^2$$

$$\mathcal{L}_\rho = -\frac{1}{4} \vec{R}_{\mu\upsilon} \cdot \vec{R}^{\mu\upsilon} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^{\,\mu}$$

$$\mathcal{L}_{\omega\rho} = \Lambda_v \left( g_\omega^2 \omega_\mu \omega^\mu \right) \left( g_\rho^2 \vec{\rho}_\mu \cdot \vec{\rho}^{\,\mu} \right)$$

$$i D_i^\mu = i \partial^\mu - g_\omega^i \omega^\mu - \frac{g_\rho^i}{2} \vec{\tau} \cdot \vec{\rho}^{\,\mu}$$

$$M_i^* = M_i - g_\sigma^i \sigma, \qquad i = p, n, \alpha, d, t, h$$

$$g_\sigma^i = A_i g_\sigma, \qquad g_\omega^i = A_i g_\omega, \qquad g_\rho^i = \left( N_i - Z_i \right) g_\rho$$

# Binding energy of clusters:

- binding energy

$$M_i = A_i m - B_i^0 - \Delta B_i$$

- dependence of density

$$\Delta B_i\left(n,T\right) = -\tilde{n}_i\left(1+\frac{\tilde{n}_i}{2\tilde{n}_i^0}\right)\delta B_i\left(T\right), \qquad \tilde{n}_i = \frac{2}{A_i}\left[Z_i n_p^{tot} + N_i n_n^{tot}\right]$$

- dependence of temperature

$$\tilde{n}_i^0\left(T\right) = \frac{B_i^0}{\delta B_i\left(T\right)}$$

# Binding energy of clusters:

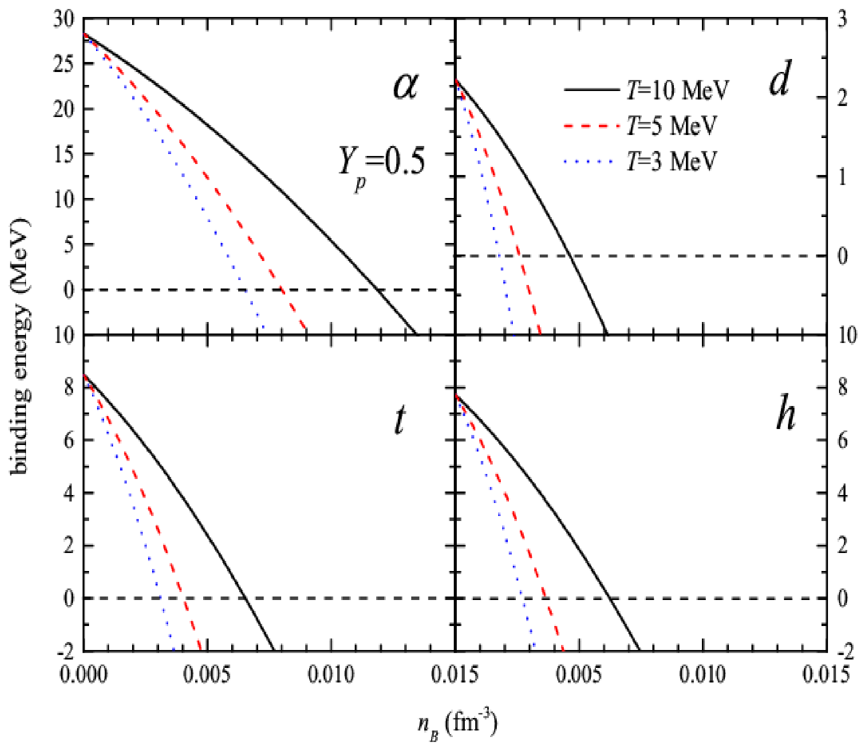
- Jastrow approach

$$\delta B_d(T)=\frac{a_{d,1}}{T^{3/2}}\left[\frac{1}{\sqrt{y_d}}-\sqrt{\pi}a_{d,3}\exp\left(a_{d,3}^2 y_d\right)\mathrm{erfc}\left(a_{d,3}\sqrt{y_d}\right)\right],\qquad y_d=1+a_{d,2}/T$$

- Gaussian approach

$$\delta B_i(T)=\frac{a_{i,1}}{\left(T+a_{i,2}\right)^{3/2}},\qquad i=\alpha,t,h$$

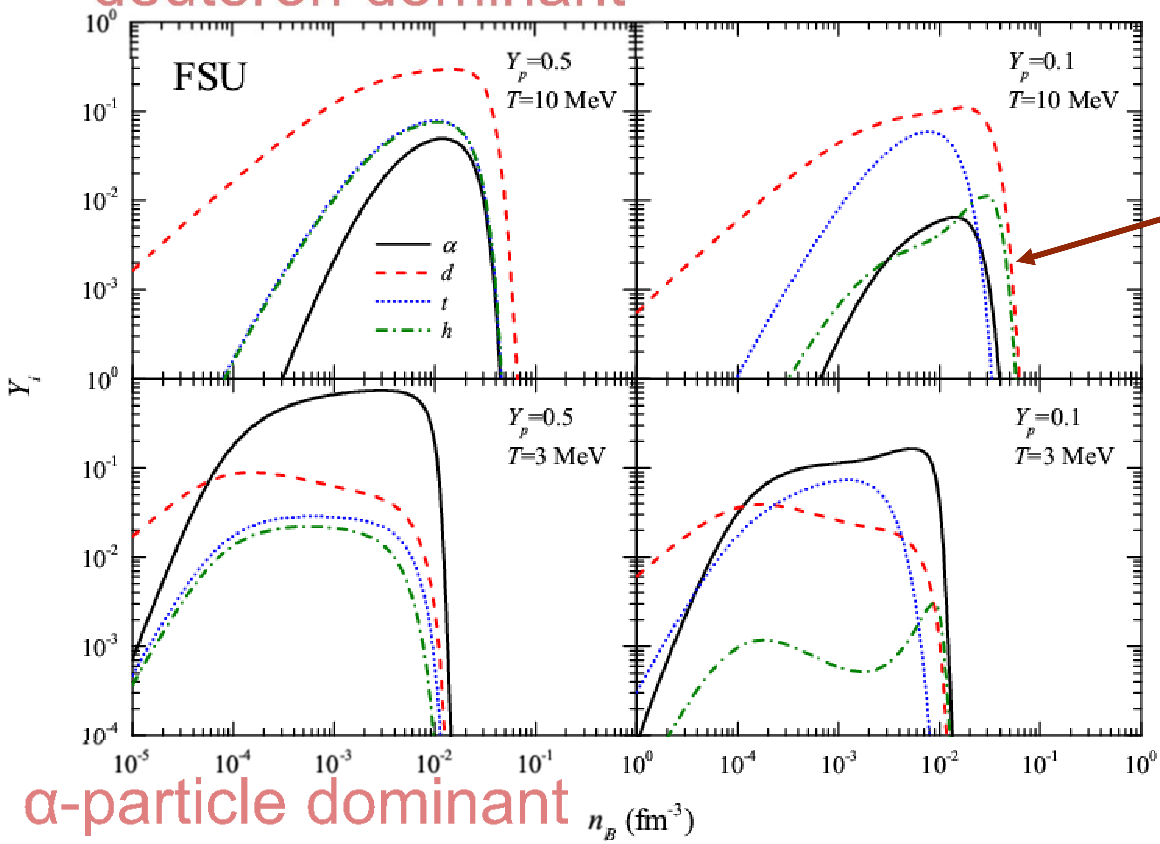
| Cluster $i$ | $a_{i,1}$ ($\mathrm{MeV}^{5/2}$ $\mathrm{fm}^3$) | $a_{i,2}$ (MeV) | $a_{i,3}$ (MeV) | $B_i^0$ (MeV) |
|---|---|---|---|---|
| $\alpha$ | 164371 | 10.6701 | - | 28.29566 |
| $d$ | 38386.4 | 22.5204 | 0.2223 | 2.224566 |
| $t$ | 69516.2 | 7.49232 | - | 8.481798 |
| $h$ | 58442.5 | 6.07718 | - | 7.718043 |



# Fraction of clusters

- deuteron is the dominant at higher temperature
- *α*-particle becomes the most dominant at low temperature

deuteron dominant



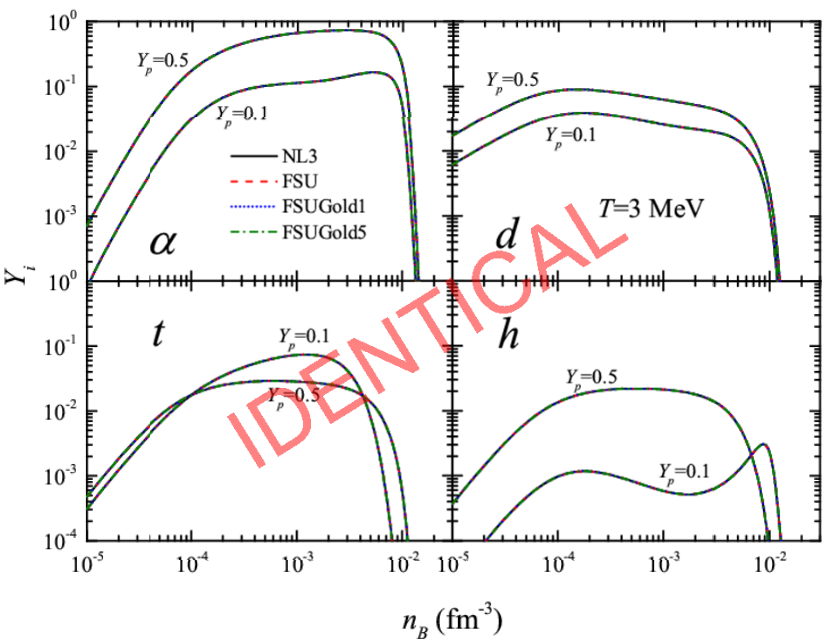

α-particle dominant

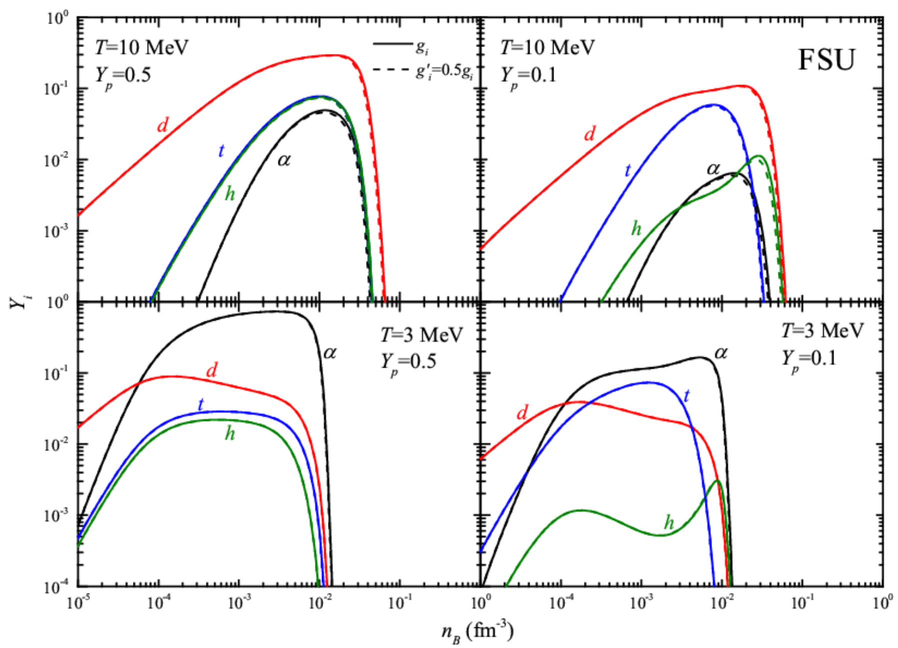
binding energies of triton and helium 3 are isospin-dependent

$$\tilde{n}_i = \frac{2}{A_i}\left[Z_i n_p^{tot} + N_i n_n^{tot}\right]$$

# Interaction-dependence of fraction

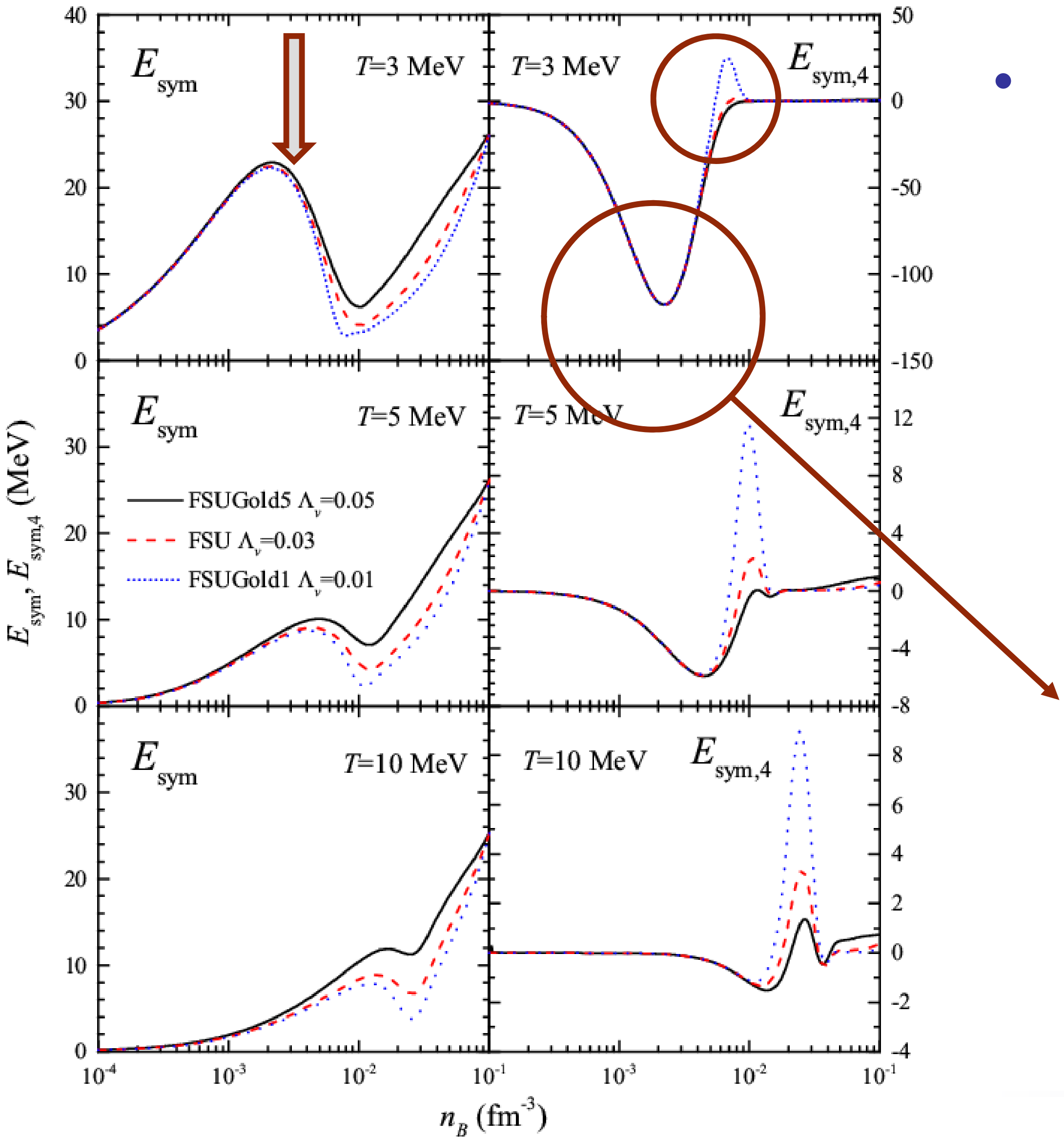
- Choosing NL3, FSU, FSUGold1 and FSUGold5 to investigate the interaction-dependence of fraction.
- Reducing the nucleon-meson couplings by half.




- Fractions of clusters are interaction-dependent.

# Interaction-dependence of $E_{sym}$




- Checking the interaction-dependence of symmetry energy.

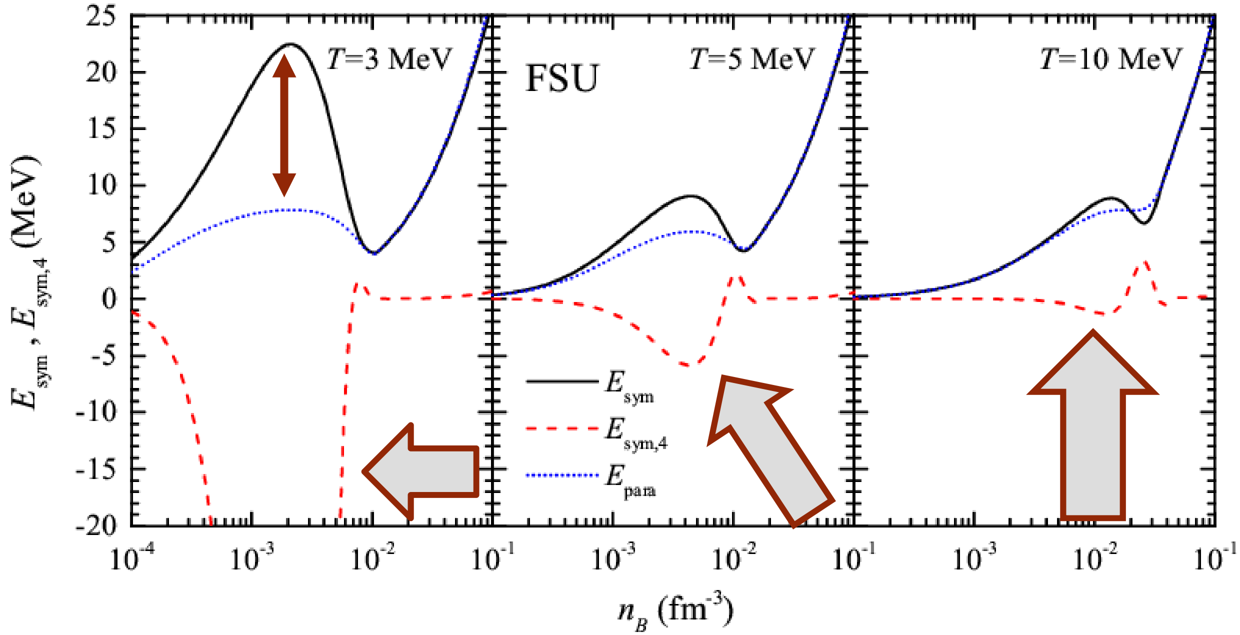
break the empirical parabolic law

# Symmetry energy

$$E_{\text{sym}}(n,T)=\frac{1}{2}\frac{\partial^2 E_{\text{int}}}{\partial\delta^2}\bigg|_{\delta=0}$$

$$E_{sym,4}(n,T)=\frac{1}{24}\frac{\partial^4 E_{\text{int}}}{\partial\delta^4}\bigg|_{\delta=0}$$

$$E_{\text{para}}(n,T)=E_{\text{int}}(n,T)\big|_{\delta=1}-E_{\text{int}}(n,T)\big|_{\delta=0}$$



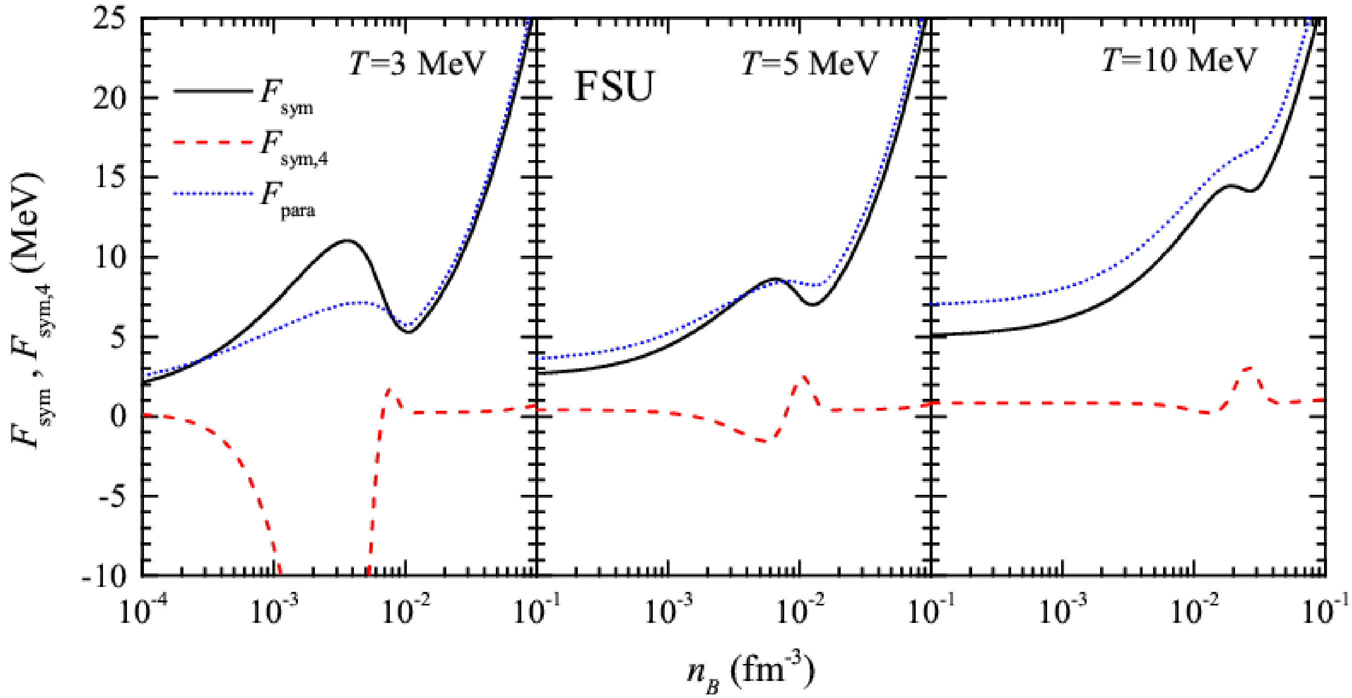

- The expansion of binding energy in isospin asymmetry is not convergent.

# Symmetry free energy

$$F_{\text{sym}}(n,T)=\frac{1}{2}\frac{\partial^2 F}{\partial\delta^2}\bigg|_{\delta=0}$$

$$F_{sym,4}(n,T)=\frac{1}{24}\frac{\partial^4 F}{\partial\delta^4}\bigg|_{\delta=0}$$

$$F_{\text{para}}(n,T)=F(n,T)\big|_{\delta=1}-F(n,T)\big|_{\delta=0}$$



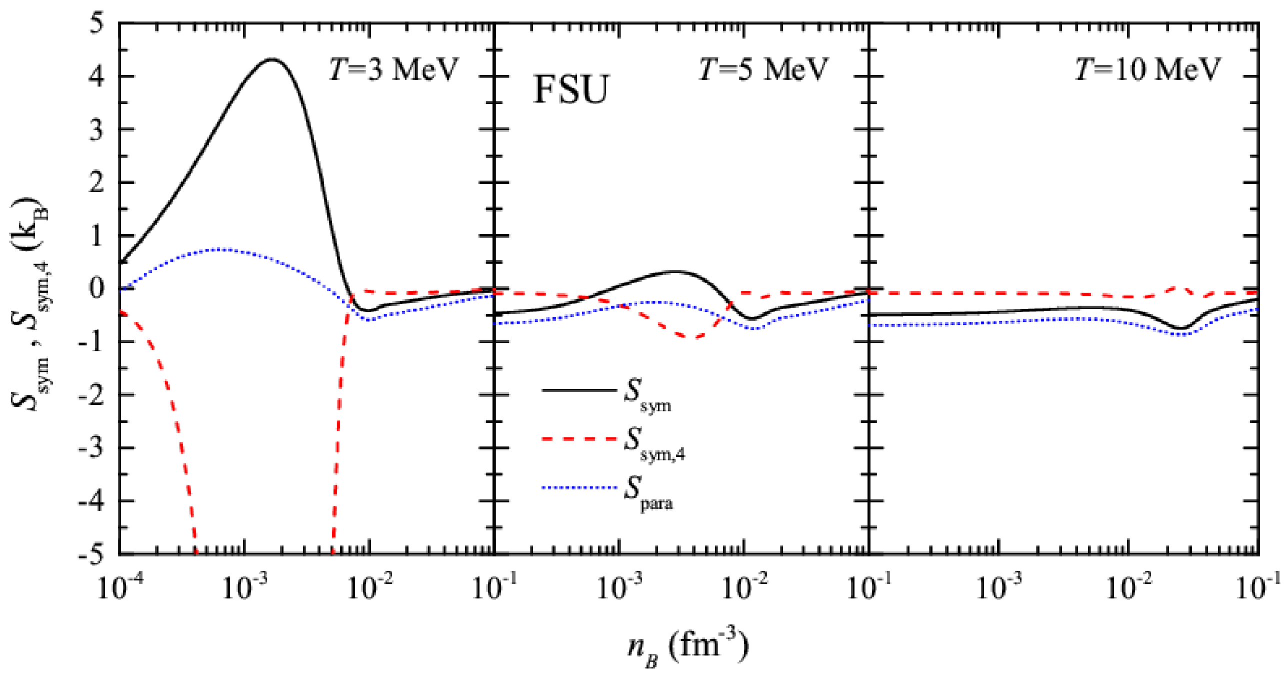

- The peak and valley is smaller than symmetry energy.

# Symmetry entropy

$$S_{\mathrm{sym}}(n,T) = \frac{1}{2}\frac{\partial^2 S}{\partial \delta^2}\bigg|_{\delta=0}$$

$$S_{sym,4}(n,T) = \frac{1}{24}\frac{\partial^4 S}{\partial \delta^4}\bigg|_{\delta=0}$$

$$S_{\mathrm{para}}(n,T) = S(n,T)\big|_{\delta=1} - S(n,T)\big|_{\delta=0}$$




- The behavior is similar with symmetry energy.

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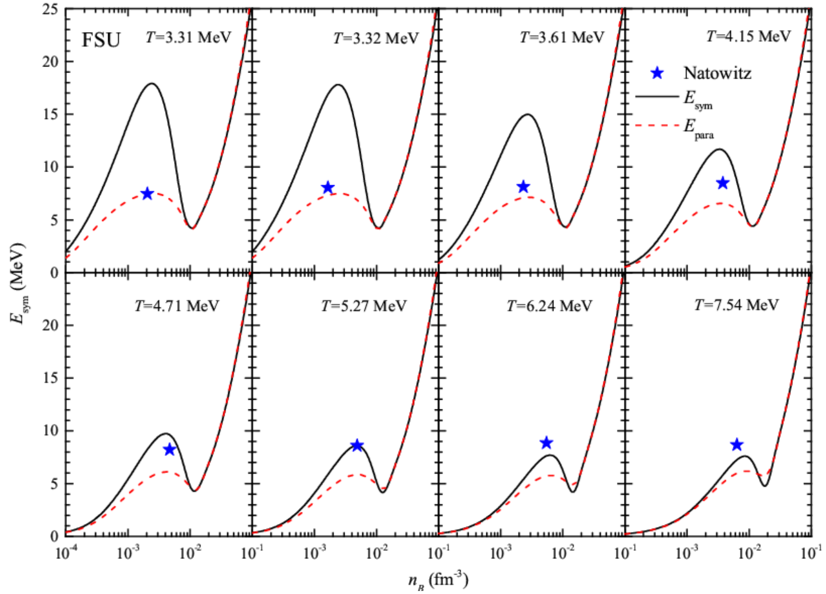
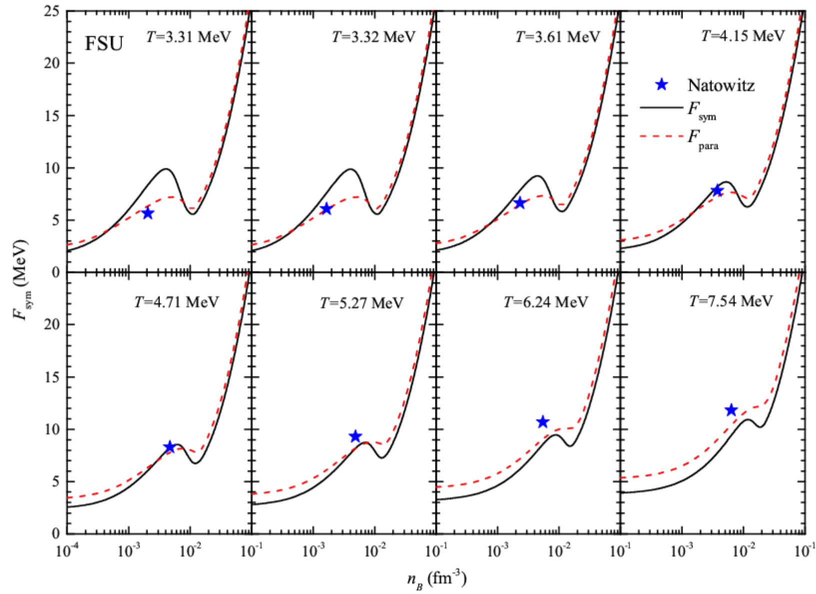
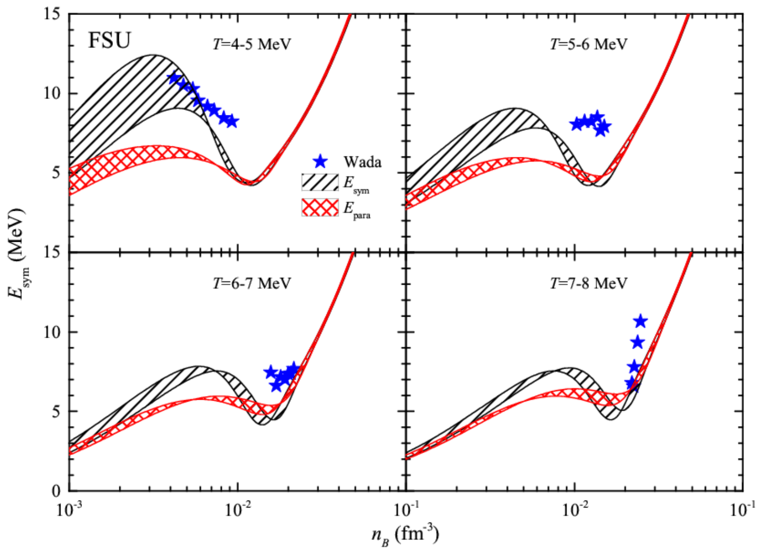
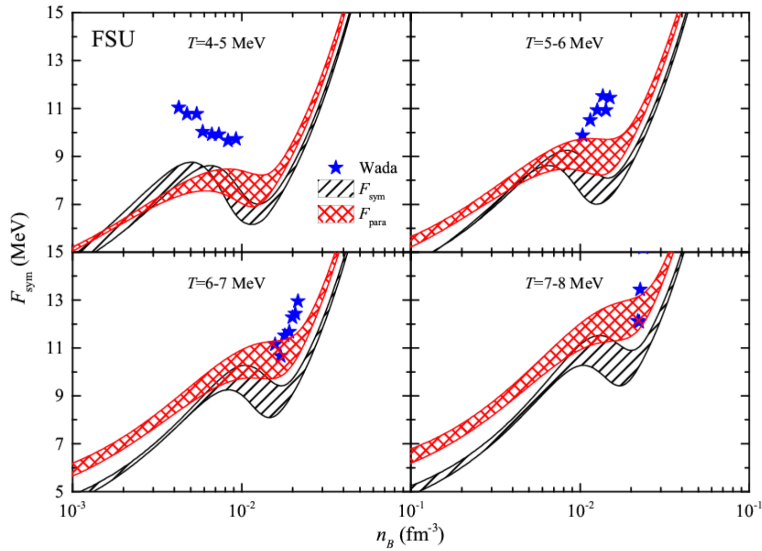
# Comparison with data



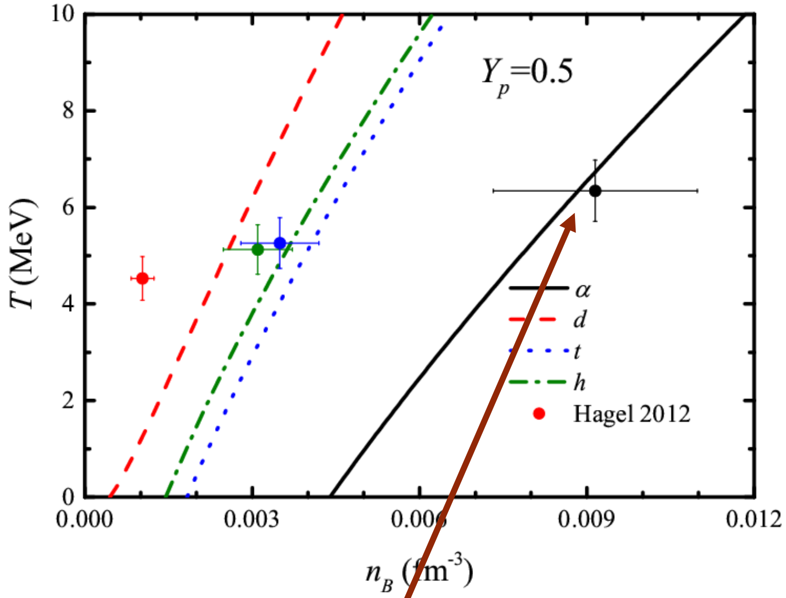



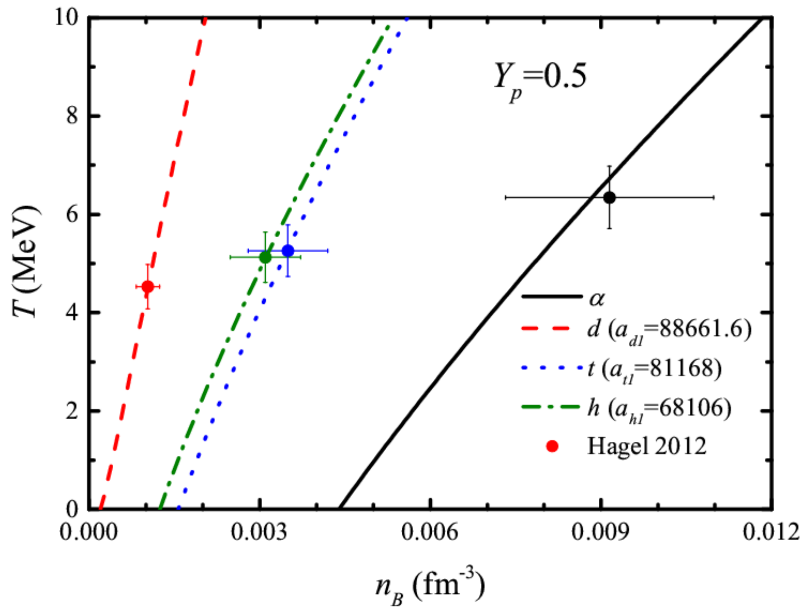




J.B. Natowitz, *et al*. Phys. Rev. Lett., **104**, 202501 (2010),
R. Wada *et al*., Phys. Rev. C **85**, 064618 (2012)

# Mott density




$\alpha$-particle gives a well agreement to the data

| Cluster $i$ | $a_{i,1}$ (MeV$^{5/2}$ fm$^3$) | $a_{i,2}$ (MeV) | $a_{i,3}$ (MeV) | $B_i^0$ (MeV) |
|---|---|---|---|---|
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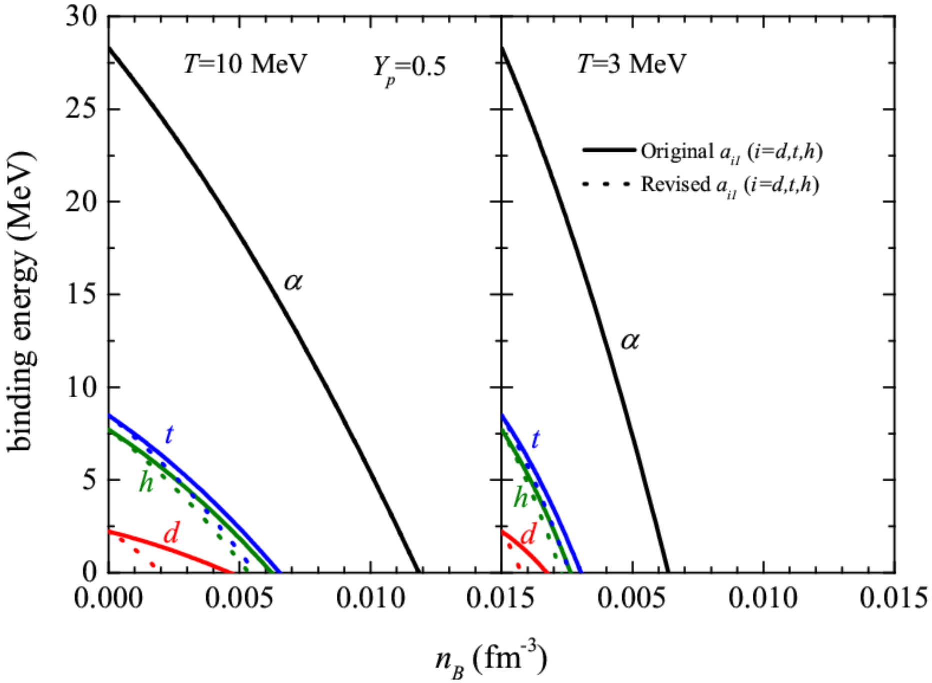
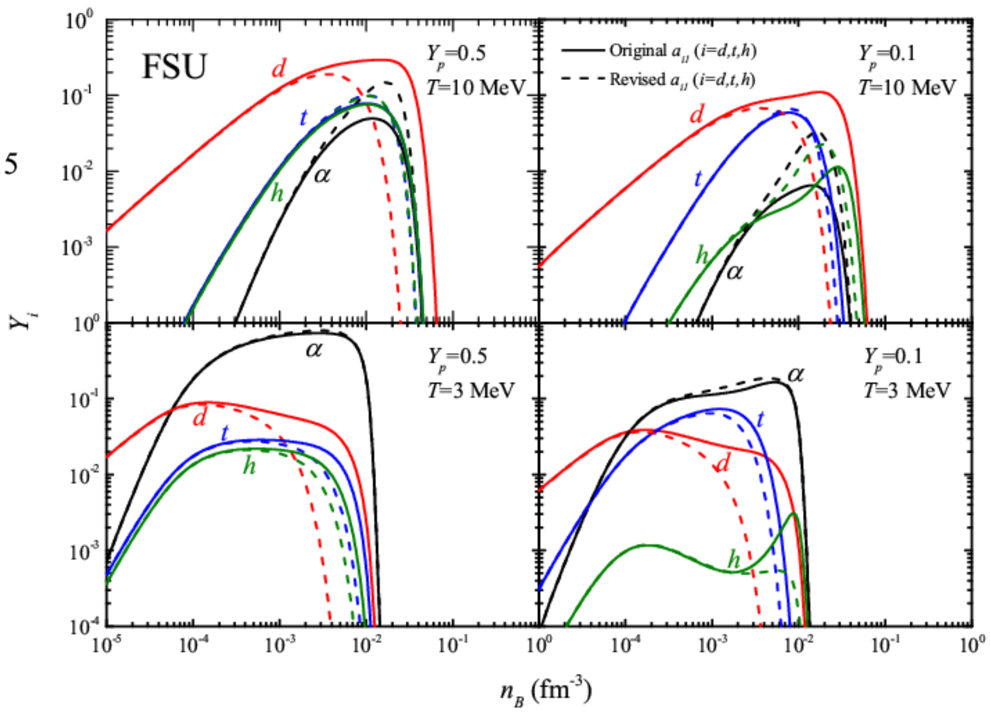
K. Hagel et al., Phys. Rev. Lett. 108, 062702 (2012)

# Revised binding energy and fraction

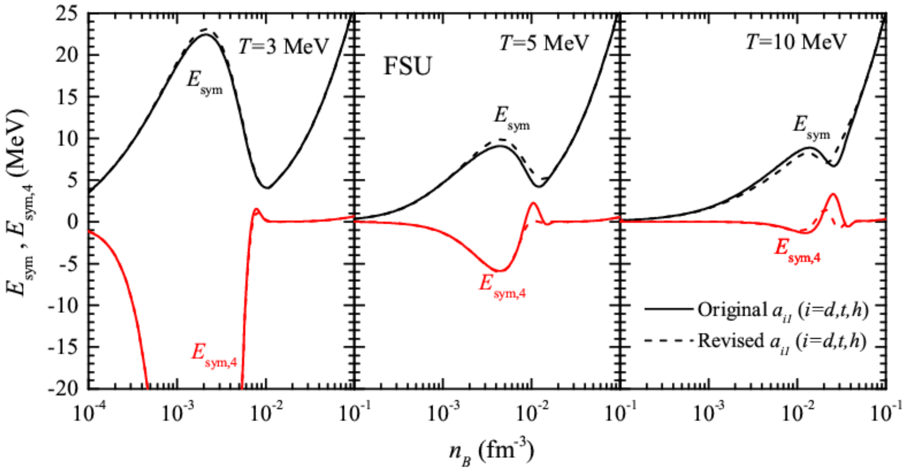


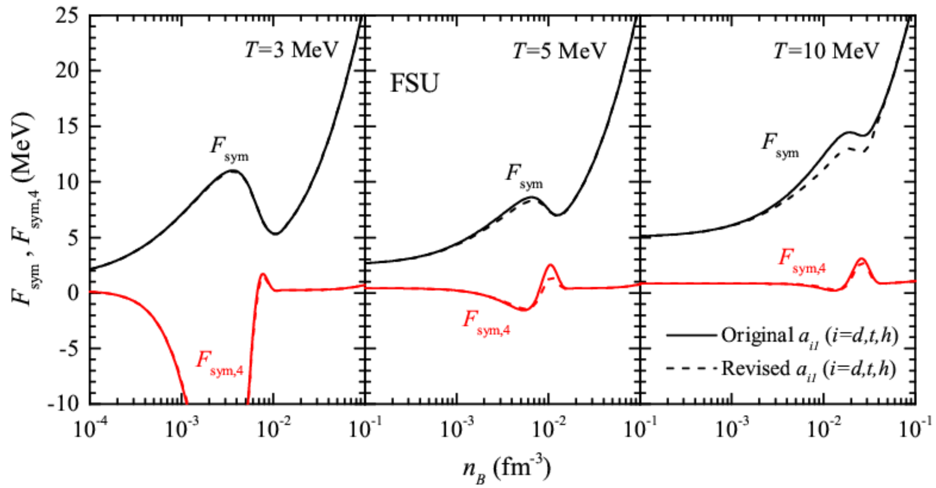

- The revised binding energies drop faster than the original one.

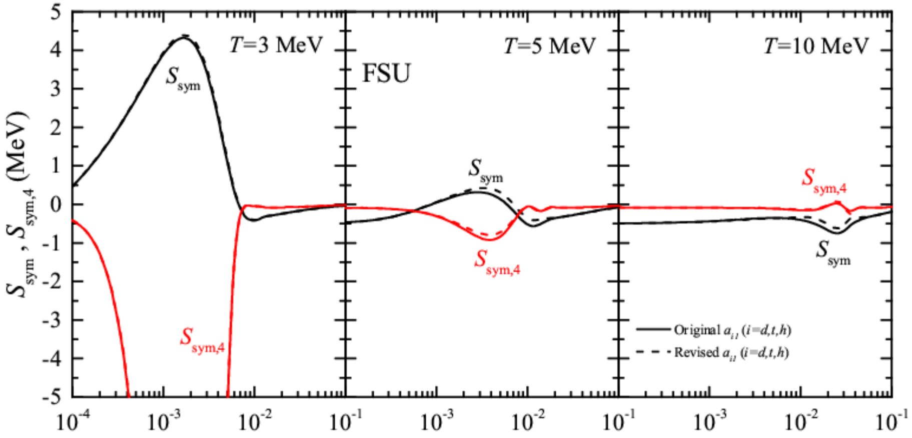
- deuteron dissolves earlier



# Revised $E_{sym}$ , $F_{sym}$ and $S_{sym}$

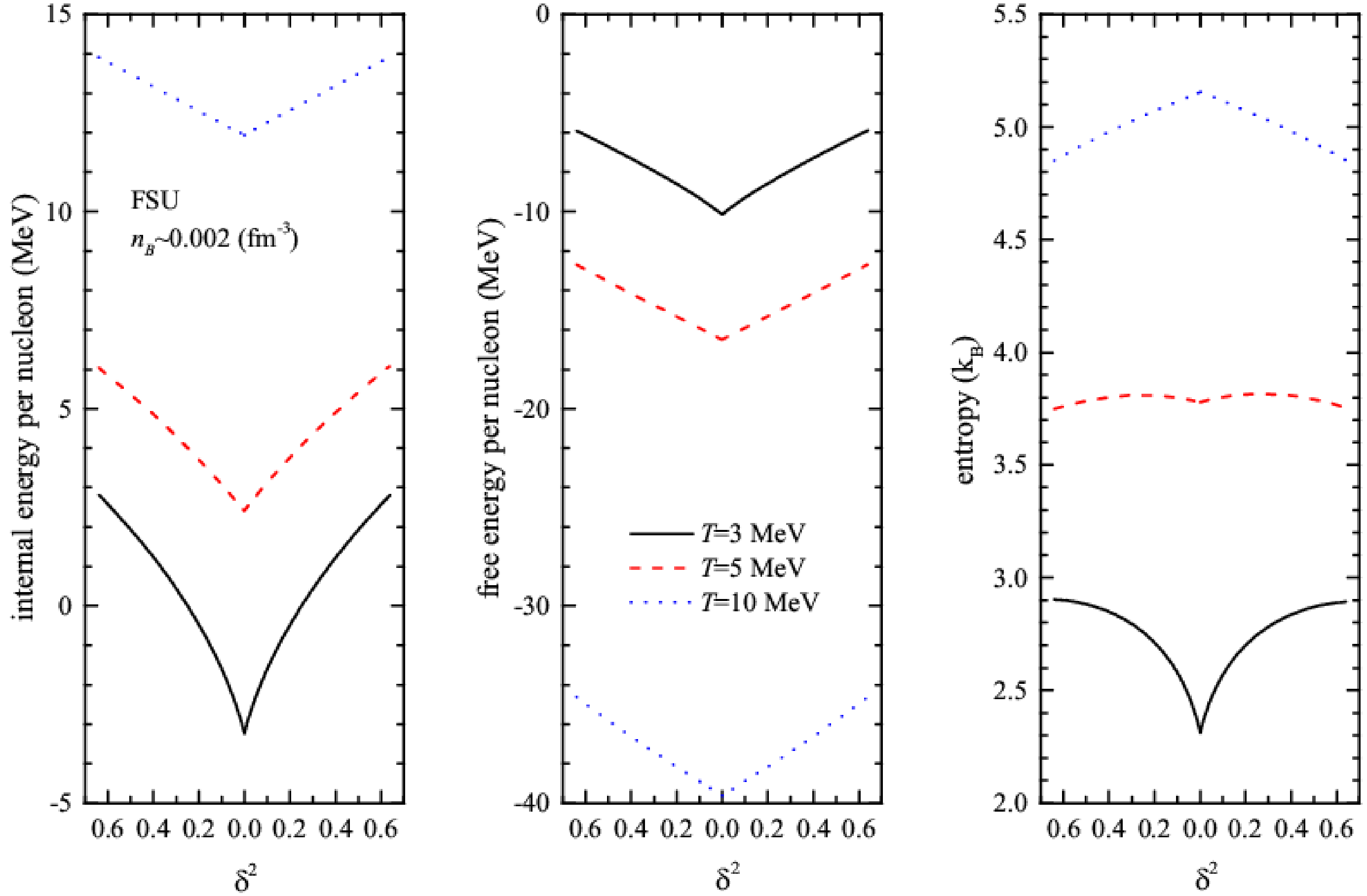



- differences can be neglected

# Parabolic approach




- parabolic approach is broken by clustering effect

# Conclusion

- We investigated the clustering effect on the symmetry energy by using the non-linear RMF model and employing the density-dependent binding energy.

- We found that the fraction of clusters is almost interaction-independent, but the $4^{th}$-order symmetry energy becomes abnormal when the value of $\Lambda_v$ is small.

- It is found that the 4th-order symmetry energy can not be neglected at low temperature.

- We also compared the numerical results to the experimental data of symmetry energy, the results are in good agreement with the data in generally.

- We revised the parameters of binding energy to fit the experimental Mott points, It is found that the revised binding energy makes a significant influence on the fraction of clusters, but not the symmetry energy.

# Thank you！